EARTHQUAKE RESPONSE OF 3-D FOUNDATIONS BY THE BOUNDARY ELEMENT METHOD

D.L. Karabalis (I)
D.E. Beskos (II)
Presenting Author: D.L. Karabalis

SUMMARY

The earthquake response of three-dimensional rigid, surface or embedded, and flexible surface foundations is numerically obtained. The foundations are of arbitrary shape and are placed on a linear elastic, isotropic and homogeneous half-space which represents the soil medium. The obliquely incident seismic waves have a general transient time variation. The problem is formulated in the time domain by the Boundary Element Method and the response is obtained by a time step-by-step integration. The main advantages of the proposed method are that, unlike frequency domain techniques, it provides directly the transient response and forms the basis for extension to non-linear problems.

INTRODUCTION

The soil-foundation interaction due to seismic wave excitation, plays a key role in the study of the general earthquake response of a structure. Most of the reported research in this area deals with surface rigid foundations subjected to vertically propagating harmonic waves. However, very few investigations have considered the effect of the flexibility and the embedment of three-dimensional foundations and/or the possibility of non-vertically propagating seismic waves. Wong and Luco (Ref. 1) and Luco and Wong (Ref. 2) studied the response of surface, rigid, massless, rectangular foundations to obliquely incident body waves (P, SV and SH) and Rayleigh waves, respectively. Their conclusion was that soil-structure interaction studies should not be limited to seismic waves with vertical incidence because additional modes of vibration, such as torsional or rocking, can be excited under the influence of obliquely incident waves. Iguchi (Ref. 3) was the first to obtain approximate solutions for rectangular flexible plates bearing on the surface of a half-space, under some simplifying assumptions. Recently, Whittaker and Christiano (Ref. 4), succeeded to provide a more general methodology for the solution of the problem of a flexible

(I) Visiting Assistant Professor, The Ohio State University, Columbus, Ohio, USA.

(II) Professor, University of Patras, Patras, Greece.

plate bearing on an elastic half-space and subjected to general seismic wave excitations. The response of three-dimensional rigid embedded foundations to obliquely incident travelling waves was studied by Dominguez (Ref. 5) using the Boundary Element Method (BEM).

It appears that the various existing methods, for the above mentioned soil-foundation interaction problems, provide results directly in the frequency (or Laplace or Fourier transformed) domain, only, and it is assumed that through a second independent step of the solution, the transient response will be obtained by a Fourier synthesis (or inverse transformation). Direct time domain solutions can be obtained by using the Finite Element Method (FEM), but its inferior handling of infinite domains and the large size of the required models are the most severe disadvantages of this, otherwise superb, numerical approach. Thus, hybrid models, providing solutions also in frequency domain, have been reported recently (Ref. 6).

The direct time domain solutions of the soil-foundation interaction problems presented in this work, are the results of a general BEM formulation in time domain (Ref. 7,8). This numerical method, compared to the FEM approach, has the advantage that discretization is required only on the boundary of the domain of interest. Thus, the dimensions of the problem are reduced by one and the radiation condition is takem into account automatically. Compared to frequency domain methods, this time domain approach, besides the fact that it provides the response in one direct step and not two, forms a basis for an extension to non-linear problems in the area of soil-structure interaction. Comparison studies presented in this work and elsewhere, (Ref. 7,8), reveal the accuracy and efficiency of the proposed methodology.

PROBLEM FORMULATION

The boundary integral equation for an elastic body of surface B, subjected to boundary displacements $u_i(\underset{\sim}{x}, t)$ and boundary tractions $t_{(n)i}(\underset{\sim}{x},t)$ can be written, under zero initial conditions and body forces, in the form, (Ref. 9)

$$\frac{1}{2}\, u_k(\underset{\sim}{\zeta},t) = \int_B \left\{ \bar{u}_{ik}\left[\underset{\sim}{x},t;\underset{\sim}{\zeta} \,\middle|\, t_{(\underset{\sim}{n})i}(\underset{\sim}{x},t)\right] - \bar{t}_{(\underset{\sim}{n})ik}\left[\underset{\sim}{x},t;\underset{\sim}{\zeta} \,\middle|\, u_i(\underset{\sim}{x},t)\right]\right\} dB(\underset{\sim}{x}) \quad (1)$$

where $\underset{\sim}{\zeta}$ and $\underset{\sim}{x}$ are points on the boundary B, $\underset{\sim}{n}$ is the unit normal and $\bar{u}_{ik}$, $\bar{t}_{(\underset{\sim}{n})ik}$ consist the Stokes fundamental solution pair expressing the propagation properties of the elastic body under consideration. The solution of Eq. (1) is obtained here for general time and space variations of the related functions, by assuming

i) a discretization in time, where the variation of the boundary tractions and displacements is constant over each time step, Fig. 1, and

ii) a spatial discretization of the boundary into a number of M rectangular elements with a constant variation of tractions and displacements over each one of them, Fig. 2.

Under these assumptions, Eq. (1) written for every boundary element R will produce the following system of linear algebraic equations

$$\frac{1}{2}\left\{u_r^{N,R}\right\} = \sum_{n=q}^{N} \sum_{s=1}^{M} \left[G^{n-q+1,s}\right] \left\{t_r^{N-n+q,s}\right\} - \left[T^{n-q+1,s}\right]\left\{u_r^{N-n+q,s}\right\} \quad (2)$$

$$R=1,\ .\ .\ ,M.$$

where N is the time step at which we seek a solution, q is the time step at which the boundary tractions $t_r^{q,s}$ and displacements $u_r^{q,s}$ were applied on the source element s, and $G^{m,s}$, $T^{m,s}$ are the discretized counterparts of the Stokes fundamental solutions $\bar{u}_{ik}$ and $\bar{t}_{(\underline{n})ik}$, respectively. The singularities of the discretized fundamental solutions, which occur when the source element becomes also a receiver, i.e., when $R \equiv s$ in Eq. (2) , can be treated analytically, (Ref. 7,8). For surface foundations and under the assumption of relaxed boundary conditions a discretization of only the contact surface between the soil and the foundation is required. For the embedded foundation case a few boundary elements outside the contact surface are required in order to obtain an acceptable accuracy.

The compatibility condition on the contact surface between the soil and foundation at the time step q can be written as, (Ref. 10),

$$\left\{u^q\right\} = \left\{u_g^q\right\} + \left\{u_r^q\right\} , \quad (3)$$

where $\{u^q\}$, $\{u_r^q\}$ represent the total and scattered displacement fields and $\{u_g^q\}$ is the discretized in time and space free-field motion. In the rigid foundation case the total displacement field u^q can be expressed by

$$\left\{u^q\right\} = \left[S\right]\left\{D^q\right\} , \quad (4)$$

where $[S]$ is the (3M x 6) rigid displacement transformation matrix, and

$$\left\{D^q\right\} = \left\lfloor \Delta_1^q\ \Delta_2^q\ \Delta_3^q\ \phi_1^q\ \phi_2^q\ \phi_3^q \right\rfloor^T \quad (5)$$

in which Δ_i^q and ϕ_i^q are the rigid body translations and rotations with respect to the coordinate axis i = 1,3. The total displacement field of a flexible foundation can be written, in terms of contact stresses, as

$$\left\{u^q\right\} = \left[K\right]^{-1}\left[A\right]\left\{t^q\right\} \quad (6)$$

where $[K]$ is the stiffness matrix of the foundation, $[A]$ is a diagonal matrix whose element A_{kk} (k = 1,..., M) express the area of the boundary element k and t^q is the vector of the total contact stresses given by

$$\left\{t^q\right\} = \left\{t^q_g\right\} + \left\{t^q_r\right\} \tag{7}$$

where t^q_g and t^q_r are the free-field and scattered traction vectors, respectively. In absence of any externally applied loads, the equilibrium equation can be written as

$$\left[S_A\right]\left\{t^q\right\} = 0 \tag{8}$$

where $\left[S_A\right]$ is the assemblage of the matrices $A_{kk}\left[\bar{S}_k\right]^T$, with $\left[\bar{S}_k\right]$ being the (3x6) rigid displacement matrix of the element k (k=1...M).

Depending on the problem at hand, i.e., rigid or flexible foundation, Eqs. (2), (3), (4), (7), (8) or (2), (3), (6), (7) and (8), respectively, viewed together, form a system of linear algebraic equations with unknowns the unspecified displacements and tractions on the contact area for each time step. The solution of this system of equations is accomplished by a step-by-step time marching, i.e., for N = q, then for N = q+1, etc.

NUMERICAL RESULTS

In absence of any foundation the free-field displacements on the surface of the half-space, i.e. $x_3 = 0$, can be expressed as, (Ref. 1),

$$\left\{u_g(x_1,x_2)\right\} = \left\{s_o\right\}\exp\left[i\,\omega\left(t - \frac{x_1}{c}\sin\theta_H - \frac{x_2}{c}\cos\theta_H\right)\right] \tag{9}$$

Where ω is the frequency of the plane incident wave, $\left\{s_o\right\} = \left[s_{x_1}\ s_{x_2}\ s_{x_3}\right]^T$ is the amplitude vector of the free-field motion u_g at the origin of the coordinate system, θ_H is the horizontal angle of incidence and $c = c_1/\cos\theta_V$ or $c = c_2/\cos\theta_V$, with c_1, c_2 being the compressional and shear wave velocities, respectively, and with θ_V being the vertical angle of incidence, Fig. 3.

In Fig. 4 (a,b,c) the dimensionless response of a square rigid surface massless foundation to an SV incident wave is plotted versus the dimensionless frequency α_o (with s_o being the amplitude of the incident wave). The results obtained by the proposed methodology, under relaxed boundary conditions, and with a 25 element mesh, are compared to those obtained in Ref. 1 where the foundation was bonded to the half-space and a 64 element discretization was used. The agreement of the two methods is excellent.

For flexible foundations the stiffness ratio, between a rectangular plate and the subgrade, is defined as

$$K = \frac{Eh^3(1-\nu_s)}{12(1-\nu_p)\mu b^3} \tag{10}$$

where E, h, ν_p and b are the modulus of elasticity, the thickness, the Poisson's ratio and the largest plan dimension of the plate, respectively, and ν_s and μ are the Poisson's ratio and shear modulus of the subgrade, respectively. In Fig. 5(a,b,c) the dimensionless response at various points of a flexible massless rectangular plate with intermediate stiffness K=0.004, subjected to a horizontally propagating S_V wave, are given. For the required stiffness matrix of the plate K, the 12 degree of freedom plate element of Ref. 11 was used. The results obtained by the proposed methodology with 25 and 49 element meshes are compared with those of Ref. 4 where a 64 element mesh was used.

The seismic wave excitation of an embedded foundation requires, the specification of both the displacements and the tractions of the free-field. Further, the variation of these fields with respect to depth, i.e. the coordinate x_3 of Fig. 2, should be considered, too. The wave excitation used in this work to study the response of embedded foundations was obtained from Ref. 12. Thus, the 60"x60"x20"rigid, massless foundation of Fig. 6, was subjected to a combination of P-SV plane wave excitation. A number of elements on the free-field outside the contact area were used, in this case only, for better accuracy. The dimensionless response of the foundation, i.e., the rigid body motion component over the corresponding amplitude of the free-field motion on the surface of the half space, is plotted in Fig. 7(a,b,c) versus the natural frequency of the soil layer corresponding to the embedment, and for various angles of incidence θ_v.

It should be emphasized at this point that results for harmonic wave excitations and simple rectangular or parallelepiped foundations were presented only for comparison purposes. The proposed time domain methodology can also provide directly the response of foundations of arbitrary shape to general transient wave excitations.

CONCLUSIONS

The application of a general time domain BEM to some representative three-dimensional soil foundation interaction problems has been demonstrated. It has been found that the proposed numerical method is both accurate and efficient compared to other existing approaches in the field. Its main advantages, compared to the FEM, is that required a minimum amount of discretization only on the boundary of the domain of interest and that the radiation condition, cause of many problems in the FEM, is taken into account automatically. Also the transient response of the foundation can be computed in one direct step and not two, as the frequency domain methods require. This time domain methodology presents the potential for accurate and efficient studies of non-linear soil-structure interaction problems.

ACKNOWLEDGMENTS

This research has been supported by a grant from the National Science Foundation under contract NSF/CEE-8024725 to the University of Minnesota.

REFERENCES

1. Wong, H.L. and Luco, J.E., "Dynamic response of rectangular foundations to obliquely incident seismic waves", Earth. Engng. Struct. Dyn., 6, 3-16 (1978).
2. Luco, J.E. and Wong, H.L., "Dynamic response of rectangular foundations for Rayleigh wave excitation", pp. 1542-1548, Proc. 6th World Conf. Earth. Engng., N. Delhi, India, 1977.
3. Iguchi, M., "A basic study on the behaviour of long dimensional size buildings during earthquakes", pp. 783-790, Proc. Japan Earth. Engng. Sym., Tokyo, Japan, 1975.
4. Whittaker, W.L. and Christiano, P., "Response of a plate and elastic half-space to harmonic waves", Earth. Engng. Struct. Dyn., 10, 255-266 (1982).
5. Dominguez, J., "Response of embedded foundations to travelling waves", Publ. No. R78-24, Dept. of Civil Engng., M.I.T., August 1978.
6. Gupta, S., Penzien, J., Lin, T.W. and Yeh, C.S., "Three-dimensional hybrid modelling of soil-structure interaction", Earth. Engng. Struct. Dyn., 10, 69-87 (1982).
7. Karabalis, D.L. and Beskos, D.E., "Dynamic response of 3-D rigid surface foundations by time domain boundary element method", Earth. Engng. Struct. Dyn., (in print).
8. Karabalis, D.L., "Dynamic response of 3-D foundations by the time domain boundary element method", Ph.D. Thesis, University of Minnesota, Minneapolis, Minnesota, 1983.
9. Eringen, A.C. and Suhubi, E.S., "Elastodynamics-Vol. II, Linear Theory", Academic Press, New York, 1975.
10. Thau, S.A., "Radiation and scattering from a rigid inclusion in an elastic medium", J. Appl. Mech., 34, No. 2, 509-511 (1967).
11. Zienkiewicz, O.C. and Cheung, Y.K., "The Finite Element Method for analysis of elastic isotropic and orthotropic slabs", Proc. Inst. Civil Eng., 28, 471-488 (1964).
12. Wolf, J.P. and Obernhuber P., "Free-field response from inclined SV and P-waves and Rayleigh-waves", Earth. Engng. Struct. Dyn., 10, 847-869 (1982).

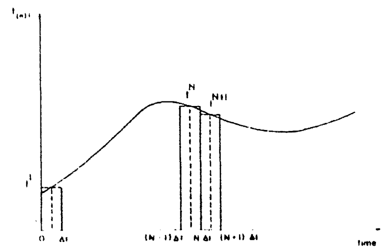

Figure 1. Approximation of the surface traction $t_{(n)i}$ by a sequence of rectangular pulses.

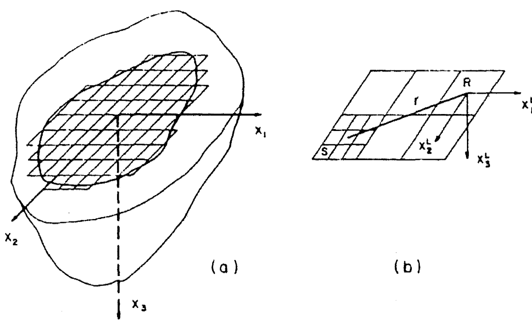

Figure 2. Geometry and discretization of a 3-D surface foundation.

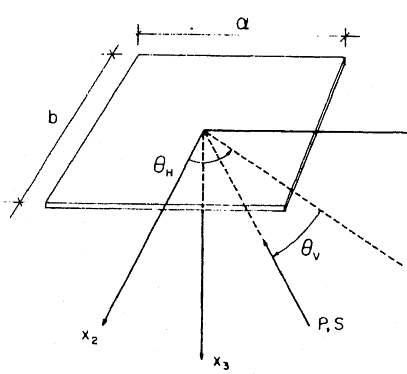

Figure 3. Oblique P and S seismic waves impinging on a surface foundation.

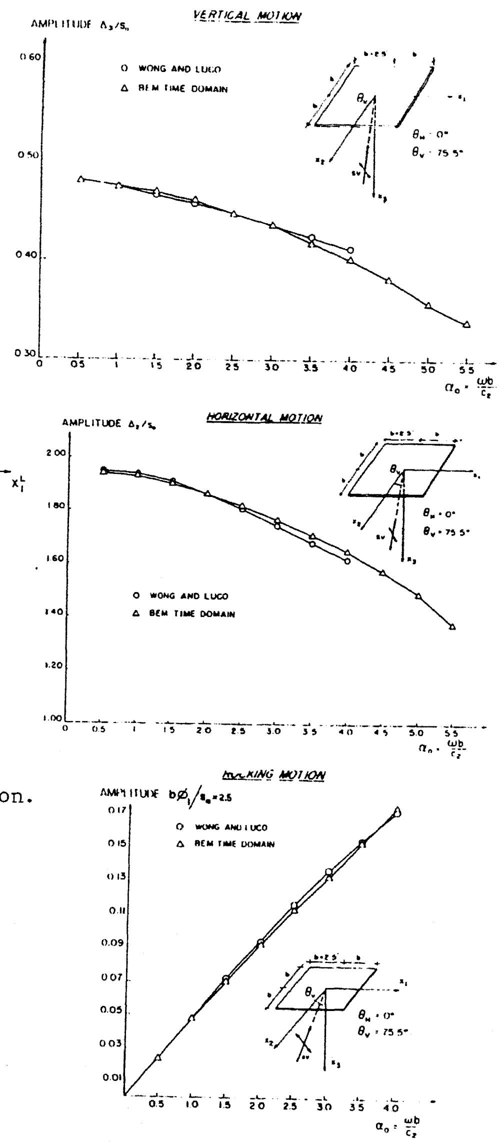

Figure 4. Vertical, horizontal and rocking harmonic response to an SV wave versus frequency for a rigid rectangular foundation.

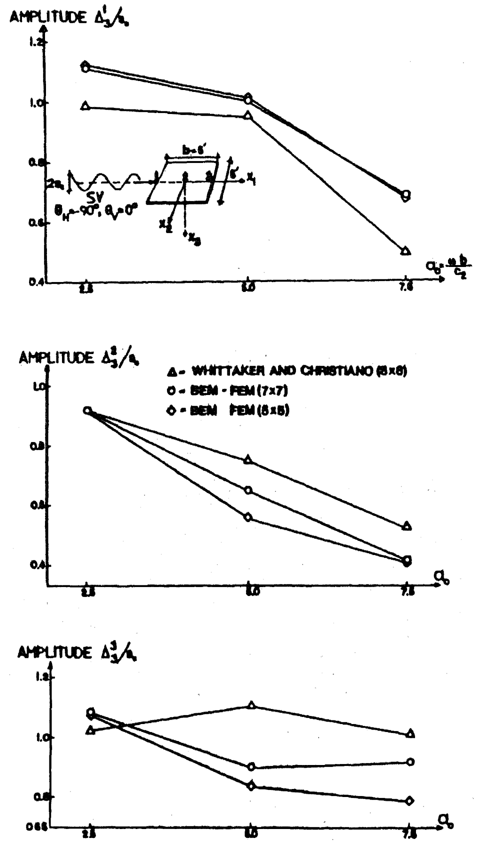

Figure 5. Flexible foundation subjected to an SV harmonic wave.

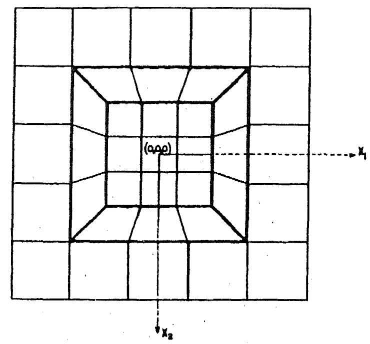

Figure 6. Discretization of a rigid embedded foundation.

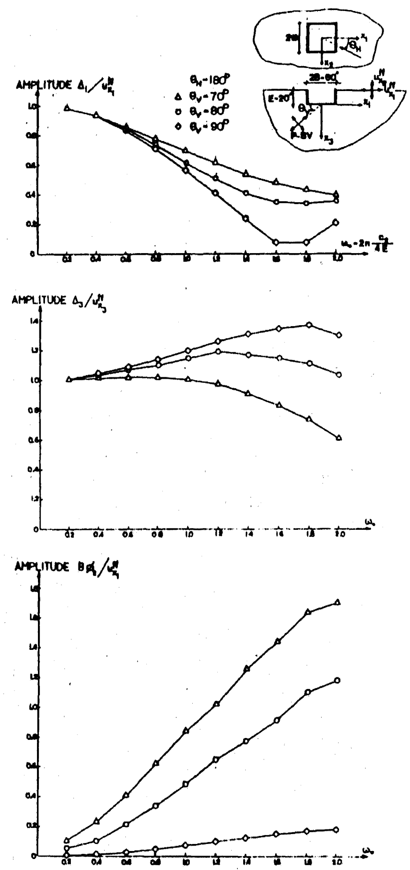

Figure 7. Horizontal, vertical and rocking harmonic response to a P-SV wave versus frequency for a rigid embedded foundation.