KINEMATIC INTERACTION OF RIGID CIRCULAR FOUNDATIONS ON LAYERED SOIL UNDER SURFACE WAVE EXCITATION

H. Werkle *

SUMMARY

The kinematic interaction of an embedded circular rigid foundation subjected to a plane wave field (i.e. P, SV, Rayleigh waves) is analysed. A semianalytic method for a horizontally layered, viscoelastic soil is applied. The results indicate that the controlling parameter for the structural response of surface foundations is the ratio of the apparent wave length of the incoming wave to the foundation diameter. For moderately embedded foundations under Rayleigh wave excitation, the embedment reduces the motion of the foundation.

INTRODUCTION

Most current analyses of soil-structure interaction are based on simplifying assumptions about the earthquake induced free-field motions; namely the assumption of vertically incoming seismic waves. In actual earthquakes obliquely incident P, SV and SH waves occur. In addition, horizontally propagating surface waves may contribute to the ground motion. It is therefore of interest to examine the response of a structure caused by a general wave field.

A basic approach to the kinematic interaction problem is simply to average the free field motion over the foundation area. The average translations and rotations are interpretated as the rigid body motion of a massless foundation (Ref. 5). The results, obtained by this procedure are indentical with those, obtained by modelling the soil by constant Winkler springs (Ref. 6). The method has been used to compute the kinematic interaction of rigid embedded and surface foundations for SH-waves.

In more detailed methods of analysis, the soil is considered as a continuum and the compatibility of stresses and displacements between soil and foundation is enforced. This approach includes the effect of the soil stiffness, damping, and mass. Analytical solutions for rectangular and circular foundations on an elastic halfspace were obtained for SH, SV, P and Rayleigh waves (Ref. 2, 3, 4, 8). Recently numerical methods, such as the finite element method, have been applied to the problem of soil-structure interaction with a wave field (Ref. 1, 9).

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* Hochtief AG, Abt. Kerntechnischer Ingenieurbau, Frankfurt a.M., Germany

In the present study a semi-analytic method which combines advantages of analytic and numerical methods is used /7/. The soil is modelled as a viscoelastic layered medium. Surface and embedded foundations are investigated under the action of wave fields in plane strain conditions, i.e. P, SV and Rayleigh waves.

SOIL MODEL AND NUMERICAL PROCEDURE

The soil is assumed to be a viscoelastic, horizontally layered continuum. It extends laterally to infinity and is bounded in the vertical direction by a rigid base at a finite depth. The determination of the soil stiffness is based on a semi-analytic solution for non-axisymmetric ring loads. A detailed derivation is presented in Ref. 7. A Fourier expansion describes the variation in the tangential direction. To describe the motion of a rigid foundation, only the terms $n = o$ (vertical) and $n = 1$ (horizontal and rocking) are needed. The displacement field is approximated by shape functions with a piecewise linear variation in vertical direction and by analytic solutions of the equations of motion to describe the variation in the horizontal direction. Observing homogeneous boundary conditions (zero stress at the surface and zero displacements at the base), yields an algebraic eigenvalue problem. The eigenmodes are generalized Rayleigh waves, and the eigenvalues are the corresponding wave numbers. These eigenmodes are identical with the Rayleigh waves propagating under plane strain conditions in a layered halfspace. The displacements, strains, and stresses of the axisymmetric system are expanded in terms of the eigensolutions. Application to the case of a ring load acting within or on the surface of the layered system yields an explicit solution for the displacement field.

The dynamic flexibility matrix of the nodal rings, at which the soil is to be connected to the structural model, contains the nodal displacements that are produced by unit loads applied at each of the nodal rings. Inversion of the flexibility matrix yields the complex dynamic stiffness matrix of the layered medium. As the ring load solutions are valid for a horizontally layered medium without an excavation for the structure, the dynamic stiffness of the excavation (modelled by axisymmetric volume elements) has to be subtracted.

The kinematic interaction of a rigid, massless foundation is analysed with the following equation, obtained by application of a substructuring principle:

$$\underline{T}^T \, \underline{K} \, \underline{T} \, \underline{\tilde{u}} = \underline{T}^T \, \underline{K}_f \, \underline{u}_f \qquad (1)$$

The vector $\tilde{\underline{u}}^T = \{\tilde{w}, \tilde{u}, \tilde{\varphi}\}$ contains the resulting rigid body motions of the foundation, the matrix $\underline{T}$ transforms $\tilde{\underline{u}}$ to a vector containing the nodal displacements at the soil-foundation interface for rigid body motion. $\underline{K}_f$ and $\underline{K}$ are the dynamic stiffness matrices of the soil with and without an excavation, respectively, and $\underline{u}_f$ contains the free field motion at the soil-foundation interface. As the free field motion generally is defined as a plane wave field, a transformation to cylindrical coordinates has to be performed. The solution of eq.(1) for the Fourier term n = o yields the vertical displacement, whereas n = 1 leads to the horizontal and rocking motion.

RESULTS

In this study a plane wave field is considered, which can be described, fig. 1, by

$$\begin{Bmatrix} u(x,z,t) \\ w(x,z,t) \end{Bmatrix} = \begin{Bmatrix} u_o(z) \\ w_o(z) \end{Bmatrix} \cdot e^{i\omega(t-\frac{x}{c})} \tag{2}$$

The kinematic interaction is described by a matrix which relates the free field displacements u_o, w_o at the soil surface to the resulting motions $\tilde{\underline{u}}$ of a massless rigid foundation:

$$\tilde{\underline{u}} = \begin{Bmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{\varphi} \end{Bmatrix} = \begin{bmatrix} S_{xx} & S_{xz} \\ S_{zx} & S_{zz} \\ R_{yx}/r & R_{yz}/r \end{bmatrix} \cdot \begin{Bmatrix} u_o \\ w_o \end{Bmatrix}\Bigg|_{z=o} \tag{3}$$

The elements of the matrix depend on the circular frequency ω and the apparent wave velocity c.

The soil model considered in this study consists of a circular foundation of radius r on an homogeneous viscoelastic layer with a depth of $h = 5 \cdot r$. The soil has a Poisson's ratio of 1/3, an internal damping of 5% and is vertically discretized in 25 layers. Fig. 3 shows the coefficients of the kinematic interaction matrix for a surface foundation versus the ratio $2r/\lambda$, where $2 \cdot r$ is the diameter of the foundation and $\lambda = 2 \cdot \pi \cdot c/\omega$ denotes the apparent wave length. The dimensionless frequency $a_o = \omega \cdot r/v_s$ (v_s = shear wave velocity) is given as a parameter. The results indicate that inertia and damping forces caused by the frequency a_o have no marked effect on the response. The important parameter is the ratio of the apparent wave length λ to the foundation diameter. The kinematic interaction coefficients of the model studied here differ only slightly from those of a square foundation on an elastic halfspace (Ref. 8).

The case of a Rayleigh wave excitation is studied for a foundation with and without embedment, the soil model being the same as before. In a layer with a finite depth Rayleigh waves can only propagate horizontally at frequencies higher than the first resonance frequency of a vertical P-wave in the layer, which occurs at $a_o = \pi/5$ in the present model. The Rayleigh waves in the layer are computed by the numerical procedure mentioned before. As foundamental mode the mode shape corresponding to the Rayleigh wave in an elastic halfspace (with an apparent velocity $c = 0.933\ v_s$ and an amplitude ratio $w_o/u_o = 1.565i$) is selected, fig. 2. The horizontal, vertical and rocking motions of the foundation are computed for three ratios of the embedment depth E to the radius r. The results, shown in fig. 4, indicate, that the embedment reduces the response of the foundation. This reflects the fact that for the given embedment ratios the horizontal free field motions vary considerably over the foundation area. The response of a circular surface foundation on a viscoelastic layer agrees well with those of a square surface foundation of equal area on an elastic half-space, fig. 4.

CONCLUSIONS

The kinematic interaction of a rigid massless embedded foundation on a layered soil subjected to a plane wave field has been analysed. The following conclusions can be drawn:

a) The response of a surface foundation depends mainly on the ratio of the apparent wave length of the free field surface motion to the foundation diameter.

b) The kinematic interaction of a surface foundation is not very sensitive to the layering of the soil.

c) Under Rayleigh wave excitation, the embedment of a foundation tends to reduce the horizontal, vertical and rocking motions of the foundation.

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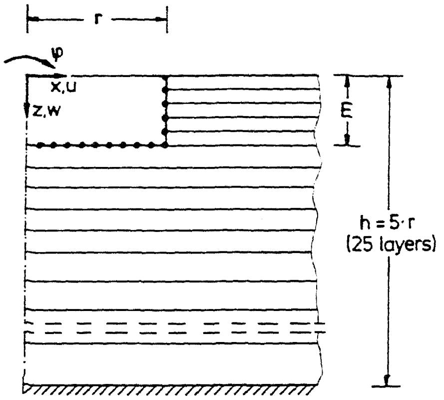

Fig. 1 :   Axisymmetric soil model

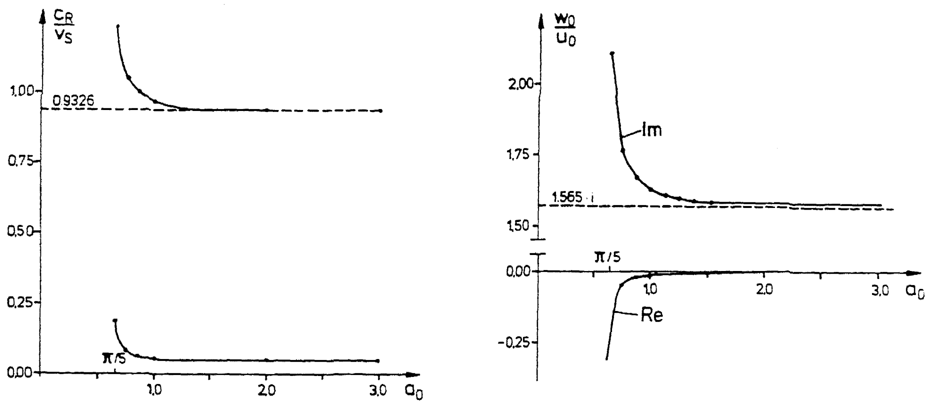

Fig. 2 :   Apparent velocity c_R and amplitude ratio for a Rayleigh wave in a viscoelastic layer, $\nu = 1/3$

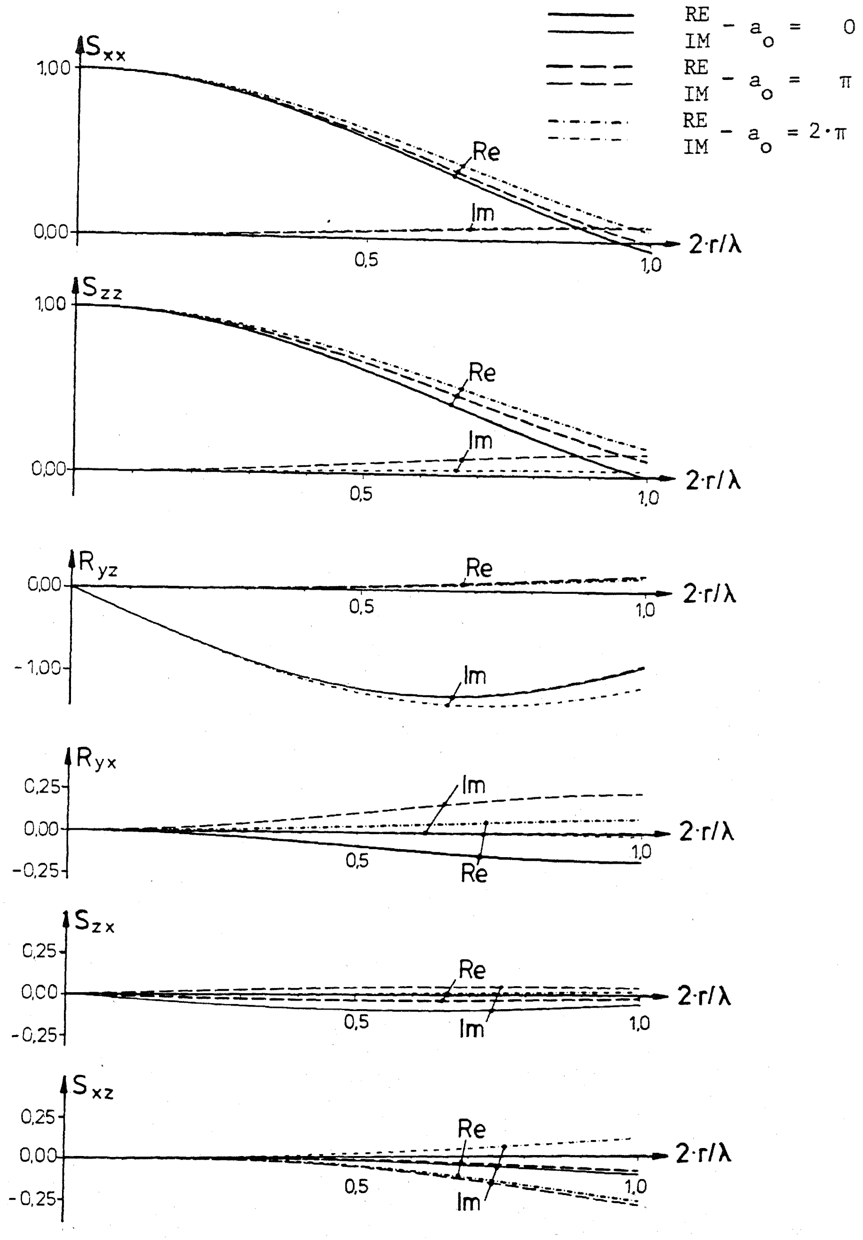

Fig. 3 : Coefficients of the kinematic interaction matrix for a surface foundation

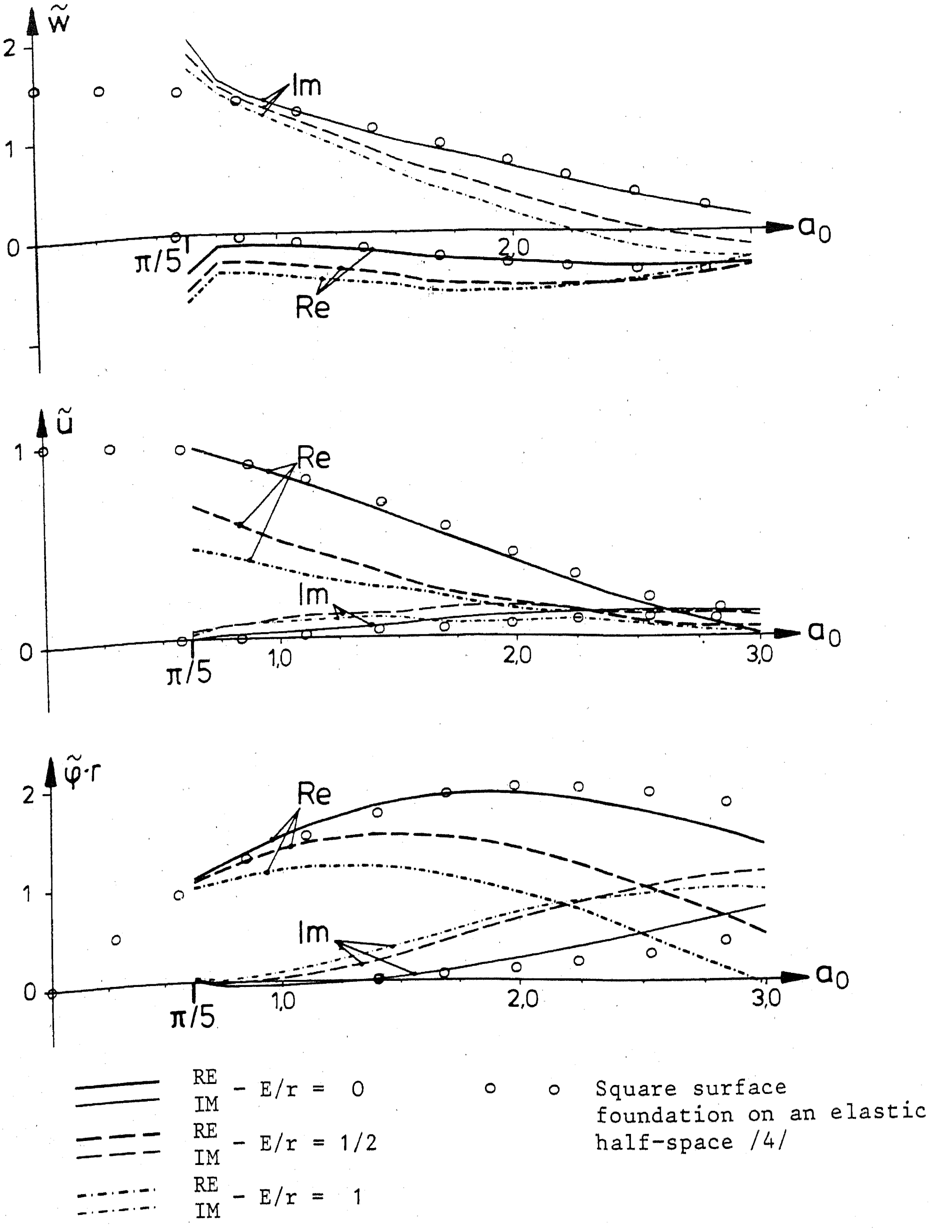

Fig. 4 : Response of a circular, embedded foundation to a Rayleigh wave in a viscoelastic layer ($\nu = 1/3$)