SOIL-STRUCTURE RESPONSE USING FIXED BASE STRUCTURAL MODES

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SUMMARY

A frequency domain solution for handling Soil-Structure-Interaction (SSI) problems, which can be implemented as a post processor of standard modal analysis programs is presented. The application results in a more realistic simulation of response with a minimum of additional computer time.

INTRODUCTION

The dynamic response of massive stiff buildings, as encountered in Nuclear Power Plants, is often controlled by SSI effects. These effects are more significant for soft and medium soils, but even for very stiff rock sites SSI-effects can affect structural response as compared with a fixed base analysis. Today both, linear and nonlinear computer programs are available to perform SSI-dynamic analysis. In present design practice only linear approaches are used, except for the determination of equivalent soil properties compatible with the strain level induced in the soil during the earthquake, for which use is made of approximations as described in reference /1/. For a linear situation, SSI-dynamic analysis methods can be classified as direct /1, 2/ and substructure methods /3, 4, 5/. The last ones provide a very flexible approach in which the problem is split in three independent parts /5, 6/: kinematic interaction motion, impedance functions and dynamic response. They can be solved independently of each other. Due to frequency variation of the impedance functions, rigorous analysis has to be performed in the frequency domain, which results in solving big systems of equations several times (one for each frequency selected to produce the system transfer function). Even if this would not be a serious problem due to recent software and hardware developments, it can lead to several hours of computer time on a CDC-176-computer for a real 3 D structural model, as the bandwidth of the Soil-Structure System is greater than that of the structure alone. In addition, the frequency domain analysis would imply the use of special unusual software for the structural engineer.

To avoid both of these inconveniences, use can be made of approximations based on a classical modal analysis as described for example in /3/. Instead of using the eigenfrequencies and eigenvectors of the structure on an elastic subgrade as in /3/, the method presented herein is developed for a fixed base situation. This method is very effective in solving SSI-problems for which the foundation can be assumed to be rigid, which can be shown to be the case in most practical applications related to NPP design.

DERIVATION OF EQUATIONS OF MOTION

In the derivation, the following symbols are used (see also Fig. 1):

$a_i(t)$	=	i-th normal coordinate
$a_i(\Omega)$	=	Fourier Transform of $a_i(t)$
β_i	=	i-th modal damping of structure on fixed base
$H_i(\Omega)$	=	relative acceleration amplification function for i-th mode
C,K,M	=	damping, stiffness and mass matrices of the structure (6n*6n)
M_i^*	=	$\emptyset_i^T M \emptyset_i$ = i-th generalized mass
K_b, M_b	=	stiffness and mass matrices of foundation refered to the centroid (6*6)
Ω	=	circular frequency
ω_i	=	i-th natural circular frequency of structure on fixed base
$\emptyset_i$	=	i-th eigenvector of structure on fixed base (6n*1)
n	=	number of nodes of structure
t	=	time variable
T	=	transformation matrix (6n*6)
T_i	=	i-th nodal transformation matrix (6*6)
U	=	vector of absolute displacements, including SSI-effects (6n*1)
$U(\Omega)$	=	Fourier Transform of U
U_b	=	vector of absolute displacements of foundation (6*1)
$U_b(\Omega)$	=	Fourier Transform of U_b
U_G	=	vector of given soil displacements (output of kinematic interaction analysis) (6*1)
$U_G(\Omega)$	=	Fourier Transform of U_G
u_i, v_i, w_i	=	translational and rotational degrees of freedom of node i (components of U)
x_i, y_i, z_i	=	coordinates of node i with respect to centroid of foundation
Y	=	vector of displacements relative to the base (6*1)
$\dot{}$, $\ddot{}$	=	derivatives with respect to time

Define the transformation matrix $T = \begin{bmatrix} T_1^T & T_2^T & \dots & T_i^T & \dots & T_n^T \end{bmatrix}^T$

with

$$T_i = \begin{bmatrix} 1 & 0 & 0 & 0 & z_i & -y_i \\ 0 & 1 & 0 & -z_i & 0 & x_i \\ 0 & 0 & 1 & y_i & -x_i & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

then $U = Y + T \cdot U_b$ (1)

where $Y = \sum a_i(t) \cdot \emptyset_i$. (2)

The equilibrium equation for the foundation mass is

$$M_b \ddot{U}_b + K_b \cdot (U_b - U_G) = P_b$$

with $P_b = -T^T M \ddot{U} = -T^T M \ddot{Y} - T^T M T \ddot{U}_b$

or rearranging $(M_b + T^T M T) \cdot \ddot{U}_b + T^T M \ddot{Y} + K_b U_b = K_b U_G$. (3)

For the superstructure the equilibrium equation is

$$M\ddot{U} + C\dot{Y} + KY = 0$$

or $M\ddot{Y} + C\dot{Y} + KY = -MT\ddot{U}_b$. (4)

Assuming that normal modes exist for the superstructure on a fixed base condition, the i-th modal equation resulting from eq.(4) and using eq.(2) is

$$\ddot{a}_i(t) + 2\beta_i \omega_i \dot{a}_i(t) + \omega_i^2 a_i(t) = - \frac{X_i^T}{M_i^*} \cdot \ddot{U}_b \quad (5)$$

or in the frequency domain

$$a_i(\Omega) = - \frac{1}{(\omega_i^2 - \Omega^2) + 2i\beta_i\omega_i\Omega} \cdot \frac{X_i^T}{M_i^*} \cdot \ddot{U}_b(\Omega), \quad (6)$$

with $X_i = T^T M \cdot \emptyset_i$

$X_i^T = \emptyset_i^T \cdot M \cdot T$.

Inserting in eq.2: $$\ddot{Y} = \sum_i H_i(\Omega) \cdot \frac{X_i^T}{M_i^*} \cdot \ddot{U}_b(\Omega) \cdot \emptyset_i \quad (7)$$

with $$H_i(\Omega) = \frac{\Omega^2}{(\omega_i^2 - \Omega^2) + 2i\beta_i\omega_i\Omega}. \quad (8)$$

Substituting in eq.(3), the reduced system of equations of the order 6*6 yields:

$$(K_b - \Omega^2 \cdot \mathcal{M}) \cdot \ddot{U}_b(\Omega) = K_b \cdot \ddot{U}_G(\Omega) \quad (9)$$

with $$\mathcal{M} = M_b + T^T M T + \sum_i H_i(\Omega) \frac{X_i \cdot X_i^T}{M_i^*} .$$

Once $\ddot{U}_b(\Omega)$ is known, the absolute nodal accelerations can be computed from

$$\ddot{U}(\Omega) = \sum \frac{H_i(\Omega)}{M_i^*} X_i^T \cdot \ddot{U}_b(\Omega) \cdot \phi_i + T\ddot{U}_b(\Omega). \qquad (10)$$

The inverse Fourier transform of eq.(10) gives the time domain response $\ddot{U}(t)$. Alternatively, $\ddot{U}_b(\Omega)$ can be obtained from eq.(9) and after inverse Fourier transformation be used as input in eq.(4).

APPLICATIONS

The application of the above described substructuring technique presuposes the solution of three independent problems:

a) Scattering Problem:

It corresponds to the determination of the seismic input motion U_G or kinematic interaction motion, defined as the motion of a rigid massless foundation embedded in the soil and excited by the incoming seismic waves. To solve this partial problem, formulations as found in /1, 2, 4/ can be used. In the present case for simplicity a superficial foundation for which U_G is identical with the free field motion, is assumed.

b) Impedance Problem

The determination of the complex matrix $K_b = K_1(\Omega) + iK_2(\Omega)$, which represents the subgrade stiffness for a rigid foundation as a function of frequency, may be computed by standard programs as the ones described in /2/ or /7/. Fig. 2 shows the soil profile selected to examine the influence of soil layering on response for a real situation. The normalized impedance functions shown in Fig. 3 were determined according to /7/ (only diagonal terms are represented and used in calulations). The adimensional frequency $a_o = \omega \cdot R/\bar{v}_s$ is determined using a shear wave velocity $\bar{v}_s$ of an equivalent half space which matches the static values of the impedances for the layered soil. For comparison, the impedances for a half space are also shown /8/.

c) The structural problem

Fig. 4 shows the mathematical model of a reactor building as typically used for design purposes. It consists of 3D beams and lumped masses. Some of the resulting modal values for a fixed base situation are also presented in Table 1.

The frequency domain analysis described in section 2 is best implemented as a post processor of the FE-Program used for the structural analysis. In the present case an interface was developed for the STARDYNE program.

Fig. 5 shows the amplification function of absolute accelerations ($\ddot{U}_G=1*\exp(i\omega t)$) from eq.(10) for two nodal points and using the first 20 structural modes. The impedance functions correspond to a half space with G = 150 MN, ρ = 2.1 t/m^3, ν = 0.45. Also, the functions are represented from an exact frequency domain solution. As it can be seen, an excellent agreement of both curves is achieved.

For the selected soil, Fig. 6 and 7 show the effect of different approaches on the absolute acceleration amplification function: Curve 0 represents the frequency domain solution from eq.(10). Curve 1, same as curve 0 but using a modified impedance with $K_2 := 0.6*K_2$. Curve 2 is a frequency independent solution where modal damping is limited to 15 % for horizontal and to 30 % for vertical modes as prescribed by German regulations /9/ for simplified analysis. Curve 3 is the same as curve 2 but without chopping modal damping values. For curves 2 and 3 half space formulas for the equivalent half space are used to compute spring and damping values for a lumped parameter representation of soil. Modal damping values were computed as weighted averages using the element potential energy in mode i and the element damping.

As an alternative representation of the approximations described above, Fig. 8 and 9 show the normalized variance of absolute acceleration as a function of frequency for a white noise stationary earthquake excitation. The curves are normalized to the value which results from application of eq.(10). Finally, the influence on typical response spectra is presented on Fig. 10 and 11. As input acceleration an artificial time history with a frequency content corresponding approximately to the NRC-spectra and normalized to 1 m/s^2 is used.

RESULTS AND CONCLUSIONS

As can be seen from Fig. 6 through 11, all of the analysed approximations strongly influence the amplitude and frequency content of structural response. Specially the frequency independent computations (case 2 and 3) produce a significant shift of the resonance frequency and may simulate highly conservative structural response (higher than a factor of 2).

As shown in this paper, these disadvantages can be avoided on the base of present standard tools with a very limited amount of additional computation by performing a more rigorous SSI-analysis.

References

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/3/ Vaish, A.K., Chopra, A.K., "Earthquake Finite Element Analysis of Structural Foundation System", Journal of the Engineering Mechanics Division, 1974

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/8/ Luco, J.E., Westmann, R.A., "Dynamic Response of Circular Footing", ASCE, Journal of the Engineering Mechanics Division, 1971

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Mode i	ω_i	β_i	M_i^*	X_i y	z	φ_x	$Ø_i$ *) v_{170}	w_{170}	$Ø_i$ v_{10}, w_{10}
1	33.0	0.07	62625	1.49	0.	-64.2	1.00	0.	0.
2	36.8	0.07	55955	1.74	0.	-41.8	-0.07	0.	0.
3	63.4	0.07	7683	0.96	0.	- 2.8	-0.01	0.	0.
4	81.2	0.07	71403	0.	1.40	0.	0.	1.00	0.
5	81.8	0.07	10868	-0.11	0.	-16.8	0.11	0.	0.
6	83.2	0.07	3600	0.36	0.	17.1	-0.66	0.	0.
7	90.3	0.07	59227	0.	1.71	0.	0.	0.07	0.

Table 1: Modal Values for Structure on Fixed Base

*) maximum value normalized to 1

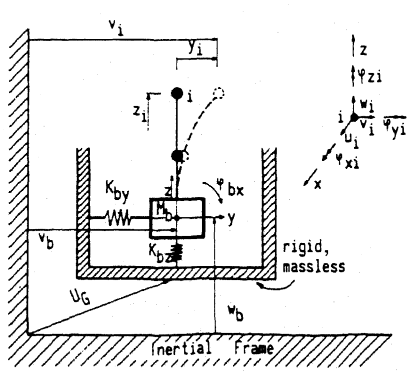

Fig. 1: Mathematical Model, Notations

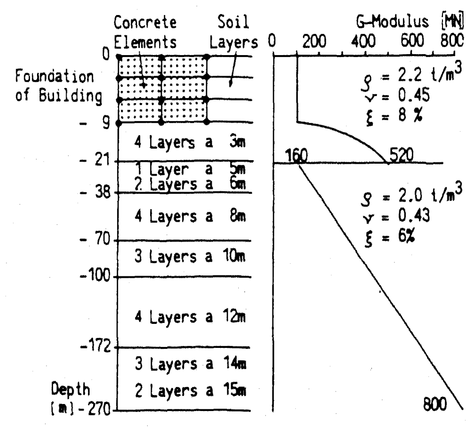

Fig. 2: Soil Profile of Layered Site. Data and Model

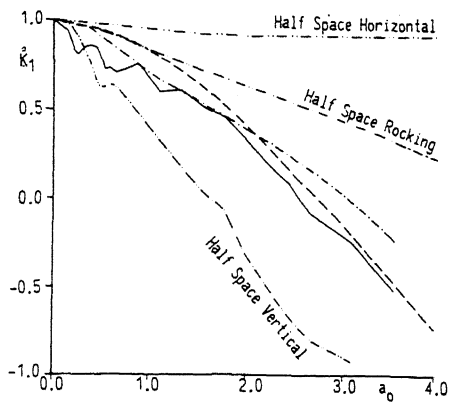

Fig. 3.1: Normalized Impedance Functions. Real Part

—— Soil Profile Horizontal ($K_1(0)$ = 8.21E07 kNm)
---- Soil Profile Vertical ($K_1(0)$ = 1.08E08 kNm)
- - - Soil Profile Rocking ($K_1(0)$ = 7.25E11 kNm)

$$\mathring{K}_1(a_0) = \frac{K_1(a_0)}{K_1(0)}$$

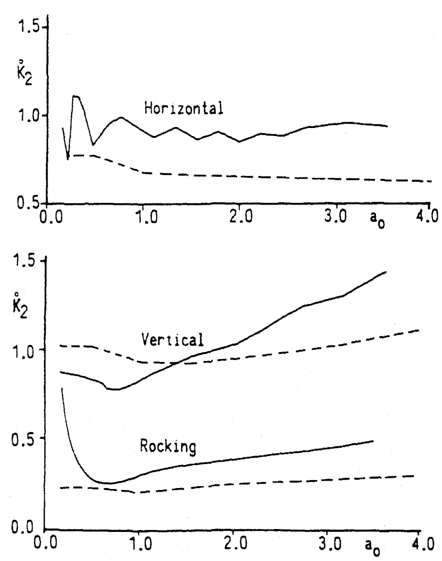

Fig. 3.2: Normalized Impedance Functions Imaginary Part

—— Soil Profile
--- Half Space

$$\mathring{K}_2(a_0) = \frac{K_2(a_0)}{K_1(0) \cdot a_0}$$

$a_0 = \frac{\omega \cdot R}{\bar{v}_s}$ R = Foundation Radius
$\bar{v}_s$ = Equivalent Shear Wave Velocity

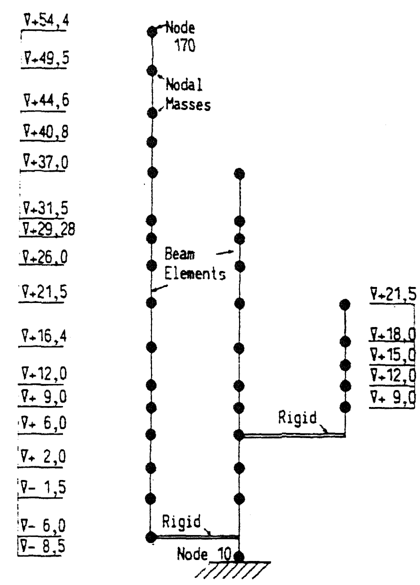

Fig. 4: Structural Model

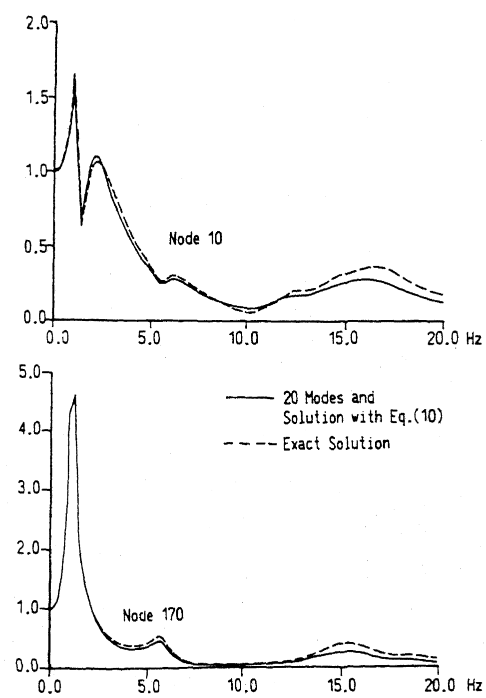

Fig. 5: Comparison Horizontal Transfer Functions

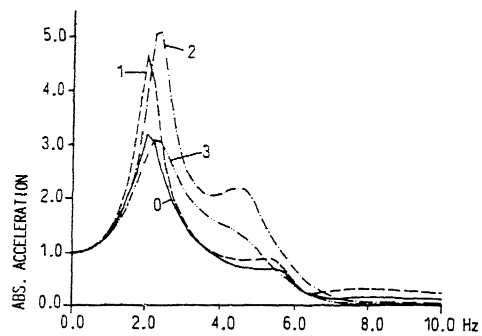

Fig. 6: Horizontal Transfer Functions Node 170

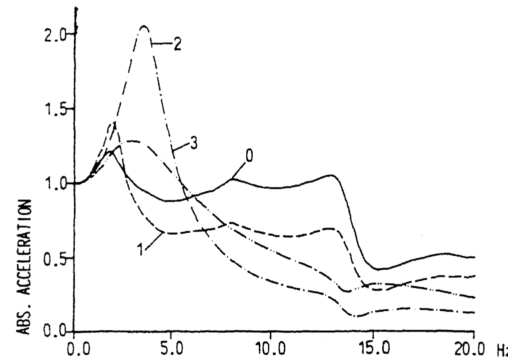

Fig. 7: Vertical Transfer Functions Node 170

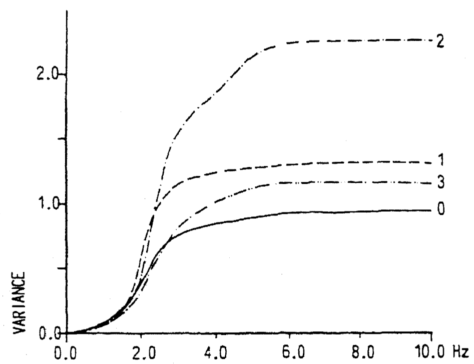

Fig. 8: Normalized Horizontal Variances Node 170

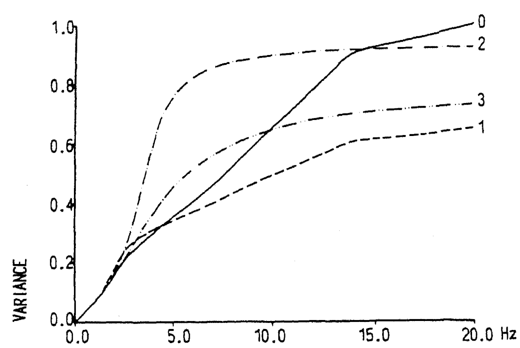

Fig. 9: Normalized Vertical Variances Node 170

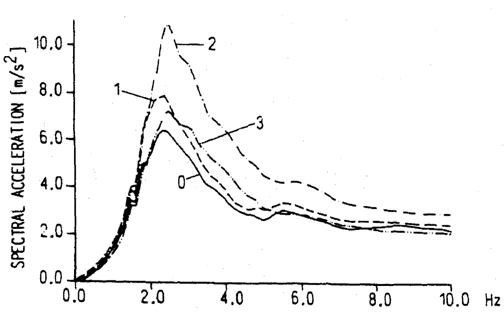

Fig. 10: Horizontal Spectra Node 170

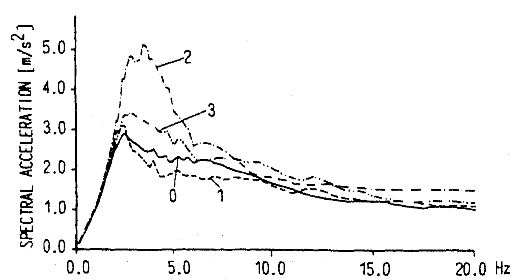

Fig. 11: Vertical Spectra Node 170

CURVE 0 : Frequency Dependent Solution eq.(10)
CURVE 1 : Frequency Dependent Solution eq.(10) with $K_2 := 0.6 \cdot K_2$
CURVE 2 : Frequency Independent Solution with Limited Modal Damping
CURVE 3 : Frequency Independent Solution with Unchopped Modal Damping