GREEN FUNCTION FOR BURIED FOUNDATION - SH WAVES INTERACTION

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SUMMARY

In this paper the response of an infinitly long, rigid, rectangular foundation, embedded in an elastic half space, to an incident SH wave with the δ-function acceleration profile is derived. The radiation and diffraction forces in the Laplace domain are obtained through the short time analysis, proposed earlier by the author, and corrected in the range of small values of the Laplace parameter s using the Born approximation. The known forces due to radiation and diffraction then yield the equation of motion, which is solved, and the results subsequently transformed back into the time domain.

INTRODUCTION

The dynamic response of a structural system may be significantly affected by the soil-structure interaction. The recognition of this fact led to intensive research of this phenomenon in the last twenty years. The problem of the surface mounted foundation was dealt with first. Due to the great complexity of the mathematical model involved, very few exact solutions were found. In most cases approximate solutions valid for a certain range of the parameters involved, e.g. low frequences, were obtained. An extensive review of the pertinent literature was given by Wong and Luco (Ref. 1). The problem of the embedded foundation was first investigated by Luco (Ref. 2) and Trifunac (Ref. 3). They employed an antiplane model consisting of a semicircular foundation embedded in an elastic half space and excited by a harmonic SH wave. Their results are exact for all frequences. Thau and Umek (Ref. 4,5) and Umek (Ref. 6) gave the response Green function of an infinitly long, rectangular foundation embedded in an elastic half space to incident plane waves. Their results are exact during the initial period of time needed for a shear wave to transverse the width of the foundation. The same model was later used by Dravinski and Thau (Ref. 7,8) and the period in which the results were exact was doubled. The other authors studying the embedded foundation problem used mostly numerical techniques of different kinds and are therefore not of special interest for this paper.

Our goal is to extend the time period in which the results obtained by Thau and Umek (Ref. 4) are valid to all times. To achieve this, what is believed to be a novel approach, not only in the dynamic soil-structure interaction but in the general scattering theory, is adopted.

DESCRIPTION OF THE PROBLEM

An infinitely long, rectangular foundation embedded in an elastic half space excited by an SH wave with the δ-function acceleration

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profile is considered. The width of the foundation is taken to be a unity and the embedment depth is arbitrary and denoted by H. The original interaction problem is subdivided into the radiation and the diffraction subproblem, as suggested by Thau (Ref. 9), in which the forces exerted on the foundation are sought. To reduce the complexity of the problems further the Laplace transform is introduced. The excellent work of Dravinski and Thau (Ref. 7) has clearly demonstrated that the exact expressions for forces in the Laplace domain for the geometry under consideration are very difficult if not impossible to obtain. Therefore we are going to derive the solutions for them, which are exact some for small and others for large values of parameter s. Combining them we will obtain a composite solution for which it is believed to be valid, at least approximately, for all s's i.e. for all times. The transformed forces, obtained this way, are then inserted into the equation of motion, which is solved. The inverse Laplace transform of the results yields the Green function for foundation - SH wave interaction.

METHOD OF SOLUTION

The crucial point of this method of solving the interaction problems is that two expressions for the forces exerted on the foundation, which are exact or at least valid for two different ranges of the parameter s, can be obtained. For our case the forces valid for large values of parameter s have already been derived by Thau and Umek (Ref. 4). Thus the radative force for a large s is:

$$\bar{F}_{R1}(s) = (2H + 1)s + 1.204 + O(\exp(-s)) \; ; \; 0.5 \leqq H \leqq 1 \tag{1}$$

The corresponding force due to diffraction then becomes:

$$\begin{aligned} \bar{F}_{D1}(s) = \bar{f}(s) \Big\{ & \tan\alpha + \cot\alpha + \tfrac{2}{3}\sec\tfrac{2}{3}(\alpha - \tfrac{\pi}{4}) \\ & + e^{-s\cos\alpha}\left(\tfrac{2}{3}\csc\tfrac{2}{3}\alpha - \tan\alpha - \cot\alpha\right) \\ & + e^{-2sH\sin\alpha}\left[\tfrac{2}{3}\sec\tfrac{2}{3}(\alpha + \tfrac{\pi}{4}) - \tan\alpha - \cot\alpha\right]\Big\} + O(\exp(-s)) \; ; \\ & 0 \leqq \alpha \leqq \tfrac{\pi}{2} \; ; \; \tfrac{1}{2} \leqq H \leqq 1 \; , \end{aligned} \tag{2}$$

where $\bar{f}(s)$ is the Laplace transform of the incident wave profile and α is the angle between the normal to the wave **front** of the incident wave and the surface of the half space.

Despite considerable effort we have not been successful in deriving analytical expresions for the forces in the range of the small values of Laplace parameter s with their domain of validity big enough to overlap or at least come close to the solutions given by equations (1) and (2). Therefore we have to settle for a numerical solution and the Born approximation was decided on. The advantages of this particular method are a reasonable numerical effort and the possibility of further improvement of the results until the desired numerical accuracy is reached. The starting point is the integral equation for a rigid inclusion in an elastic space given by Pao and Mow (Ref. 1o).

Denoting the Laplace transform of the only in the antiplane problem nontrivial displacement component by $\overline{w}$ we obtain for the diffraction subproblem the following equation:

$$\int_C G(r,r') \frac{\partial \overline{w}(r')}{\partial n'} dc' = \overline{w}^{(i)} \; ; \quad r,\ r' \text{ on } C \tag{3}$$

and for the radiation subproblem respectively:

$$\int_C G(r,r') \frac{\partial \overline{w}(r')}{\partial n'} dc' = \overline{W}(s)\Big[\int_C \frac{\partial G(r,r')}{\partial n'} dc' - 0.5 \Big] \; ; \quad r,\ r' \text{ on } C, \tag{4}$$

where $G(r,r') = (2\pi)^{-1} K_o (s|r - r'|)$ is the Green function of the antiplane full space problem, the curve C is the perimeter of the foundation and its miror picture with respect to the surface of the half space, n is the unit outer normal, $\overline{w}^{(i)}$ is the Laplace transform of the incident wave and $\overline{W}(s)$ is the Laplace transform of the foundation motion. The zero order Born approximation of the equations (3) and (4) is defined as:

$$\frac{\partial \overline{w}_o}{\partial n} = \Big(\int_C G(r,r') \, dc' \Big)^{-1} \overline{w}^{(i)}(r) \tag{5}$$

respectively

$$\frac{\partial \overline{w}_o}{\partial n} = \overline{W}(s) \Big(\int_C G(r,r') \, dc' \Big)^{-1} \Big(\int_C \frac{\partial G(r,r')}{\partial n'} dc' - 0.5 \Big) \tag{6}$$

Multiplying the strains on the left hand side of the equations (5) and (6) by the shear modul of the half space, the stresses along the foundation perimeter are obtained. Integrating them yields the forces exerted on the foundation in the diffraction and the radiation subproblems.

Once the zero order Born approximations are known, we can consecutively calculate the hiher order approximations. Assuming that n-th order Born approximation is known the (n+1)-th order approximation for the diffraction is given by:

$$\frac{\partial \overline{w}_{n+1}}{\partial n} = \Big(\int_C G(r,r') \, dc' \Big)^{-1} \Big[\overline{w}^{(i)}(r) - \int_C G(r,r') \frac{\partial \overline{w}_n}{\partial n'} dc' \Big] \tag{7}$$

and the radiation by:

$$\frac{\partial \overline{w}_{n+1}}{\partial n} = \Big(\int_C G(r,r') \, dc' \Big)^{-1} \Big\{ \overline{W}(s) \Big[\int_C \frac{G(r,r')}{n'} dc' - 0.5 \Big] - \int_C G(r,r') \frac{\partial \overline{w}_n}{\partial n'} dc' \Big\} \tag{8}$$

From consecutively higher orders of the Born approximation of the strains, the corresponding approximations for the radiation and diffraction forces are calculated. They form series, which are believed to converge to the exact solutions. The convergence of these series is greatly improved by the use of the Richardson extrapolation.

In the way just described the forces, exerted on the foundation in the radiation and diffraction subproblems are determined for as many discrete values of s as desired. It is understood however that all these values of s lie in an interval where s is small and the Born approximation converges. So we have succeeded in obtaining the radiation and diffraction forces exerted on the foundation valid for two different ranges of parameter s, which was our goal set at the beginning of this paragraph. We now form the expressios for the forces valid for all s's as:

$$\bar{F}_R(s) = \bar{F}_{R1}(s) + \bar{E}_R(s) \tag{9}$$

and

$$\bar{F}_D(s) = \bar{F}_{D1}(s) + \bar{f}(s)\,\bar{E}_D(s) \tag{10}$$

Where $\bar{F}_{R1}(s)$ and $\bar{F}_{D1}(s)$ are the forces defined by equations (1) and (2) respectively and $\bar{E}_R(s)$ and $\bar{E}_D(s)$ are the correction terms to be determined from the results obtained from the Born approximation. For this purpose we write:

$$\bar{E}_R(s) = \sum_i A_i \exp(-\alpha_i s) \; ; \; \bar{E}_D(s) = \sum_i B_i \exp(-\beta_i s) \tag{11}$$

where A_i, B_i, α_i and β_i are the coefficients to be determined. This is done by fitting the equations (9) and (10) to the data resulting from the Born approximation in the least square sense. Since the correction terms should apply for the times $t \geqq 1$ only, the constraints that $\alpha_i \geqq 1$ and $\beta_i \geqq 1$ have to be taken into account. In doing so the inverses of the equations (9) and (10) are exact during the time interval $0 \leqq t \leqq 1$ and yield a valid expresion for all other times. Balancing the forces due to diffraction, radiation and inertia the equation of motion is formulated as:

$$(\, ms^2 + \bar{F}_{R1}(s) + \bar{E}_R(s)\,)\;\bar{W}(s) = \bar{F}_{D1}(s) + \bar{f}(s)\,\bar{E}_D(s) \tag{12}$$

where m is the total mass of the foundation. This equation is now solved and transformed back to the time domain. This yields:

$$W(t) = W_1(t) - G_R(t) * W(t) + f(t) * G_D(t) \tag{13}$$

The first term on the right hand side of the equation (13) is the short time response of the foundation exact for $0 \leqq t \leqq 1$ as obtained by Thau and Umek (Ref. 4). It is given by:

$$W_1(t) = L^{-1}\left[\overline{F}_{D1}\ (ms^2 + \overline{F}_{R1})^{-1}\right] \tag{14}$$

The second and the third term are convolution integrals where

$$G_R(t) = L^{-1}\left[\overline{E}_R(s)\ (ms^2 + \overline{F}_{R1})^{-1}\right] \tag{15}$$

and

$$G_D(t) = L^{-1}\left[\overline{E}_D(s)\ (ms^2 + \overline{F}_{R1})^{-1}\right] \tag{16}$$

Evaluating the second term one has to note that this convolution integral can be arranged in such a way that only displacements of the foundation for times less or equal to t-1 are required. Therefore evaluating the equation (13) for the increasing times, known functions only appear on its right hand side.

NUMERICAL RESULTS AND CONCLUSIONS

For the purpose of the numerical calculation two different embedment depths of H=1 and H=0.5 have been decided on. The angle between the direction in which the incident wave propagates and the surface of the half space has been taken to be $\alpha = 0.25\pi$. For these values first the equations (5) and (6) and later for the increasingly higher orders of the Born approximation the equations (7) and (8) have been evaluated. In calculating the pertinent integrals, the trapezoidal rule and a specially developed formula taking into account the singularities of the Green's function and its derivative have been employed. The values of the radiative and diffractive forces have been estimated using the Richardson extrapolation technique. Four terms have been taken in each of the series expansions for correction terms in equation (11) and a NAG-library routine to determine the unknown coefficients. The radiative force for H=0.5 is shown in Figure 1. The dash line indicates the Laplace transform of the force obtained by Thau and Umek (Ref. 4), the small circles the results obtained by the Born approximation and the solid line the force given by the equation (9). An incident wave with the δ-function acceleration profile has been chosen and the acceleration response of the foundation has been determined and plotted in Figure 2. Here again the dash line indicates the results obtained by Thau and Umek (Ref. 4) and the solid line the results of the present work.

On the basis of the example given the proposed method could be considered as successful. The long time limit of the velocity response for the foundation with the embedment depth H=0.5 e.g. is 0.98, and comes very close to the physically correct limit, which is 1.0. The computational effort is also adequate and without considerable difficulties. The standard mathematical library routins have been used throughout our work with the only exception of the evaluation of the integrals involving the Green's function and its derivative, where a special formula has been derived. It is also believed that the extension to other geometries is straightforward.

However before anything more precise could be said about the method, more examples will have to be studied and more expansions like

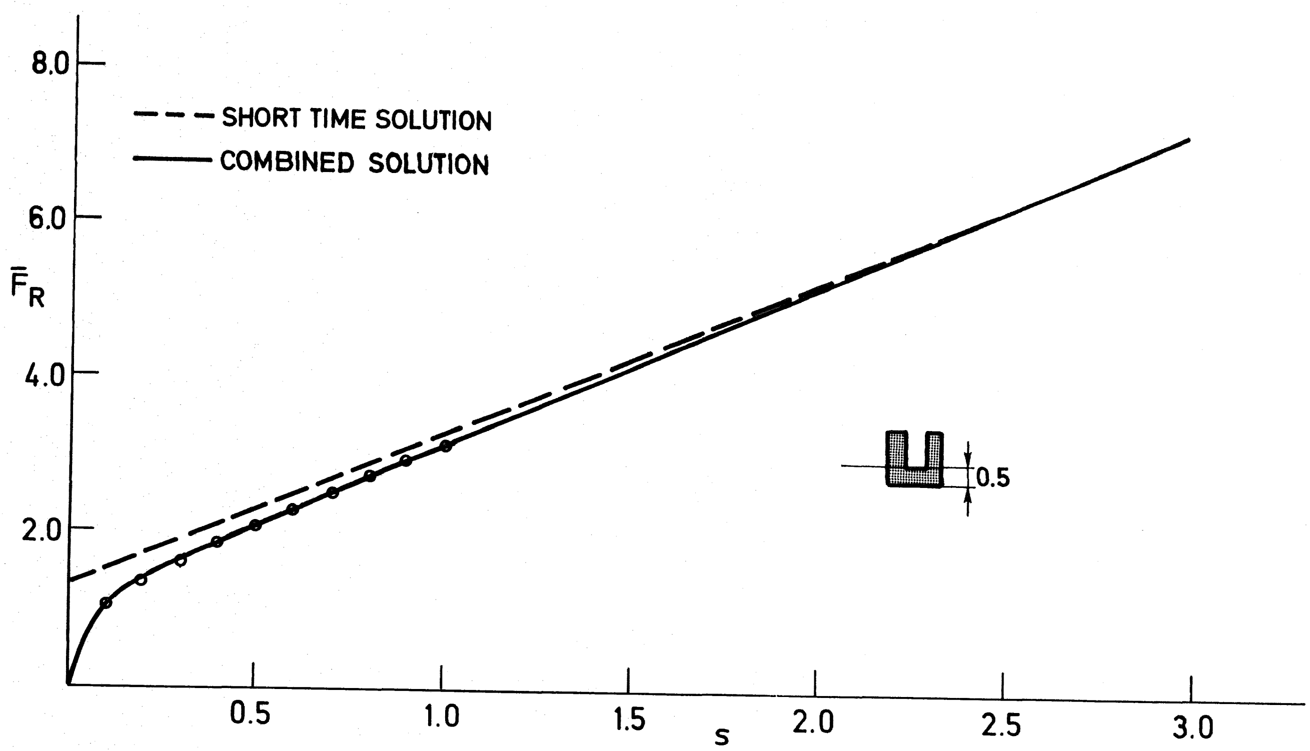

Figure 1. Laplace Transforms of Radiative Forces for Embedded Foundation; Embedment Depth = 0.5

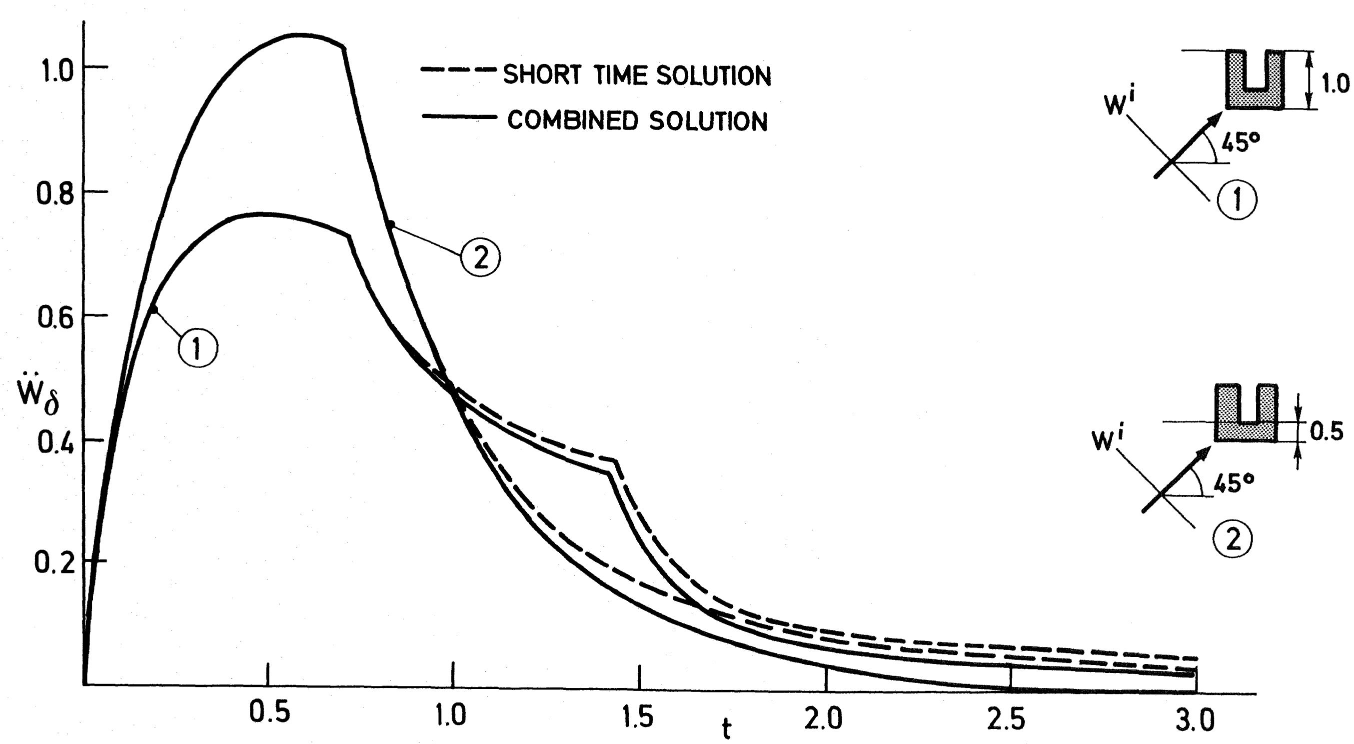

Figure 2. Acceleration Responses of Embedded Foundations for 45° Incident SH-Wave

those in equations (11) will have to be tried.

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