EARTHQUAKE RESPONSE ANALYSIS OF TANKS INCLUDING HYDRODYNAMIC AND FOUNDATION INTERACTION EFFECTS

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SUMMARY

This paper presents a procedure to formulate and solve theoretically the dynamic interactions of soil and liquid-tank system subjected to earthquake excitations based on Rayleigh-Ritz method. Considering the complex stiffness of the foundation and the tanks as a thin elastic circular cylindrical shell, the solution of boundary value problems including the sloshing effects for the internal pressure is presented as the superposition of the elastic, translational and rocking pressures, respectively. The interaction forces are considered as the generalized forces and the equations of motion are expressed by several unknowns using the eigen vectors of rigid foundation system for the displacements.

INTRODUCTION

Sufficient attention should be given to the dynamic interactions of liquid-tank-soil system. The dynamic analysis of the system by F.E.M. has been recently developed, but, using F.E.M. for the analysis of this system, a large number of freedom are necessary in many cases on account of discretization of liquid-tank-soil system by finite elements. To overcome this problem, the substructure method such as that which A.K.Chopra et al. (Ref.1) have recently presented for the analysis of gravity dams may be effective. But for the analysis of circular cylindrical tanks, as the assumed displacement functions could approximate sufficiently the actual displacement mode because of simplicity of the shape, the dynamic interaction forces would be obtained explicitly as the impedance functions to tanks.

Rayleigh-Ritz approach to liquid-tank-soil problem is able to present the interaction forces as generalized forces impedance functions, which are obtained in the explicit forms using the assumed displacement functions.

The principle assumptions adopted here are as follows:

1. Linear elastic shell theory and linear flow theory are assumed.
2. The foundation is considered as an elastic halfspace.
3. Rocking motion rotates about a horizontal axis perpendicular to the plane of vibration and the motion is infinitesimal.
4. Internal liquid is ideal fluid.

GOVERNING EQUATIONS

System Considered

The model treated in this paper is made up of liquid-circular

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cylindrical shell-a rigid circular disk footing system in Fig.1. Using circular cylindrical coordinate system (x, ϑ, r), the displacements are represented by u, v, w. It is assumed that the footing of the shell is translated by the free-field horizontal ground displacement $Z_g(t)$ which yield the sway displacement $Z_0(t)$ and the rocking rotation $\psi(t)$ of the footings. The interaction forces between the footing and the elastic halfspace are presented by the base shear V(t) and moment M(t).

The equations expressing dynamic equilibrium of the shell are expressed by the Lagrange's equations as

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\eta}_q}+\frac{\partial S}{\partial \eta_q}+\frac{\partial F}{\partial \dot{\eta}_q}=N_q+N_{Dq}, \quad (q=1,2,\cdots,3N) \tag{1.a}$$

in which $\eta_q(t)$ is the generalized coordinate of the shell. In addition, the two equations expressing the equilibrium of the footing in translation and rotation are

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{Z}_0}+V(t)=N_0,$$
$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\psi}}+M(t)=N_R, \tag{1.b,c}$$

In these equations, T=Ts + Tb is the sum of the kinematic energies of the shell and the footing which are expressed as the functions of the total displacements of the shell and the displacements of the footing, respectively. S is the strain energy of the shell derived from the elastic displacements and F is the disspation function due to viscous damping of the shell. Nq ,No and N_R are the generalized forces to the shell which are derived from the internal liquid pressure. N_{Dq} is the generalized damping force derived from the internal liquid pressure.

Displacements of The System

The total displacements of the shell with respect to a fixed axis u^t, v^t, w^t are expressed as the sum of the elastic displacements of the shell u(x,t),v(x,t),w(x,t), the two interaction displacements at the base and the free field ground motion:

$$u^t = \cos\vartheta\{u(x,t) - a\Psi(t)\},$$
$$v^t = \sin\vartheta\{v(x,t) - x\Psi(t) - Z_0(t) - Z_g(t)\},$$
$$w^t = \cos\vartheta\{w(x,t) + x\Psi(t) + Z_0(t) + Z_g(t)\}. \tag{2.a-c}$$

where a is the radius of the shell, the first terms of the braces are the elastic displacements of the shell. Based on Rayleigh-Ritz method, these displacements can be expressed as

$$u(x,t) = \sum_{n=1}^{N} f_n'(x)C_n(t)$$
$$v(x,t) = \sum_{n=1}^{N} f_n(x)B_n(t)$$
$$w(x,t) = \sum_{n=1}^{N} f_n(x)A_n(t) \tag{3.a-c}$$

in which $f_n(x)$ is the flexural vibration mode of a cantilevered beam, f'n(x)=dfn/dx and Cn(t), Bn(t), An(t), $(n=1,2,\cdots,N)$ are generalized coordinates corresponding to $\eta_q(t)$, $(q=1,2,\cdots,3N)$. The displacements of the footings are given by

$$u_b = -\cos\theta r\psi(t),$$
$$v_b = -\sin\theta(Z_0(t)+Z_g(t)),$$
$$w_b = \cos\theta(Z_0(t)+Z_g(t)). \tag{4.a-c}$$

Frequency Response Functions

For harmonic ground acceleration $\ddot{Z}g(t)=e^{i\omega t}$, the generalized displacements and forces can be expressed in terms of their complex frequency responses; $\eta_q(t) = \overline{\eta}_q(\omega)e^{i\omega t}$, $Zo(t) = \overline{Z}o(\omega)e^{i\omega t}$, $\psi(t) = \overline{\psi}(\omega)e^{i\omega t}$, $V(t) = \overline{V}(\omega)e^{i\omega t}$, $M(t) = \overline{M}(\omega)e^{i\omega t}$, and internal hydrodynamic pressure is $P(x,r,\vartheta,t) = \overline{P}(x,r,\vartheta,\omega)e^{i\omega t}$.

$\overline{V}(\omega)$, $\overline{M}(\omega)$ may be obtained from the complex stiffnesses of the rigid circular disk on an elastic halfspace for the case of shear or dilatational waves propagating vertically. In this paper, using the results obtained by Veletsos and Wei (Ref.2) and neglecting the effects of cross terms of stiffnesses, V(t) and M(t) may be expressed as

$$V = K_0 k_0(i\omega)Z_0 ,$$
$$M = K_R k_R(i\omega)\psi , \qquad (5.a,b)$$

where $Ko = 8\rho_g V_s^2 a_b/(2-\nu_g)$, $K_R = 8\rho_g V_s^2 a_b^3/3(1-\nu_g)$ are the static stiffnesses of the disk, ρ_g, ν_g are, respectively, the mass density and Poisson's ratio of material of the halfspace and Vs is the shear velocity in the halfspace.

Generalized Forces of Liquid

Representing the boundary value problem between the liquid and the shell by the pressure P, the following relations are given

$$\nabla^2\overline{P}(x,r,\vartheta,\omega) = \frac{\partial^2}{\partial x^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\vartheta^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}\ \overline{P} = 0 , \qquad (6)$$

$$\frac{\partial\overline{P}}{\partial\vartheta}(x,r,0,\omega) = \frac{\partial\overline{P}}{\partial\vartheta}(x,r,\pi,\omega) = 0 , \quad \left|\frac{\partial\overline{P}}{\partial r}(x,0,\vartheta,\omega)\right| < M ,$$
$$\frac{\partial\overline{P}}{\partial x}(0,r,\vartheta,\omega) = -\cos\vartheta\, r\ddot{\overline{\Psi}}(\omega)$$
$$\frac{\partial\overline{P}}{\partial r}(x,a,\vartheta,\omega) = \ddot{\overline{w}}^t = \cos\vartheta\{\ddot{\overline{w}}(x,\omega) + x\ddot{\overline{\Psi}}(\omega) + \ddot{\overline{Z}}_0(\omega) + 1\}$$
$$\ddot{\overline{P}}(h,r,\vartheta,\omega) + i\mu\dot{\overline{P}}(h,r,\vartheta,\omega) + g\frac{\partial\overline{P}}{\partial x}(h,r,\vartheta,\omega) = 0 \qquad (7.a\text{-}e)$$

where M is constant, μ is the damping coefficient of the free surface due to sloshing, g is the acceleration of gravity, h is the height of free surface.

Because the governing equation as well as the boundary conditions are linear, $\overline{P}$ is able to be expressed by the superposition of the complex frequency response functions $\overline{P}n(x,r,\omega)$, $\overline{P}o(x,r,\omega)$ and $\overline{P}r(x,r,\omega)$ which are associated respectively with the elastic response function $\ddot{\overline{w}}(x,\omega)$, translational response function $\ddot{\overline{Z}}o(x,\omega)+1$ and rocking response function $\ddot{\overline{\psi}}(x,\omega)$ and becomes

$$\overline{P} = -\rho_L a\cos\vartheta\left\{\sum_{n=1}^{N}\overline{P}_n(x,r,\omega)\ddot{\overline{A}}_n(\omega) + \overline{P}_0(x,r,\omega)[\ddot{\overline{Z}}_0(\omega) + 1] + \overline{P}_R(x,r,\omega)\ddot{\overline{\Psi}}(\omega)\right\} \qquad (8)$$

The solutions of these boundary value problems are obtained as (Ref.3,4).

$$\overline{P}_n(x,r,\omega) = \sum_{k=0}^{K} D_{1k}R_k(r)\cos(\xi_k x) - \sum_{j=1}^{J} H_j(i\omega)\frac{E_{1j}}{\cosh(\lambda_{1j}\frac{h}{a})}J_1(\lambda_{1j}\frac{r}{a})\cosh(\lambda_{1j}\frac{x}{a})$$

$$\overline{P}_0(x,r,\omega) = \frac{r}{a} - \sum_{j=1}^{J} H_j(i\omega)\frac{2J_1(\lambda_{1j}\frac{r}{a})}{(\lambda_{1j}^2 - 1)J_1(\lambda_{1j})}\frac{\cosh(\lambda_{1j}\frac{x}{a})}{\cosh(\lambda_{1j}\frac{h}{a})} , \quad H_j(i\omega) = \frac{-\omega^2 + 2i\xi_j\omega}{\omega_j^2 + 2i\xi\omega_j\omega - \omega^2} ,$$

$$\overline{P}_R(x,r,\omega) = a\left[\sum_{j=1}^{J} G_j J_1(\lambda_{1j}\frac{r}{a})\left\{\sinh(\lambda_{1j}\frac{x}{a}) - H_j(i\omega)\tanh(\lambda_{1j}\frac{h}{a})\cosh(\lambda_{1j}\frac{x}{a})\right\}, \quad \xi_i = \mu/2\omega_i ,\right.$$
$$+ \sum_{j=1}^{J} J_1(\lambda_{1j}\frac{r}{a})(H_j(i\omega) - 1).$$

$$\left\{\frac{\cosh(\lambda_{1i}\frac{h}{a}) + 1}{\sinh(\lambda_{1i}\frac{h}{a})}G_i + \frac{2}{\lambda_{1i}^2(\lambda_{1i}^2 - 1)J_1(\lambda_{1i})}\right\} \cosh(\lambda_{1i}\frac{x}{a}) + \sum_{k=1}^{K} F_k R_k(r)\sin(\xi_k x) \Big] \tag{9.a-c}$$

in which ρ_L is the mass density of liquid; J_1 ,I_1 are, respectively, Bessel function and modified Bessel function of the 1st kind; ω_j, ξ_j are, respectively, circular frequency and damping constant of the j-th liquid sloshing.

When the effects of the waves at the free surface of liquid are neglected, the solutions are obtained by substituting $\omega_j = 0$ in these equations.

The generalized forces of the liquid pressure can be derived from the virtual work δWp. δWp is expressed in the constrained coordinates

$$\delta W_P = \int_0^{2\pi}\int_0^{h} P(x,a,\theta,t)\delta w' a dx d\theta + \int_0^{2\pi}\int_0^{a} P(0,r,\theta,t)\delta \chi a dr d\theta, \tag{10}$$

where dw' is the virtual displacement of w' and $\delta\chi = r\cos\delta\psi$ is the virtual rotational displacement of the footing. δWp can also be expressed in the generalized coordinate as

$$\delta W_P = \sum_{q=1}^{N'} N_q \delta\eta_q + N_0 \delta Z_0 + N_R \delta\psi. \tag{11}$$

Letting equation (10) be the equal of equation (11), the generalized forces of liquid are obtained. The generalized damping force N_{Dq} is able to be derived from the virtual work related to a damping force P which may be assumed to be proportional to the elastic equivalent added mass $\overline{M}^n = -\overline{P}_n(x,r,\omega)/\rho_s h_s$.

$$N_{Dq} = 2\alpha\cos\vartheta \sum_{n=1}^{N} \overline{P}_n(x,r,\omega)\overline{A}_n(\omega) \tag{12}$$

Equation of Motion For The System

Substituting the above displacements and forces in the equations (1.a-c), the equations of motion then becomes

$$[M_s]\{\ddot{Q}\} + [C_s]\{\dot{Q}\} + [k]\{Q\} + \{\Gamma_{s0}\}(\ddot{\overline{Z}}_0 + 1) + \{\Gamma_{sR}\}\ddot{\overline{\Psi}} = \{N_q\} + \{N_{Dq}\}$$

$$\{\Gamma_{s0}\}^T\{\ddot{Q}\} + \Gamma_{b0}(\ddot{\overline{Z}}_0 + 1) + \Gamma_{b0R}\ddot{\overline{\Psi}} + \frac{\overline{V}}{\rho_s h_s \pi a} = N_0$$

$$\{\Gamma_{sR}\}^T\{\ddot{Q}\} + \Gamma_{b0R}(\ddot{\overline{Z}}_0 + 1) + \Gamma_{bR}\ddot{\overline{\Psi}} + \frac{\overline{M}}{\rho_s h_s \pi a} = N_R \tag{13.a-c}$$

in which $\{Q\} = [A_1,\dots,A_N, B_1,\dots,B_N, C_1,\dots,C_N]$, $[Ms]$, $[Cs]$, $[K]$ are the mass, damping and stiffness matrices of the shell, $\{\Gamma_{s0}\}$, $\{\Gamma_{sR}\}$ are the effective load coefficient vectors, Γ_{b0}, Γ_{bR} are respectively the total mass and the total moment of inertia associated with the shell and the footing and Γ_{b0R} is their cross term.

The equations (13.a-c) is a set of 3N+2 algebraic equations. If the normal modes of an associated liquid-shell system on an rigid base are used in the elastic displacements, equations(3.a-c), the number of unknown are reduced. However, on account of sloshing at the free surface of liquid, the repeated analysis of free vibration would be required for many values of excitation frequencies.

Effects of Free Surface

Then, for the examination of the effects of sloshing to the system, numerical examples are shown in Fig.2.a-b. But for the simplicities of calculations, the tanks are assumed to be rigid in these equations.

Table 1 shows the parametric values of the model which has the height ratio to radius L/a=3.849. Fig.2a considers the effects of sloshing and Fig.2b ignores the effects. In Fig.2.a, the peak at ω=1.81 rad/sec is shown corresponding to 1st mode of sloshing and there are two other small peaks.

These peaks are excited at the same frequency with those in Fig.2.b. and the magnitudes are almost equal, that is, the effects of coupling between the impulsive pressure and convective pressure are negligible in this rigid tank-soil system.

SYSTEM IGNORING WAVES ON FREE SURFACE

Fundamental Modes

If the effects of free surface of liquid are ignored, the equations of motion will be reduced by Ritz concept in Ref. 5 shown by Chopra et al. for the interaction problems between buildings and foundations. In this study, the Ritz vectors of the system are obtained by the analysis of shell including liquid on rigid foundation in which the effects of sloshing are ignored.

The equations of free vibration are expressed as

$$[K]\{Q\} = \Delta[[M_S] + [M_L]]\{Q\} \tag{14}$$

in which Δ , $[M_L]$ are nondimensional natural frequency and the added mass matrix of the liquid-shell system. The orthogonality between the modes is represented as

$$\{Q^{(n)}\}^T[K]\{Q^{(n)}\} = \Delta_n\{Q^{(n)}\}^T[M_s+M_L]\{Q^{(n)}\}$$
$$= \Delta_n[M_S^{(n)}+M_L^{(n)}], \tag{15}$$

in which $[M_S^{(n)}]$, $[M_L^{(n)}]$ are respectively generalized mass of shell and liquid.

Reduced Equation of Motion

For the system ignoring the effects of sloshing, the displacements are expressed as

$$u(x,t) = \sum_{n=1}^{N'} \eta_n(t)u_n(x)$$
$$v(x,t) = \sum_{n=1}^{N'} \eta_n(t)v_n(x)$$
$$w(x,t) = \sum_{n=1}^{N'} \eta_n(t)w_n(x) \tag{16.a-c}$$

where $u_n(x)$, $v_n(x)$, $w_n(x)$ are the fundamental modes of equation (15) and $N \leq N'$. Substituting the equation (16.a-c) in equation (2.a-c), and considering $\omega_i = 0$ in equations (6)-(10), the generalized forces of hydrodynamic pressures are expressed as

$$N_n = -\rho_s h_s a\pi[[M_L^{(n)}]\ddot{\bar{\eta}}_n + \Gamma_{30}^{(n)}(\ddot{\bar{Z}}_0 + \ddot{\bar{Z}}_g) + \Gamma_{3R}^{(n)}\ddot{\bar{\Psi}}], \quad (n=1,2,\ldots N')$$
$$N_0 = -\rho_s h_s a\pi\left[\sum_{q=1}^{N'}\Gamma_{03}^{(q)}\ddot{\bar{\eta}}_n + \Gamma_{00}(\ddot{\bar{Z}}_0 + \ddot{\bar{Z}}_g) + \Gamma_{0R}\ddot{\bar{\Psi}}\right],$$
$$N_R = -\rho_s h_s a\pi\left[\sum_{q=1}^{N'}\Gamma_{R3}^{(q)}\ddot{\bar{\eta}}_n + \Gamma_{R0}(\ddot{\bar{Z}}_0 + \ddot{\bar{Z}}_g) + \Gamma_{RR}\ddot{\bar{\Psi}}\right]. \tag{17.a-c}$$

where

$$[M_L^{(n)}] = \beta\int_0^h P_n(x,a)w_q(x)dx, \quad \beta = \rho_L a/\rho_S h_S,$$

$$\Gamma_{30}^{(n)} = \Gamma_{03}^{(n)} = \frac{1}{2}\beta\left\{\int_0^h P_0(x,a)w_q(x)dx + \int_0^h P_n(x,a)dx\right\},$$

$$\Gamma_{3R}^{(n)} = \Gamma_{R3}^{(n)} = \frac{1}{2}\beta\left\{\int_0^h P_R(x,a)w_q(x)dx + \int_0^h P_n(x,a)xdx + \int_0^a P_n(0,r)r^2dr\right\},$$

$$\Gamma_{0R} = \Gamma_{R0} = \frac{1}{2}\beta\left\{\int_0^h P_R(x,a)dx + \int_0^h P_0(x,a)xdx + \int_0^a P_0(0,r)r^2dr\right\},$$

$$\Gamma_{00} = \beta\int_0^h P_0(x,a)dx, \quad \Gamma_{RR} = \beta\left\{\int_0^h P_R(x,a)xdx + \int_0^a P_R(0,r)r^2dr\right\}.$$

In these equations, β is density ratio of liquid to shell, Γ_{30}, Γ_{03} are the generalized loads of liquid due to the interaction between the translational and elastic displacements, Γ_{3R}, Γ_{R3} are due to the interaction between the rocking and elastic displacements and Γ_{0R}, Γ_{R0} are due to the the interaction between the translational and rocking displacements, which are almost equal each other, respectively. Then, in the above equations they have been respectively expressed as half of the sum.

Substituting equations (16), (17) into equation (1) and using the orthogonality condition, equation (15), the equations of motion are expressed as

$$[M_S^{(n)}+M_L^{(n)}]S_n(i\omega)\overline{\eta}_n - \omega^2(\Gamma_{S0}^{(n)}+\Gamma_{30}^{(n)})\overline{Z}_0 - \omega^2(\Gamma_{SR}^{(n)}+\Gamma_{3R}^{(n)})\overline{\Psi} = -(\Gamma_{S0}^{(n)}+\Gamma_{30}^{(n)}), \quad (n=1,2,\ldots,N')$$

$$-\omega^2\sum_{n=1}^{N'}(\Gamma_{S0}^{(n)}+\Gamma_{30}^{(n)})\overline{\eta}_n - \omega^2(\Gamma_{b0}+\Gamma_{00})\overline{Z}_0 - \omega^2(\Gamma_{b0R}+\Gamma_{0R})\overline{\Psi} + \frac{K_{VV}(i\omega)}{\rho_S h_S \pi a}\overline{Z}_0 = -(\Gamma_{b0}+\Gamma_{00})$$

$$-\omega^2\sum_{n=1}^{N'}(\Gamma_{SR}^{(n)}+\Gamma_{3R}^{(n)})\overline{\eta}_n - \omega^2(\Gamma_{b0R}+\Gamma_{0R})\overline{Z}_0 - \omega^2(\Gamma_{bR}+\Gamma_{RR})\overline{\Psi} + \frac{K_{MM}(i\omega)}{\rho_S h_S \pi a}\overline{\Psi} = -(\Gamma_{b0R}+\Gamma_{0R}) \qquad (18.a\text{-}c)$$

in which

$$\Gamma_{S0}^{(q)} = \int_0^L(-v_q+w_q)dx\,, \quad \Gamma_{SR}^{(q)} = \int_0^L(-au_q-xv_q+xw_q)dx,$$

$$\Gamma_{b0} = 2\left(\int_0^L dx+\beta\int_0^{a_b} r\,dr\right) \quad \Gamma_{b0R} = 2\int_0^L x\,dx\,, \qquad \Gamma_{bR} = \int_0^L(a^2+2x^2)dx+\beta\int_0^{a_b} r^3 dr.$$

These equations of liquid-shell-soil system are represented by $N'+2$ generalized coordinates.

The complex frequency responses for any excitation frequency are decided by the equation (18.a-c) and the responses to arbitrary ground acceleration $\ddot{Z}g(t)$ can be FFT algorithm.

NUMERICAL ANALYSIS

To show the effectiveness of this procedure, responses of liquid-tank-soil system are presented for the model as shown in Table 1. For the assumption of rigid circular disk footing, the numerical model is ristricted to a tall tank. The material properties of the halfspace are assumed for two cases; one case has the shear velocity Vs=400m/sec, and another case has Vs=150m/sec.

Frequency Response

Fig.3.a-c show the frequency response accelerations of the shell. In Fig.3.a, the elastic response $\ddot{w}$ decrease when Vs becomes small, but in Fig. 3.b-c, the rocking responses at the top of the shell wall and translating responses show the contrary results with $\ddot{w}$.

In Fig.4, the responses using the modal analysis are shown, where the system has the static foundation stiffness. The results in Fig.3.a and Fig.4 are similar to each other.

Response to Arbitrary Ground Motion

In Fig.5-6, the responses to the ground motion recorded at the EL-

CENTRO(NS) 1940 are presented based on the above frequency responses. In Fig.5a-b, the total accelerations of the shell are compared with the results for fixed rigid foundation. While the effects of the shear wave velocity are not brought out, the responses increase compared to fixed case. This may be explained as the increments of rocking response are greater than the reductions of the elastic response for considering the radiation damping. The rocking effects are noticeable compared to the translating effects of the foundation. Because $\ddot{v}^t$, $\ddot{w}^t$ are almost same values in these figures, the distortions of circular section are little.

In Fig.6.a-b, the response pressures to the above ground motion are shown. In these figures, Pr,Pe show the impulsive pressure produced by the rigid motion, i.e. $(\ddot{Z}o + \ddot{Z}g)$, $\ddot{\psi}$, and the elasic motion, i.e. $\ddot{w}$, respectively, and the total impulsive pressure P is shown as the root mean square of these pressures. The pressure indicated by (FIX) is the total impulsive pressure P for fixed rigid foundation. In Fig.6.a, where Vs=400m/sec, Pr is almost the same value with Pe. However, in Fig.6.b, where Vs=150m/sec, Pe is considerably small compared to Pr. This may be explained as the effect of radiation damping decreases the elastic pressure, but increases the pressure due to rocking motion. Consequently, the total pressure P becomes greater than the pressure P(Fix).

CONCLUSIONS

Basesd on Rayleigh-Ritz method, a procedure for complex frequency response analysis of the liquid-tank-soil system including sloshing at the free surface of liquid has been presented, in which the interaction forces of the system are considered as the generalized forces. This procedure reduces the freedoms compared with F.E.M. and for the case of neglecting the sloshing, the number of generalized coordinate are more reduced.

Numerical examples show that the effectiveness of radiation damping decreases the elastic responses but increases the rocking and translating responses and total acceleration consequently increases compared to the case of fixed rigid foundation.

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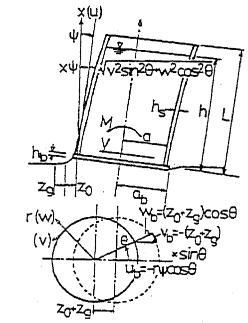

Fig.1 Shell Model.

Table 1. Numerical examples of model.

MODEL	a(cm)	ℓ(cm)	h_s(cm)	h/ℓ	h_b/a	a_b/a
1	550.0	2117.	1.500	0.9074	0.0727	1.05

$E=2.1\text{x}10^{6}$ (kg/cm^2) , $\nu=0.3$
$\rho_s=8.163\text{x}10^{-6}$ $(kg.sec^2/cm^4)$, $\nu_g=0.45$
$\rho_L=1.020\text{x}10^{-6}$ $(kg.sec^2/cm^4)$, $\rho_g=1.837\text{x}10^{-6}(kg.sec^2/cm^4)$
$\rho_b=2.449\text{x}10^{-6}$ $(kg.sec^2/cm^4)$,

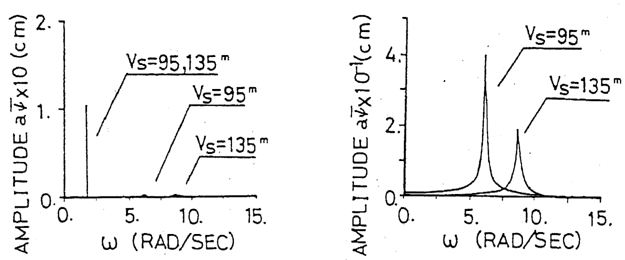

Fig.2a-b Frequency Responses
(a-Consider Sloshing, b-Ignore Sloshing)

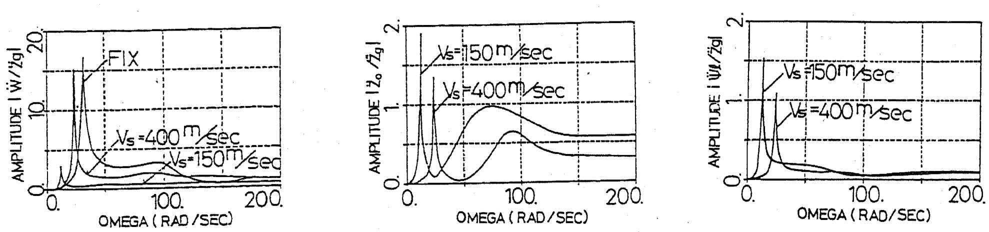

Fig.3a-c Frequency Responses (x=L).

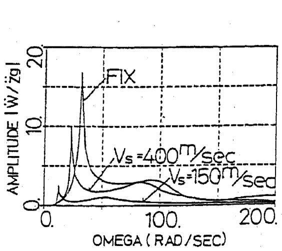

Fig.4 Frequency Response (x=L).

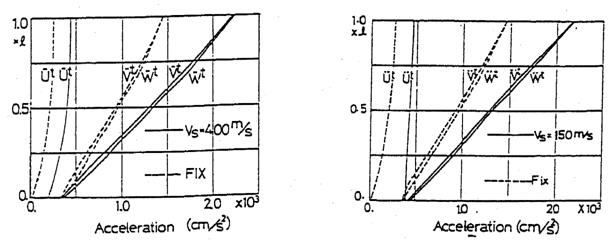

Fig.5a-b RMS.Response Accelerations of Shell

Appendix of Eqs.(9)

$$\omega_1^2 = g\lambda_{j1}\tanh(\lambda_{11}\tfrac{h}{a})/a \quad , \quad \frac{\partial J_1(\lambda_{11})}{\partial X} = 0$$

$$G_1 = \frac{2}{\lambda_{11}(\lambda_{11}^2-1)J_1(\lambda_{11})}\left\{\sum_{k=1}^{K}(-1)^k\frac{2\lambda_{11}^2}{(\xi_k a)^2+\lambda_{11}^2} - 1\right\},$$

$$F_k = (-1)^{k+1}\frac{2}{\xi_k a},$$

$$D_{1k} = \frac{2}{h(1+\delta_{0k})}\int_0^h w_n(x)\cos(\xi_k x)dx \ , \ R_k(r) = \frac{I_1(\xi_k r)}{\xi_k a\frac{\partial I_1(\xi_k a)}{\partial r}},$$

$$E_{11} = \frac{2\lambda_{11}^2}{(\lambda_{11}^2-1)J_1(\lambda_{11})}\sum_{k=0}^{K}(-1)^k\frac{D_{1k}}{(\xi_k a)^2+\lambda_{11}^2}, \quad \xi_k = \frac{k\pi}{h}.$$

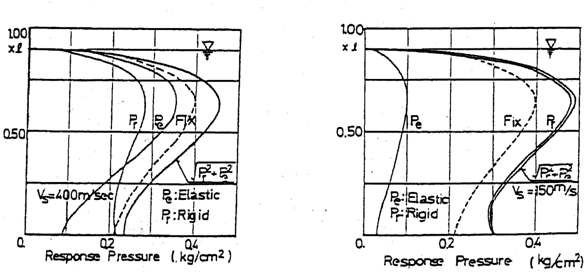

Fig.6a-b Hydrodynamic Wall Pressures