SOIL FOUNDATION STRUCTURE INTERACTION
OF FRAMED STRUCTURES

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SUMMARY

Analysis of a framed structure as an integrated system consists of three media: soil, foundation, and super structure is presented. Soil is modeled as an elastic half space - EHS, then its stiffness matrix is developed. Stiffness matrices of the three media are utilized to formulate the global stiffness matrix of the entire system. Using the advanced direct method, the governing matrix equation is written. Tractions, stresses, and displacements at nodal points and in structural elements are calculated by means of a computer program. Effect of shear wall panels may be considered. Vertical and seismic forces based on equivalent static loads are applied. Interaction of three media is illustrated by an example problem considering the variation of their stiffnesses.

INTRODUCTION

The problem of soil structure interaction has been a great stimulant for design engineers during the last four decades. Simulation of soil by discrete, elastic, and vertical springs (Winkler Model) has been used. Recently, this model has been improved considering the cohesion of soil (Ref.1). Another approach of soil modeling is to represent the soil as an elastic half space body-EHS. This model was used by DeBeer (Ref.2), and Cheung and Zeinkiewicz (Ref.3).

Winkler's model is based on the assumption that the reaction pressure at the soil-foundation interface is proportional to the displacement at the contact surface. This assumption implies a complete lack of continuity in soil. It is concluded that such concept does not realize the interaction. On the other hand, the EHS model which considers the soil properties was successfully applied in the analysis of plates on elastic foundation (Ref.3). EHS model which realizes the soil-foundation interaction better than Winkler's model is utilized in this study.

This paper presents a study on soil-foundation-structure interaction of two-dimensional framed structures-SFSI2D. It is part of a research project initiated at Yarmouk University. The major objective is to re-examine the available methods of analysis and to develop methods of solution to this problem under static, dynamic, and seismic loads. The study in this paper employs the direct stiffness method where the finite element method is used for soil, foundation, and shear panels and classical stiffness method for structural members such as beams and

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columns. The calculations have been carried out by means of a computer program. SFSI2D is illustrated using an example problem where stiffness variants of the three components of the integrated system are considered.

METHOD OF ANALYSIS

The matrix displacement or the direct method that uses the stiffness properties of various elements of the structural system that includes the EHS body of soil is employed. A set of simultaneous equations relating displacements to loads at the nodal points expressed in the governing matrix equation of the form

$$\{Q\}=[K]\{D\} \tag{1}$$

The values of stiffness influence elements in the global stiffness matrix reflect the cumulative stiffnesses of the elements meeting at nodal points. The direct method employes the frame analysis for the super structure and finite element technique for both the foundation, shear wall panels, and EHS soil body. Equilibrium and compatibility conditions are enforced in developing the governing matrix equation.

Stiffness Matrices

The stiffness matrices of a structural element (or member) considering both axial and bending stiffnesses, a rectangular plane-stress element representing the shear wall panels, and a uniaxial finite element representing the foundation structure have been used and can be found in many texts of matrix analysis of structures. Transformation matrices have also been used to transform from local to global co-ordinate axes in which the overall stiffness matrix is developed.

Stiffness Matrix of Soil

The soil is treated as an elastic, homogeneous, isotropic body extending indefinately below the foundation, i.e. modeled as an elastic half space-EHS, then the vertical displacements at a point, n, within the half space (having co-ordinates r and Θ) or at a point, m, due to a point load, Q_o, as shown in Fig. 1 can be expressed by the following expressions (Ref.4):

$$\delta_{no}=\frac{Q_o}{2\pi r}\,\frac{1+\nu}{E}\{\cos^2\Theta+2(1-\nu)\}\sin\Theta \tag{2}$$

$$\delta_{mo}=\frac{Q_o}{2\pi r}\,\frac{1-\nu^2}{E} \tag{3}$$

The flexibility influence coefficients, f_{ij}, for a rectangular area as shown in Fig.2 can be formulated and expressed as follows:

$$f_{ij}=\frac{(1-\nu^2)}{\pi.E.X_{ij}}\,;\ i\neq j \tag{4}$$

$$f_{ij}=\frac{2(1-\nu^2)}{a_o\pi.E}\,C_{ij};\ i=j \tag{5}$$

where $X_{ij}=X_j-X_i$ and

$$C_{ij}=\ell n\{\alpha+\sqrt{\alpha^2+1}+\alpha.\ell n\{\beta+\sqrt{\beta^2+1}\} \qquad (6)$$

in which $\alpha= a/b$, and $\beta= b/a$

The stiffness matrix of soil is obtained by inverting the flexibility matrix. A computer subroutine used in the computer program is given below.

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C WORKING SUBROUTINE TO DEVELOP STIFFNESS MATRIX OF SOIL : EHS MODEL
      SUBROUTINE KSOIL (F,ENU,E,A,A1,B,N,X)
      DIMENSION F(N,N), X (N)
      PI=ATAN (1.0)*4.0
      C=(1.0-ENU*ENU)/PI/E
      DO 1 I=1,N-1
      DO 1 J=I+1,N
      F(I,J)=C/(X(J)-X(I))
      DO 2 I=1,N
1     IF(I.EQ.1.OR.I.EQ.N)A=A1
      ALFA=A/B
      ALFA2=ALFA*ALFA
      BETA=1.0/ALFA
      BETA2=BETA*BETA
      CIJ=LOG(ALFA+SQRT(ALFA2+1.)+ALFA*LOG(BETA+SQRT(BETA2+1.))
2     F(I,I)=2*C*CIJ/A
      CALL MATINV(F,N,IPIVOT,INDEX,PIVOT)
      RETURN
      END
```

EXAMPLE

A four story, reinforced concrete, framed building is considered. The scheme is shown in Fig.3. The structure is calssified as regular (Ref.5).

Properties: $E_c=30.5GP_a, \nu_c=0.2, E=\text{variable}, \nu=0.3$

Loads/Floor: D=1950 KN; L=1125 KN

Loading cases: Vertical and seismic combinations.

The natural period, T=0.605 sec.(Ref.6) and the vibration modes are shown in Fig.4.

Design Seismic Forces: $F_i=C_d.\gamma_i.W_i$ where

$$C_d=I.A.S.\alpha.(T2/T)^{\beta}.\frac{1}{k};\gamma_i=\frac{h_i\Sigma W_i}{\Sigma W_i h_i}$$

Contact pressures at the soil-foundation interface for two cases of loading (vertical and seismic) are graphically shown in Figs.6 and 7. Code-numbers for nodal points and elements are shown in Fig.5.

PARAMETRIC STUDY

The soil-foundation-structure interaction is illustrated by considering stiffness variants of soil, foundation, and super structure. The stiffness of soil varies with its modulus of elasticity; the foundation stiffness varies with its thickness and modulus of elasticity of concrete, and the stiffness of the super structure changes according to whether we have shear-wall panels or we vary the number of floors. This parametric study has incorporated these stiffness variants. The composite behaviour of the foundation and the subgrading soil may be expressed by the modular rigidity, μ, as nondimensional quantity which is equal to $nt^3/12$; $n=E_c/E$.

From the plots of Figs. 6 and 7, and the results of this study, the following observations can be made:

1) For soft days, i.e. for low values of modulus of elasticity of soil, the soil pressure distribution at the contact surface tends to be uniform as the classical solutions except at the free edges where the soil exhibits very high pressure values which require special attention in the design procedure.

2) For medium stiff and very stiff soils, the soil pressure distribution exhibits high values beneath the column boads. Stresses at the free ends and between the columns decrease.

3) The shape of soil pressure distribution is not altered by increasing the number of stories or having shear walls.

4) For a value of μ approximately greater or equal to 25, a uniform pressure distribution can be obtained; as a result, the soil foundation interaction depends mainly on the relative stiffness between the foundation and its subgrading soil.

5) It was also observed that stresses under the column positions are nearly equal to twice of the average values in classical design.

6) For seismic forces based on equivalent static loading, numerical results of soil pressure may have tensile stresses which is not realistic, however, such finding should be considered by the design engineer. Horizontal displacements increase rapidly with the decrease of the modulus of elasticity of soil. These observations are limited to the given example and restricted to static loads.

7) Shear wall panels influence the distribution of tractions in the members of the super structure. Moreover, the values of drift decreased five times relative to the same structure without shear walls.

CONCLUSIONS

Analysis of framed structures considering the soil as part of the integrated structural system was presented. Simulation of soil by EHS model realized the soil-foundation interaction. Soil pressures, tractions and displacements were easily computed.

As a static design approach, the method of analysis could be satisfactory for up to 20 story buildings. Design engineers should avoid using cantilever parts in the foundation. Composite behaviour of the entire structural system could be adjusted to exhibit soil pressure given by trivial analysis, simply by keeping the modular rigidity equal to 25. Effect of shear wall panels are easily considered. The presented study could easily be extended to consider isolated footings as well as pile foundations.

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NOTATION

$\{Q\},\{D\}$ = Column vectors of external nodal forces and displacements, respectively.
$[K]$ = Global stiffness matrix.
E, E_c = Modulii of elasticity of soil and concrete, respectively.
ν, ν_c = Poisson's ratios of soil and concrete respectively.
D,L = Dead and Live Loads.
W_i = Total gravity load at floor i.
F_i = Horizontal design force at floor i.
C_d = Design Seismic Coefficient.
γ_i = Distribution Factor.
I = Importance Factor.
A = Peak Ground acceleration, (=0.3g).

S = Site Coefficient.
α = Spectral Amplification Factor.
T_2 = Transition Period and Equal to 0.4 sec.
T = Natural Period.
β = Parameter of the Elastic Response Spectrum.
k = Behaviour Factor.
h_i = Height of a floor i.
t = Thickness of foundation slab(raft).
μ = Modular Rigidity and Equal to $nt^3/12$.
n = Modular Ratio and Equal to E_c/E.

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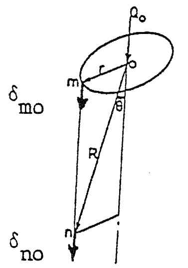


Figure 1. Vertical Displacements at Points m and n due to a Point Load Q_o.

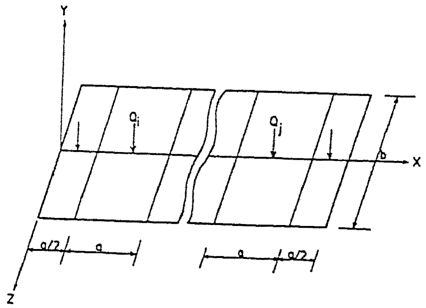


Figure 2. Point Loads on Rectangular Areas.

Vertical Loading: $Q=360$ kN
$F_1=F_2=F_3=F_4=0$
Seismic Loading: $Q=190$ kN
$F_1=14.67$ kN
$F_2=25.67$ kN
$F_3=36.67$ kN
$F_4=47.67$ kN
Dimensions in meters.

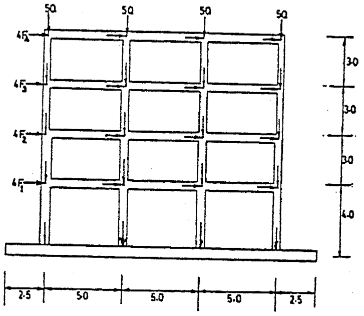
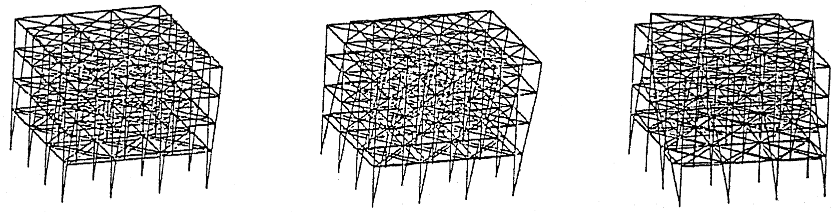


Figure 3. Plane Frame and Loading

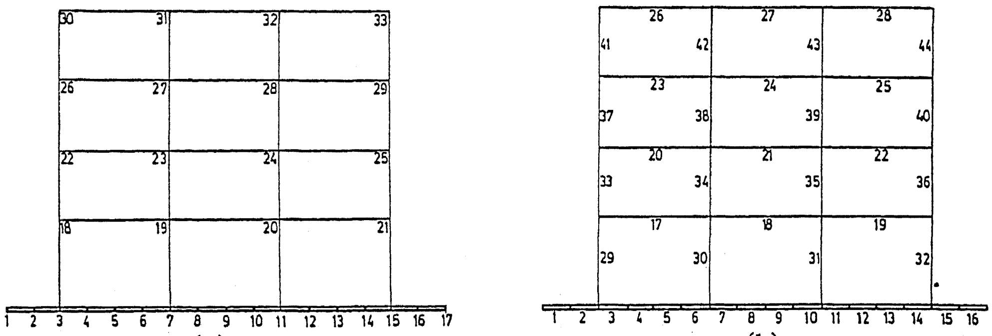


Mode 1
Frequency = 1.65 Hz.

Mode 2
Frequency = 1.65 Hz.

Mode 3
Frequency = 2.08 Hz.

Figure 4. Vibration Modes of the Structure (Ref 6).



(a) (b)

Figure 5. Code-numbers, (a) for joints, (b) for structural elements.

Figure 6. Soil Pressure for Vertical Loading

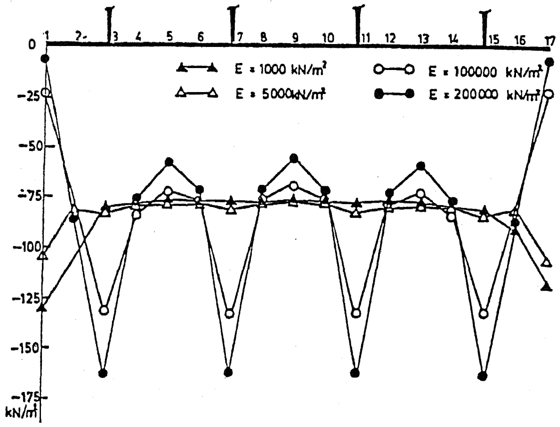


Figure 7. Soil Pressure for Seismic Loading.