DYNAMIC INTERACTION OF A RIGID FOUNDATION EMBEDDED IN TWO-LAYERED VISCO-ELASTIC MEDIUM

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SUMMARY

This study deals with the harmonic responses of a rigid massless foundation of arbitrary shape embedded in the surface layer of two-layered visco-elastic medium. The Boundary Element Method is employed to formulate the compliance matrix and the foundation input motion, in which the Green functions for two-layered visco-elastic medium are utilized. As the results of this study, the compliance matrix of a hemispherical foundation is obtained and it has become clear that the thickness of the surface layer and the viscosity affect significantly the behavior of the foundation.

INTRODUCTION

It has been shown that soil-foundation interaction problems are characterized by the compliance matrix of the foundation and the foundation input motion to various free field motion (Ref. 1). Many studies which have dealt with surface foundations are available, but with respect to embedded foundations, the investigations are few. In recent years, B.E.M. (Boundary Element Method) has been shown to be a very powerful tool to study this kind of problem, especially for the investigation of embedded foundations (Ref. 2). Authors have already presented the efficiency of B.E.M. to study the dynamic interaction of a rigid hemispherical foundation embedded in an elastic half-space subjected to obliquely incident SH, P and SV waves and Rayleigh wave (Ref. 3). The impedance matrix and the foundation input motion were presented in that work and compared with other analytical and numerical results available to show its validity.

Although the studies using B.E.M. have been increasing in number, most of them are restricted to the foundation supported on or embedded in an elastic half-space. In this study, the harmonic responses of a rigid foundation of arbitrary shape embedded in the surface layer of two-layered voigt type visco-elastic medium are studied by B.E.M.. Following the expression of B.E.M., the compliance matrix and the foundation input motion are formulated as the functions of a dimensionless frequency parameter a_0. Then the Green functions for the surface layer of two-layered visco-elastic medium are presented referring to the method proposed by Harkrider (Ref. 4). They are composed of two solutions, i.e. fundamental solutions for full space and the homogeneous solutions which are needed to satisfy the free surface condition, the interface condition and the radiation condition completely. According to this partition of the Green functions and their properties, the way of estimating the influence functions is discussed. Finally, the compliance matrix of a

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hemispherical foundation is presented and the effects of layering, viscosity and embedment are discussed.

FORMULATION OF THE PROBLEM

In this section, the compliance matrix and the foundation input motion for the embedded foundation of arbitrary shape are formulated following the expression of B.E.M.. The harmonic time dependence exp[$i\omega t$] is assumed implicitly and Einstein summation convention for indices is used. Superscripts are employed to identify the layer such that u_i^m means u_i in the m-th layer, $k_{\alpha_0}^m$, $k_{\alpha v}^m$, $k_{\beta_0}^m$, $k_{\beta v}^m$, $c_{d_0}^m$, c_{dv}^m, $c_{s_0}^m$, c_{sv}^m, ν_m, ρ_m, μ_v^m and so forth.

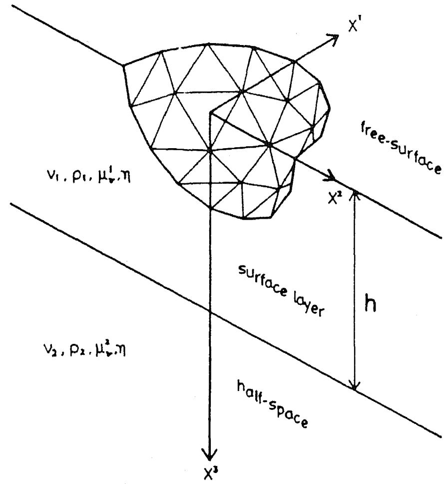

Fig. 1 Description of the model (ν_m:poisson's ratio, ρ_m:mass density, μ_v^m:Lame's constant, η:viscosity parameter)

The voigt type visco-elastic body is governed by the differential equation of motion

$$(\lambda_v + \mu_v)\, u_{j,ji} + \mu_v\, u_{i,jj} + \rho\omega^2 u_i = 0\,, \quad (1)$$

$$\lambda_v = \lambda_r + i\omega\lambda_i \quad , \qquad \mu_v = \mu_r + i\omega\mu_i \quad ,$$

where λ_v and μ_v are Lame's constants of complex number, ρ is mass density, ω is a circular frequency, u_j is the x^j component of a displacement vector, (,j) denotes a partial differentiation with respect to x^j , and i is imaginary unit. The viscosity parameters η_d and η_s are introduced following Kobori et al. (Ref. 5),

$$\eta_d = \frac{c_{s0}^1\,(\lambda_i + 2\mu_i)}{a\,(\lambda_r + 2\mu_r)} \quad , \qquad \eta_s = \frac{c_{s0}^1\,\mu_i}{a\,\mu_r} \quad ,$$

in which a is reference length (later a radius of a hemisphere), c_{s_0} is a real part of $c_{sv} = (\mu_v/\rho)^{\frac{1}{2}}$. Using these parameters, wave velocities c_{dv} and c_{sv} , wave numbers $k_{\alpha v}$ and $k_{\beta v}$ are related to those of elastic body (real number) as follows,

$$c_{dv} = c_{d_0} g_d^{-\frac{1}{2}}, \qquad c_{sv} = c_{s_0} g_s^{-\frac{1}{2}} \quad , \qquad c_{d0} = \{(\lambda_r + 2\mu_r)/\rho\}^{\frac{1}{2}} \quad , \qquad c_{s0} = \{\mu_r / \rho\}^{\frac{1}{2}},$$

$$k_{\alpha v} = k_{\alpha_0} g_d^{\frac{1}{2}} \;, \qquad k_{\beta v} = k_{\beta_0} g_s^{\frac{1}{2}} \quad , \qquad k_{\alpha_0} = \omega / c_{d0} \;, \qquad k_{\beta_0} = \omega / c_{s0} \quad ,$$

$$g_d = (\lambda_r + 2\mu_r)/(\lambda_v + 2\mu_v) = (1 + i a_0 \eta_d)^{-1} \;, \qquad g_s = \mu_r/\mu_v = (1 + i a_0 \eta_s)^{-1} \;, \qquad a_0 = k_{\beta_0}^1 a \;.$$

In the following λ_v is set equal to μ_v (i.e. poisson's ratio $\nu = 1/4$) to make $\eta_d = \eta_s = \eta$ and $g_d = g_s = g$. The model in this study is defined in Fig. 1. A visco-elastic layer of thickness h with properties ρ_1, μ_v^1 , ν_1 and η rests on visco-elastic half-space with properties ρ_2, μ_v^2, ν_2 and η .

Following Shaw (Ref. 6), eq.(1) is converted to the expression of Boundary Integral Equation,

$$c(\vec{x})u_k(\vec{x}) - \bar{u}_k(\vec{x}) + \int_\Gamma t_{kl}^*(\vec{x},\vec{y})\, u_l(\vec{y})\, d\Gamma(\vec{y}) = \int_\Gamma u_{kl}^*(\vec{x},\vec{y})\, t_l(\vec{y})\, d\Gamma(\vec{y}) \quad , \qquad (2)$$

in which $u_k(\vec{x})$ and $t_k(\vec{x})$ are the x^k component of the displacement and the traction at the point $\vec{x}$ respectively, $\bar{u}_k(\vec{x})$ is the x^k component of the free field motion evaluated at the contact surface, $u^*_{kl}(\vec{x},\vec{y})$ and $t^*_{kl}(\vec{x},\vec{y})$ are the Green functions which denote the x^l component of the displacement and the traction at the point $\vec{y}$ produced by a point force in the x^k direction applied at the point $\vec{x}$. Γ denotes the contact surface and $\vec{x}$ and $\vec{y}$ are the points on Γ. $c^*(\vec{x})$ is a coefficient which depends on the smoothness of the surface at the point $\vec{x}$. Using n constant elements, eq.(2) is discretized into the expression of B.E.M. (Ref. 7),

$$c^*_i\, u_k(i) - \bar{u}_k(i) + T^*_{kl}(i,j)u_l(j) = U^*_{kl}(i,j)t_l(j) \quad , \tag{3}$$

where i and j denote a number varying from 1 to n, $u_k(i)$ and $t_k(i)$ are the x^k component of the displacement and the traction at the center of the i-th element and $U^*_{kl}(i,j)$ and $T^*_{kl}(i,j)$ are the influence functions defined by

$$U^*_{kl}(i,j) = \int_{\Gamma_j} u^*_{kl}(\vec{x}_i,\vec{x}_j)\, d\Gamma(\vec{x}_j) \quad , \qquad T^*_{kl}(i,j) = \int_{\Gamma_j} t^*_{kl}(\vec{x}_i,\vec{x}_j)\, d\Gamma(\vec{x}_j) \quad , \tag{4a,b}$$

where Γ_j is the area of the j-th element, and $\vec{x}_i$ is the centroid of the i-th element. Estimating eq.(4) at every boundary element and assembling them yield a system of linear algebraic equations,

$$[c^*]\,\{u\} - \{\bar{u}\} + [T^*]\,\{u\} = [U^*]\,\{t\} \ . \tag{5}$$

Thus, the displacement and the tractions evaluated at the centers of the elements are connected by eq.(5). Since the Green functions used here satisfy the free surface conditions completely, the surface on which the discretization is needed is limited to the contact surface of the foundation and the soil.

In order to formulate the compliance matrix and the foundation input motion in dimensionless form, a replaced concentrated force vector $\{Q\}$ defined by $\{Q\}_i = \{t\}_i\, \Gamma_i$,

is introduced. Then eq.(5) is deformed to

$$(k^1_{\beta_0} / \mu^1_v)[G]\,\{Q\} = [H]\,\{u\} - \{\bar{u}\} \quad , \tag{6}$$

in which matrices [G] and [H] are defined by

$$[G]\,\{Q\} = \mu^1_v / k^1_{\beta_0}\, [U^*]\,\{t\} \quad , \qquad [H] = [c^*] + [T] \ . \tag{7}$$

Since the foundation is a rigid body, the displacements at the contact surface are determined by the rigid body motion and the generalized force vector is obtained by integrating tractions at the contact surface. These vectors are evaluated at the origin and defined by

$$\{U\} = (\ U_1\ U_2\ U_3\ a\varphi_1\ a\varphi_2\ a\varphi_3\ )^T \quad , \qquad \{T\} = (\ T_1\ T_2\ T_3,\ M_1/a\ M_2/a\ M_3/a\ )^T \quad , \tag{8a,b}$$

where U_i and T_i are the x^i component of the displacement and the force respectively, φ_i and M_i are the rotation and the moment about x^i axis respectively. The displacement and the replaced concentrated force vector at a point $\vec{x}$ on the contact surface are related to $\{U\}$ and $\{T\}$ by the relations,

$$\{u(\vec{x})\} = [A(\vec{x})]\{U\} \ , \qquad \{T\} = [A(\vec{x})]^T\,\{Q(\vec{x})\} \ , \tag{9a,b}$$

where the superscript ^{T} denotes the transpose and [A] is a matrix defined by

$$[A(\vec{x})] = \begin{bmatrix} 1 & 0 & 0 & 0 & x_3/a & -x_2/a \\ 0 & 1 & 0 & -x_3/a & 0 & x_1/a \\ 0 & 0 & 1 & x_2/a & x_1/a & 0 \end{bmatrix} .$$

Estimating eq.(9) at every center of the element and substituting them into eq.(6) yield the following key expression,

$$\{U\} = (1/\mu_r^1 a)[C]\{T\} + \{S^*\} \quad , \tag{10}$$

where [C] is the compliance matrix and $\{S^*\}$ is the foundation input motion which are defined by

$$[C] = a_0 g_s \, [[A^T][G]^{-1}[H][A]]^{-1} \quad , \tag{11}$$

$$\{S^*\} = (1/a_0 g_s)\,[C][A^T][G]^{-1}\{\bar{u}\} \quad . \tag{12}$$

Estimating the compliance matrix [C] and the foundation input motion $\{S^*\}$, soil-foundation interaction problem is fully characterized.

GREEN FUNCTIONS

Green functions for the surface layer of two-layered visco-elastic medium are defined as the displacement and the traction in the surface layer generated by the concentrated force which is located also in the surface layer. These Green functions are derived following the method proposed by Harkrider (Ref. 4), where the surface waves in multi-layered elastic medium generated by arbitrary sources located in the medium are studied using transfer matrix method similar to that employed by Haskell (Ref. 8). In this method, the layer in which the source is located is partitioned into two layers by introducing imaginary interface. Using transfer matrix expression and substituting the continuity condition at the actual interface, the incontinuity condition at the imaginary interface, the free surface condition and the radiation condition, the displacements at the free surface are evaluated in the Fourier-Bessel inverse transformation form. From them, the potentials of the surface layer above the imaginary interface can be obtained.

In order to obtain the Green functions, these potentials are divided into two parts; source potentials which denote the fundamental solutions for full space and the homogeneous potentials which denote the effects of the free surface and the layering. Moreover, the fundamental solutions generated by the mirror point source are also separated from the homogeneous potentials. Green functions are derived from these potentials and expressed in the summation form, i.e. the sum of fundamental solutions u_{kl}^F for the source and u_{kl}^M for the mirror point source both of which have explicit form as shown below and the homogeneous solutions u_{kl}^H which are expressed in Fourier-Bessel inverse transformation.

$$u_{kl}^F(\vec{x}_i,\vec{x}_j) = \frac{1}{4\pi\mu_v^1}\left[\delta_{kl}\frac{1}{R}\exp[-ik_{\beta v}^1 R] + \frac{1}{(k_{\beta v}^1)^2}\frac{\partial^2}{\partial x^k\,\partial x^l}\left\{\frac{1}{R}\left(\exp[-ik_{\beta v}^1 R]-\exp[-ik_{\alpha v}^1 R]\right)\right\}\right] , \tag{13}$$

$$u_{kl}^M(\vec{x}_i,\vec{x}_j) = \frac{1}{4\pi\mu_v^1}\left[\delta_{kl}\frac{1}{R_m}\exp[-ik_{\beta v}^1 R_m] + \frac{1}{(k_{\beta v}^1)^2}\frac{\partial^2}{\partial x^k\,\partial x^l}\left\{\frac{1}{R_m}\left(\exp[-ik_{\beta v}^1 R_m]-\exp[-ik_{\alpha v}^1 R_m]\right)\right\}\right] , \tag{14}$$

$$R = |\vec{x}_i - \vec{x}_j| \;;\; \vec{x}_j = (x_j^1, x_j^2, x_j^3) , \qquad R_m = |\vec{x}_i - \vec{x}_j^m| \;;\; \vec{x}_j^m = (x_j^1, x_j^2, -x_j^3) ,$$

$$u_{11}^H = u_3^H + u_4^H(1-2\cos^2\vartheta), \quad u_{21}^H = u_{12}^H , \quad u_{31}^H = u_1^H\cos\vartheta ,$$

$$u_{12}^H = -u_4^H\sin 2\vartheta , \quad u_{22}^H = u_3^H + u_4^H(1-2\sin^2\vartheta) , \quad u_{32}^H = u_1^H\sin\vartheta ,$$

$$u_{13}^H = u_5^H\cos\vartheta , \quad u_{23}^H = u_5^H\sin\vartheta , \quad u_{33}^H = u_2^H \;; \tag{15}$$

$$u_i^H = \int_0^\infty \bar{u}_i^H\, d\zeta , \qquad \bar{u}_1^H = Q_V J_1(a\zeta)\times k_{\beta_0}^1/4\pi\mu_v^1 ,$$

$$\bar{u}_2^H = W_V J_0(a\zeta)\times k_{\beta_0}^1/4\pi\mu_v^1 , \qquad \bar{u}_3^H = Q_{H1} J_0(a\zeta)\times k_{\beta_0}^1/4\pi\mu_v^1 ,$$

$$\bar{u}_4^H = Q_{H2} J_2(a\zeta)\times k_{\beta_0}^1/4\pi\mu_v^1 , \qquad \bar{u}_5^H = W_H J_1(a\zeta)\times k_{\beta_0}^1/4\pi\mu_v^1 ,$$

$$Q_V = (\zeta^2/Fg)(-\Delta_{v_1} e^{-r\alpha_1 b} - \Delta_{v_2} e^{r\alpha_1 b} + \omega_{v_1} e^{-r\beta_1 b} - \omega_{v_2} e^{r\beta_1 b}) \quad ,$$

$$W_V = (r_{\alpha_1}\zeta/Fg)(-\Delta_{v_1} e^{-r\alpha_1 b} + \Delta_{v_2} e^{r\alpha_1 b}) + (\zeta^3/F r_{\beta_1} g)(\omega_{v_1} e^{-r\beta_1 b} + \omega_{v_2} e^{r\beta_1 b}) \quad ,$$

$$Q_{H_1} = (Q_{HA} + Q_{HB})/2 \; , \qquad Q_{H_2} = (Q_{HA} - Q_{HB})/2 \; , \qquad Q_{HB} = \zeta E_{H_2} \; ,$$

$$Q_{HA} = (-\zeta^3/F r_{\alpha_1} g)(\Delta_{H_1} e^{-r\alpha_1 b} + \Delta_{H_2} e^{r\alpha_1 b}) - (\zeta r_{\beta_1}/Fg)(\omega_{H_1} e^{-r\beta_1 b} - \omega_{H_2} e^{r\beta_1 b}) \quad ,$$

$$W_H = (\zeta^2/Fg)(\Delta_{H_1} e^{-r\alpha_1 b} - \Delta_{H_2} e^{r\alpha_1 b} + \omega_{H_1} e^{-r\beta_1 b} + \omega_{H_2} e^{r\beta_1 b}) \quad ,$$

$$r_{\alpha_1} = \sqrt{\zeta^2 - \xi^2 g} \; , \quad r_{\beta_1} = \sqrt{\zeta^2 - g} \; , \quad r_{\alpha_2} = \sqrt{\zeta^2 - \sigma^2 \xi^2 g} \; , \quad r_{\beta_2} = \sqrt{\zeta^2 - \eta^2 g} \; ,$$

$$\xi = \sqrt{(1-2\nu_1)/(2-2\nu_1)} \; , \quad \sigma = \sqrt{\rho(1-2\nu_2)\nu_1/\{\mu(1-2\nu_1)\nu_2\}} \; , \quad \eta = \sqrt{\rho/\mu} \; , \quad b = k^1_{\beta_0} x^3 \; ,$$

$$\rho = \rho_2/\rho_1 \; , \quad \mu = \mu^2_r/u^1_r \; .$$

The equations for constants of u^H_{kl} are omitted for space limitation. t^F_{kl}, t^M_{kl} and t^H_{kl} are computed from u^F_{kl}, u^M_{kl} and u^H_{kl} respectively. The Green functions are valid in the surface layer irrespective of the location of the observation point above or beneath the imaginary interface. The partition of the Green functions contributes to the estimation of the influence functions.

INFLUENCE FUNCTIONS

The influence functions U^*_{kl} and T^*_{kl} are defined as integration of the Green functions over an element. When $U^*_{kl}(i,i)$ and $T^*_{kl}(i,i)$ are evaluated, the principal values must be obtained, for u^F_{kl} and t^F_{kl} have the singularity of the order of r^{-1} and r^{-2}, respectively. In order to evaluate $U^*_{kl}(i,j)$ and $T^*_{kl}(i,j)$ $(i \neq j)$ and to integrate u^M_{kl}, t^M_{kl}, u^H_{kl} and t^H_{kl} in $U^*_{kl}(i,i)$ and $T^*_{kl}(i,i)$, ordinary Gaussian quadrature can be utilized.

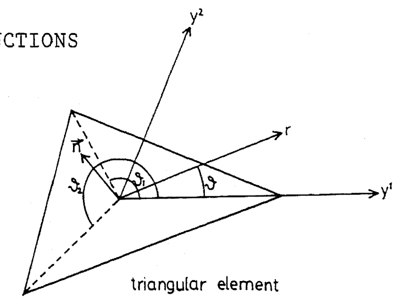

Fig. 2 Local coordinate systems
$(y^1 - y^2,\ r - \vartheta)$ $\quad$ ($\vec{n}$:outer normal)

The principal values are evaluated referring to the local coordinate system shown in Fig. 2; y^1-y^2 is an orthogonal Cartesian coordinate system and $r - \vartheta$ is a polar coordinate system, both of which are located in the plane of the element and have the origin at its centroid. The terms $\exp[-ik^1_{\alpha v} r]$ and $\exp[-ik^1_{\beta v} r]$ are expanded in Taylor expansion series around r=0,

$$\frac{4\pi\mu^1_v}{k^1_{\beta v}} u^F_{kl}(x_i,x_j) \simeq \Big[\Big\{\frac{1+\gamma^2}{2}\frac{1}{k^1_{\beta v} r} - \frac{i(2+\gamma^3)}{3} - \frac{(3+\gamma^4)}{8} k^1_{\beta v} r + \frac{i(4+\gamma^5)}{30}(k^1_{\beta v} r)^2$$

$$+ \frac{5+\gamma^6}{144}(k^1_{\beta v} r)^3 - \frac{i(6+\gamma^7)}{840}(k^1_{\beta v} r)^4\Big\}\delta_{kl}$$

$$+\Big\{\frac{1-\gamma^2}{2}\frac{1}{k^1_{\beta v} r} + \frac{1-\gamma^4}{8} k^1_{\beta v} r - \frac{i(1-\gamma^5)}{15}(k^1_{\beta v} r)^2 - \frac{(1-\gamma^6)}{48}(k^1_{\beta v} r)^3 + \frac{i(1-\gamma^7)}{210}(k^1_{\beta v} r)^4\Big\}$$

$$\times\Big\{\cos^2\vartheta \frac{\partial y^1}{\partial x^k}\frac{\partial y^1}{\partial x^l} + \cos\vartheta \sin\vartheta \Big(\frac{\partial y^1}{\partial x^k}\frac{\partial y^2}{\partial x^l} + \frac{\partial y^1}{\partial x^l}\frac{\partial y^2}{\partial x^k}\Big) + \sin^2\vartheta \frac{\partial y^2}{\partial x^k}\frac{\partial y^2}{\partial x^l}\Big\}\Big] \quad , \quad (16)$$

then the integration with respect to r is carried out in the dimensionless polar coordinate system,

$$\int_\Gamma d\Gamma = \lim_{\varepsilon \to 0} \frac{1}{(k^1_{\beta_0})^2}\int_0^{2\pi}\int_\varepsilon^{\bar{R}_0(\vartheta)} (k^1_{\beta_0} r)\, d(k^1_{\beta_0} r)\, d\vartheta \quad , \qquad (17)$$

where $\bar{R}_0 = k^1_{\beta_0}\bar{R}$, $\bar{R}(\vartheta)$ is the value of r estimated at the edge. The integration with respect to $k^1_{\beta_0}r$ is carried out analytically,

$$\lim_{\varepsilon\to 0} \frac{4\pi\mu^1_v}{k^1_{\beta_0}} \int_{\varepsilon}^{\bar{R}_0} u^F_{kl}\,(k^1_{\beta_0}\,r)\,d(k^1_{\beta_0}\,r)$$

$$\simeq \frac{k^1_{\beta_0}}{k^1_{\beta v}} \Big[\{ \frac{1+\gamma^2}{2}\bar{R}_1 - \frac{i(2+\gamma^3)}{6}\bar{R}_1^2 - \frac{3+\gamma^4}{24}\bar{R}_1^3 + \frac{i(4+\gamma^5)}{120}\bar{R}_1^4 + \frac{5+\gamma^6}{720}\bar{R}_1^5 - \frac{i(6+\gamma^7)}{5040}\bar{R}_1^6 \}\delta_{kl}$$

$$+ \{ \frac{1-\gamma^2}{2}\bar{R}_1 + \frac{1-\gamma^4}{24}\bar{R}_1^3 - \frac{i(1-\gamma^5)}{60}\bar{R}_1^4 - \frac{(1-\gamma^6)}{240}\bar{R}_1^5 + \frac{i(1-\gamma^7)}{1260}\bar{R}_1^6\}$$

$$\times\{\cos^2\vartheta\frac{\partial y^1}{\partial x^k}\frac{\partial y^1}{\partial x^l} + \cos\vartheta\,\sin\vartheta\,(\frac{\partial y^1}{\partial x^k}\frac{\partial y^2}{\partial x^l} + \frac{\partial y^1}{\partial x^l}\frac{\partial y^2}{\partial x^k}) + \sin^2\vartheta\frac{\partial y^2}{\partial x^k}\frac{\partial y^2}{\partial x^l}\}\Big] \quad . \qquad (18)$$

Then the integration with respect to ϑ is carried out numerically considering the dependence of $\bar{R}_0$ on ϑ.

The integration of t^F_{kl} is carried out similarly for u^F_{kl}. For $\vec{r}$ and $\vec{n}$ are mutually orthogonal, the integral with respect to $k^1_{\beta_0}r$ is estimated as

$$\frac{4\pi}{(k^1_{\beta_0})^2}\int_{\varepsilon}^{\bar{R}_0} t^F_{kl}\,(k^1_{\beta_0}\,r)\,d(k^1_{\beta_0}\,r)$$

$$\simeq \Big[\{ \gamma^2\log(k^1_{\beta_0}\,r) + (\frac{1}{4} - \frac{1}{2}\gamma^2 + \frac{3}{4}\gamma^4)\frac{(k^1_{\beta v}r)^2}{2}$$

$$+ i(-\frac{2}{15} + \frac{\gamma^3}{3} - \frac{8}{15}\gamma^5)\frac{(k^1_{\beta v}r)^3}{3}\}(\cos\vartheta\frac{\partial y^1}{\partial x^k} + \sin\vartheta\frac{\partial y^2}{\partial x^k})\,n_l$$

$$+ \{-\gamma^2\log(k^1_{\beta_0}\,r) - \frac{(1+\gamma^4)}{4}\frac{(k^1_{\beta v}r)^2}{2} + i(\frac{1}{5} + \frac{2}{15}\gamma^5)\frac{(k^1_{\beta v}r)^3}{3}\}$$

$$\times(\cos\vartheta\frac{\partial y^1}{\partial x^l} + \sin\vartheta\frac{\partial y^2}{\partial x^l})\,n_k\Big]_{\varepsilon}^{R_0} \qquad (19)$$

Considering the integral

$$\lim_{\varepsilon\to 0}\int_{\varepsilon}^{\bar{R}(\vartheta)}\int_0^{2\pi}\cos\vartheta/r\,dr\,d\vartheta = \lim_{\varepsilon\to 0}\int_0^{2\pi}[\log r]_{\varepsilon}^{\bar{R}(\vartheta)}\cos\vartheta\,d\vartheta = \int_0^{2\pi}\log\bar{R}(\vartheta)\cos\vartheta\,d\vartheta \quad , \qquad (20)$$

the integration with respect to ϑ is carried out numerically.

COMPLIANCE MATRIX FOR A HEMISPHERICAL FOUNDATION

In this section, the numerical results are presented for the compliance matrix for a rigid hemispherical foundation of radius a embedded in two-layered visco-elastic medium; $\rho_2/\rho_1 = 1.0$, $\mu^2_v/\mu^1_v = 4.0$, $\nu=1/4$, $\eta=0.2$, h/a=2.0, 4.0 and ∞ . In Fig. 3, components of the compliance matrix are shown as functions of a dimensionless frequency a_0 with different values of layer thickness ratio h/a. The results for the square foundation supported on the same medium obtained by Kobori et al. (Ref. 5) are also shown.

Comparison between the compliance functions for a half-space with $\eta=0.0$ and $\eta=0.2$ indicates that

1) Every component of the compliance matrix with $\eta=0.2$ exhibits larger phase difference than that without viscosity.
2) The property of c_{11} is similar to that of c_{33} and c_{44} is similar to c_{66}.
3) The effects of η on the rotational components c_{44} and c_{66} are more remarkable than on c_{11} and c_{33}.

With respect to the layering effect,

1) Some peaks which seem to correspond to natural frequencies of the system are

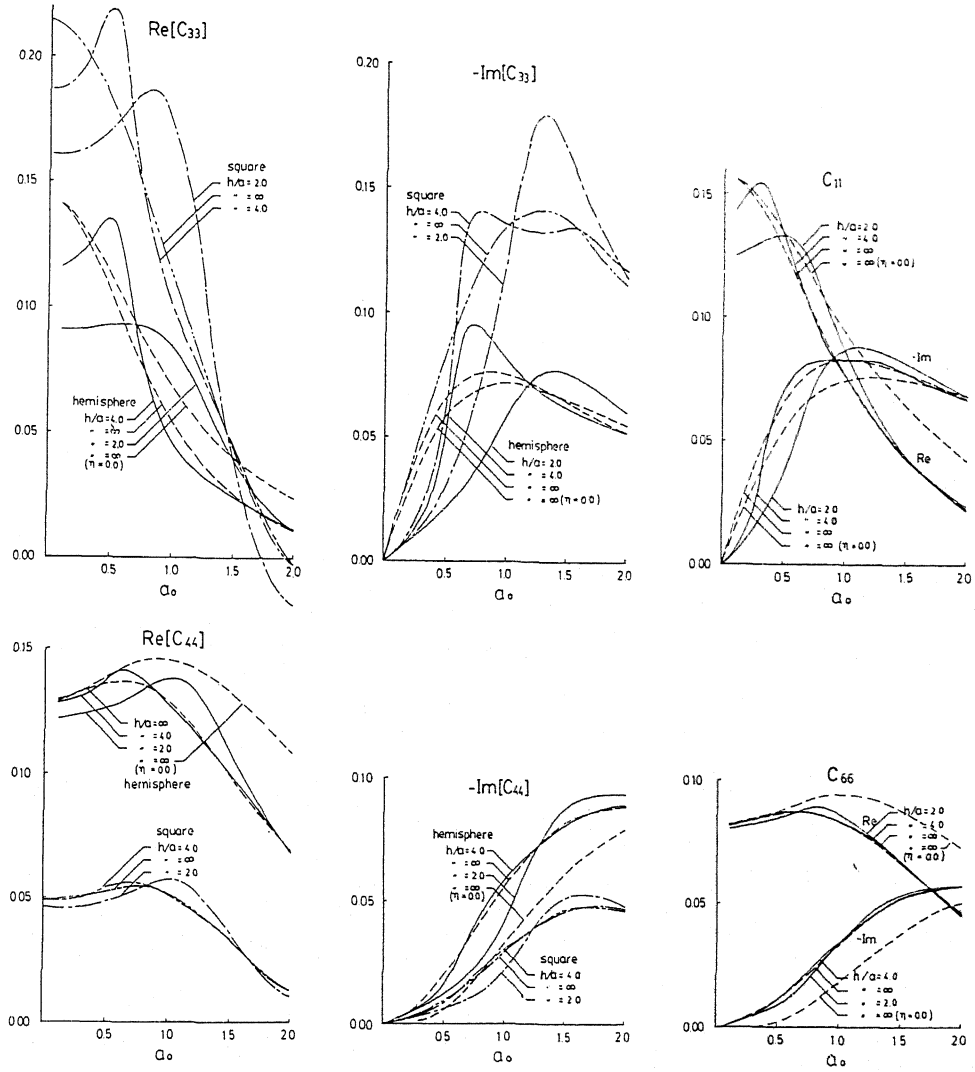

Fig. 3 Compliance matrix for a hemispherical foundation

($\nu_1=\nu_2=1/4$, $\rho_2/\rho_1=1.0$, $\mu_r^2/\mu_r^1=4.0$, $\eta_1=\eta_2=0.2$, h/a=2.0, 4.0 and ∞)

observed in contradiction to the case of a half-space.

2) For thinner surface layer, the peaks tend to be located at higher frequency.
3) The asymptotic property of the compliance functions for two-layered medium with h/a=4.0 to that for a half-space in higher frequency shows siginificant dependence on the type of applied force. For c_{33}, the asymptotic property is not so obvious. For c_{11} and c_{44}, this property is more remarkable and for c_{66} the difference is hardly seen.

Comparing the results for a hemispherical foundation with that for a square foundation, the effects of embedment are discussed.

1) Every compliance function for a hemispherical foundation has similar property to that for a square foundation; the peaks are located at similar frequencies.
2) Irrespective of the layering, $|c_{33}|$ for a square foundation is larger than that for a hemispherical foundation, in contradiction to the case of $|c_{44}|$.

CONCLUSIONS

1) The effects of the embedment are not significantly affected by the thickness of the surface layer.
2) The effects of layering is significant, especially for the location of the peaks of the compliance functions. The asymptotic property of the compliance functions for two-layered medium to that for a half-space strongly depends on the type of applied force.

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