DYNAMIC RESPONSE CHARACTERISTICS OF AN ELASTIC FOUNDATION EMBEDDED IN A LAYERED MEDIUM

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SUMMARY

This paper presents an analysis of a dynamic behavior of an elastic foundation embedded into a visco-elastic layered medium. Incident, harmonic SH waves are supposed to propagate obliquely upwards. The problem is formulated by means of boundary integral method and numerical results are obtained in the sense of least square. It is pointed out that scattering and diffraction of incident waves increase to a deeply embedded foundation and that displacement response of a surface layer is largely affected by a deep foundation.

INTRODUCTION

In recent years, it has been accepted that the evaluation of soil-structure interaction phenomena is important for the analysis of structural safety for earthquakes. There were reported many researches dealing with soil-structure interaction through various approaches. However, as to an embedded foundation, relatively a small number of results(Refs. 1 to 4) is only available on account of difficulties in theoretical analysis.

In this paper, an elastic foundation embedded into a visco-elastic, layered medium is supposed. The foundation which is infinitely long to the antiplane direction is subjected to incident, plane, harmonic SH waves propagating obliquely upwards to the surface. The analysis is treated as a two-dimensional case and is carried out based on the boundary integral method(Ref. 5). The problem is formulated in terms of a set of Fredholm integral equations of the first kind and integrations are made along paths outside the boundary in order to avoid the singularities of integral(Refs. 6 and 7). Using Green's functions represented as solutions for a point source, boundary conditions are satisfied discretely at finite points on the boundary in the sense of the least squares.

MATHEMATICAL MODEL AND METHOD OF ANALYSIS

An elastic foundation embedded into a layered medium subjected to incident SH waves is shown in Fig. 1. Symbols in the figure mean the following quantities: ρ_j is mass density of medium denoted by j, (j = 1, 2, 3), μ_j^* ($= \mu_j + \mu_j'$) is complex shear modulus, γ is incident angle of SH waves, b and D are half width and depth of foundation, H is thickness of surface layer.

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Incident field of visco-elastic, layered medium for plane SH waves is represented with omission of time factor $e^{i\omega t}$ as follows:

$$u_1^i = \frac{e^{ink\alpha_0 H\cos\gamma'} e^{-ik\alpha_0 x\sin\gamma}}{\cos(k\alpha_0 H\cos\gamma) + i\lambda\sin(k\alpha_0 H\cos\gamma)} (e^{ik\alpha_0 z\cos\gamma} + e^{-ik\alpha_0 z\cos\gamma})$$

$$u_2^i = \{e^{ink\alpha_0 z\cos\gamma'} + \frac{\cos(k\alpha_0 H\cos\gamma)-i\lambda\sin(k\alpha_0 H\cos\gamma)}{\cos(k\alpha_0 H\cos\gamma)+i\lambda\sin(k\alpha_0 H\cos\gamma)}$$

$$\times e^{i2nk\alpha_0 H\cos\gamma'} e^{-ink\alpha_0 z\cos\gamma'}\} e^{-ink\alpha_0 x\sin\gamma'} \qquad (1)$$

where $c_j = \sqrt{\mu_j / \rho_j}$, $k_j = \omega / c_j$, $(j = 1,2)$

$$\alpha_0 = b\omega / c_1 = bk_1$$

$$\mu = \mu_2 / \mu_1 , \quad n = c_1 / c_2 = k_2 / k_1$$

$$\sin\gamma / k_1 = \sin\gamma' / k_2 , \quad \lambda = \rho_1 c_1 \cos\gamma / \rho_2 c_2 \cos\gamma'$$

$$d = \mu_1' / \mu_1 = \mu_2' / \mu_2 , \quad k = 1 / \sqrt{1+d}$$

And c_j and k_j means shear wave velocity and wave number, α_0 means non-dimensional frequency, γ' means incident angle in a surface layer, d is a coefficient of viscosity.

Incident waves are diffracted by the presence of an embedded foundation. Diffraction field can be expressed as a single layer potential at curves C_1 and C_3 which are defined in the foundation and in the layered medium respectively shown in Fig. 2. If $\sigma_1(r_0)$ and $\sigma_3(r_0)$ are assumed as the single layer densities on C_1 and C_3, respectively, diffraction field is represeted by use of Green's functions for each medium. And displacements u_1, u_2 and u_3 of surface layer, half space and foundation are expressed in eq. (2) as total field defined by sum of incident field and diffraction field.

$$u_1 = u_1^i + u_1^d = u_1^i + \int_{C_1} G_1(r,r_0)\sigma_1(r_0)dS_1$$

$$u_2 = u_2^i + u_2^d = u_2^i + \int_{C_1} G_2(r,r_0)\sigma_1(r_0)dS_1$$

$$u_3 = u_3^d = \int_{C_3} G_3(r,r_0)\sigma_3(r_0)dS_3 \qquad (2)$$

where $G_1(r,r_0)$, $G_2(r,r_0)$ and $G_3(r,r_0)$ are Green's functions for a layered medium and a half space and expressed in the following equations:

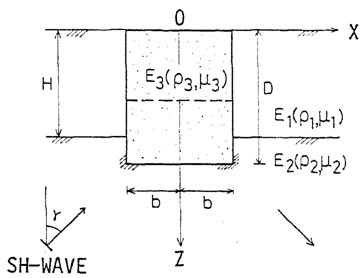

Fig. 1 Mathematical Model

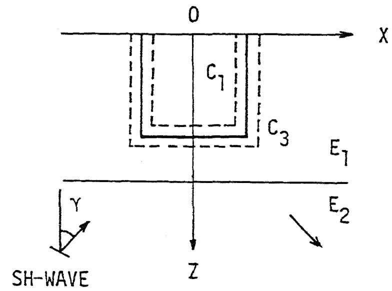

Fig. 2 Path of Integration

$$G_1(r,r_0) = \frac{-1}{2\pi}\int_0^\infty \frac{\lambda_1 S_1(\xi) + \mu\lambda_2 T_1(\xi)}{\lambda_1[\lambda_1\sinh(\lambda_1\alpha_0 H) + \mu\lambda_2\cosh(\lambda_1\alpha_0 H)]} \times \cos[\alpha_0(x-x_0)\xi]d\xi \qquad \text{for} \quad z_0 < H$$

$$= -\frac{1}{\pi}\int_0^\infty \frac{\mu e^{-\lambda_2\alpha_0(z_0-H)}\cosh(\lambda_1\alpha_0 z)}{\lambda_1[\lambda_1\sinh(\lambda_1\alpha_0 H) + \mu\lambda_2\cosh(\lambda_1\alpha_0 H)]} \times \cos[\alpha_0(x-x_0)\xi]d\xi \qquad \text{for} \quad z_0 > H$$

$$G_2(r,r_0) = -\frac{1}{\pi}\int_0^\infty \frac{e^{-\lambda_2\alpha_0(z-H)}\cosh(\lambda_1\alpha_0 z_0)}{\lambda_1[\lambda_1\sinh(\lambda_1\alpha_0 H) + \mu\lambda_2\cosh(\lambda_1\alpha_0 H)]} \times \cos[\alpha_0(x-x_0)\xi]d\xi \qquad \text{for} \quad z_0 < H$$

$$= \frac{-1}{2\pi}\int_0^\infty \frac{\lambda_1 S_2(\xi) + \mu\lambda_2 T_2(\xi)}{\lambda_2[\lambda_1\sinh(\lambda_1\alpha_0 H) + \mu\lambda_2\cosh(\lambda_1\alpha_0 H)]} \times \cos[\alpha_0(x-x_0)\xi]d\xi \qquad \text{for} \quad z_0 > H$$

$$G_3(r,r_0) = \frac{i}{4}[H_0^{(2)}(m\alpha_0 R) + H_0^{(2)}(m\alpha_0 R')]$$

$$S_1(\xi) = \frac{1}{2}[e^{\lambda_1\alpha_0 H}\{e^{-\lambda_1\alpha_0|z-z_0|} + e^{-\lambda_1\alpha_0(z+z_0)}\} + e^{-\lambda_1\alpha_0 H}\{e^{-\lambda_1\alpha_0|z-z_0|} + e^{-\lambda_1\alpha_0(z+z_0)}\}]$$

$$T_1(\xi) = \frac{1}{2}[e^{\lambda_1\alpha_0 H}\{e^{-\lambda_1\alpha_0|z-z_0|} + e^{-\lambda_1\alpha_0(z+z_0)}\} - e^{-\lambda_1\alpha_0 H}\{e^{-\lambda_1\alpha_0|z-z_0|} + e^{-\lambda_1\alpha_0(z+z_0)}\}]$$

$$S_2(\xi) = \frac{1}{2}\,[e^{\lambda_1 \alpha_0 H}\{e^{-\lambda_2 \alpha_0 |z-z_0|} - e^{-\lambda_2 \alpha_0 (z+z_0-2H)}\}$$

$$- e^{-\lambda_1 \alpha_0 H}\{e^{-\lambda_2 \alpha_0 |z-z_0|} - e^{-\lambda_2 \alpha_0 (z+z_0-2H)}\}]$$

$$T_2(\xi) = \frac{1}{2}\,[e^{\lambda_1 \alpha_0 H}\{e^{-\lambda_2 \alpha_0 |z-z_0|} + e^{-\lambda_2 \alpha_0 (z+z_0-2H)}\}$$

$$+ e^{-\lambda_1 \alpha_0 H}\{e^{-\lambda_2 \alpha_0 |z-z_0|} + e^{-\lambda_2 \alpha_0 (z+z_0-2H)}\}] \qquad (3)$$

where $\lambda_1 = \sqrt{\xi^2 - k^2}$, $\lambda_2 = \sqrt{\xi^2 - n^2k^2}$

$$R = \sqrt{(x-x_0)^2 + (z-z_0)^2} \quad , \quad R' = \sqrt{(x-x_0)^2 + (z+z_0)^2}$$

$$m = k_3 \,/\, k_1 = c_1 \,/\, c_3$$

and $H_0^{(2)}(\cdot)$ denotes Hankel's function of the second kind and order zero, $r_0(x_0,z_0)$ and $r(x,z)$ are position vectors of a point of a source and of an arbitrary point in the medium.

Substituting these equations into continuity condition of displacement and stress at the boundaries of a foundation, a surface layer and a half space, the Fredholm integral equation of second kind are derived for the unknown single layer densities $\sigma_1(r_0)$ and $\sigma_3(r_0)$ But the solution of these integral equations can not be obtained easily, a suitable numerical procedure is used here. The single layer densities are assumed at points of an adequate number on the path of integration. And the condition of continuity of displacement and stress is satisfied also at points of a finite number on the boundary. Then the problem is deduced to simultaneous algebraic equations and numerical solution is obtained for less unknown quantities than the number of the continuity condition with the least square method. Substituting the numerical solution of $\sigma_1(r_0)$ and $\sigma_3(r_0)$ into eq. (2), displacements of a foundation, a surface layer and a half space can be evaluated.

RESULTS AND CONCLUSIONS

In Figs. 3(a) and 3(b), Green's function for a layered medium is shown with respect to vertical coordinate z. Point Q denotes a source point of a harmonic SH wave. A solid line and a broken line represent responses of a half space medium ($\mu = 1$) and an elastic layer with depth $H = 4$ on a rigid base ($\mu = \infty$), respectively. Real part of numerical solution seems to be extremely large at point Q nevertheless imaginary part is shown as a smooth curve. Fig. 4 shows displacement response of point P defined in the figures subjected to a point source at Q to the nondimensional frequency. Natural vibration characteristics of a surface layer are shown obviously as several peaks and dips in the frequency response for a large ratio μ of shear modulus of a surface layer to that of a base medium.

Numerical solution of displacement response of an embedded foundation and a layered medium along z axis (x = 0) are shown in Figs. 5(a) to 5(d). Dotted curve represents response of a layered medium with no foundation (D = 0), namely the incident field. For a shallow foundation (D ≤ 1.0), diffraction and scattering of incident waves by a foundation have a little effect on response of a surface layer. However, a deep foundation causes a large scattering effect in a surface layer, displacement response in a base medium of a half space is affected scarecely. These features can be seen commonly in both real and imaginary parts and for different frequencies. Fig. 6 shows displacement response at the center and edge of the surface of foundation and on the surface of layer near the foundation (x = 2) with respect to frequency of incident waves. In lower frequency range, displacement amplitude of a foundation is as large as that of a surface layer and have a resonant peak corresponding to the fundamental natural vibration of a coupled foundation-layered medium systems. On the other hand, response of a foundation for a higher frequency is suppressed and shows no evident peak yet response of a surface layer varies with frequency.

Next figure shows a comparison of frequency response with depth of foundation at the center of foundation surface in Fig. 7(a) and on the surface of layer near the foundation (x = 2) in Fig. 7(b). A parameter Ms means the ratio of shear modulus of foundation to that of surface layer. Although the response of a shallow foundation (D ≤ 2.0) is largely affected by the frequency characteristics of a surface layer, vibrational characteristics of foundation itself or of a base medium of a half space seem to come out for a deep foundation (D ≥ 4.0). On the surface of a layer near the foundation, vibrational characteristics of a surface layer is predominant widely without partial exception for a deep foundation.

Figs. 8(a) to 8(c) represent influence of depth of surface layer on the frequency response at the center of surface and bottom of foundation and on the surface of a layer near the foundation. Dotted curves mean the special case D = H = 2.0, when the bottom of foundation contact with a base medium of a half space and response seems a peculiar one. Resonant frequencies of a foundation and a surface layer decrease with depth H of a surface layer.

As regards the incident angle of SH waves, comparison of response at the center of surface of foundation is represented in Figs. 9(a) to 9(c) for D = 0 to D = 2.0, respectively. Amplitude of foundation displacement and effective input motion to the foundation become small with the angle because of the energy dispersion by the scattering of incident waves by the foundation. And increase of incident angle causes generally the resonant frequency of higher modes large. Response of foundation is decreased in high frequency range irrespective of the incident angle.

Fig. 10 represents distribution of displacement response of surface of a foundation and a surface layer. With increase of incident angle, amplitude of displacement becomes large in front of the foundation and small at behind of it. And it is emphasized by a deep foundation in Fig. 10(b). These phenomena can be considered to be an amplification of response by the energy accumulation and conversely an isolation of input motion by the presence of foundation. In Figs. 11(a) and 11(b), distribution of response amplitude is shown along the surface and the boundary of foundation at the layered medium. The curves are corresponding to a foundation embedded in a half space (μ = 1.0) in a layered medium (μ = 6.0) and in a surface layer on a rigid base (

$\mu = \infty$). Although response increases generally to the free surface ($z = 0$) along the vertical boundary of a foundation, variation of shear modulus of a layered medium seems to have a little effect on response distribution.

From the results abovementioned, concluding remarks are pointed out as follows:

1. A deeply embedded foundation causes large scattering and diffraction of the incident waves, and affects displacement response distribution of a surface layer near the foundation.

2. Though responses of the coupled foundation-layered medium system are amplified evidently at the fundamental natural frequency, natural modes of the higher order seems to be obscure.

3. Responses of the coupled system decrease for a large incident angle to the vertical plane because of the energy dispersion by the foundation.

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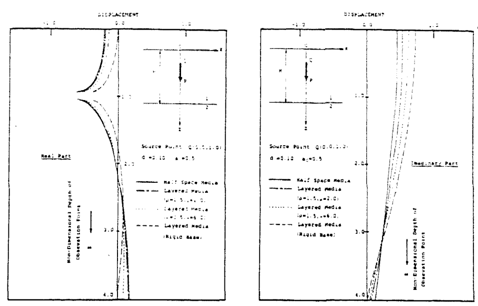
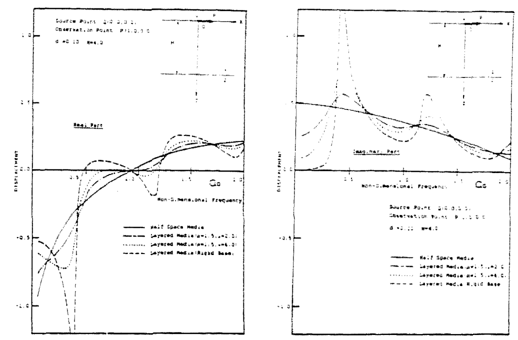


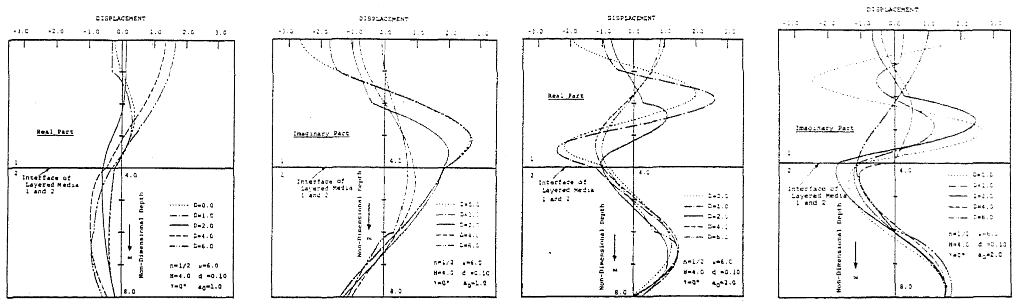
(a) (b)

Fig. 3 Displacement to Point Source at Q



(a) (b)

Fig. 4 Displacement at P versus Frequency



(a) (b) (c) (d)

Fig. 5 Displacement of Embedded Foundation and Layered Medium along z-axis

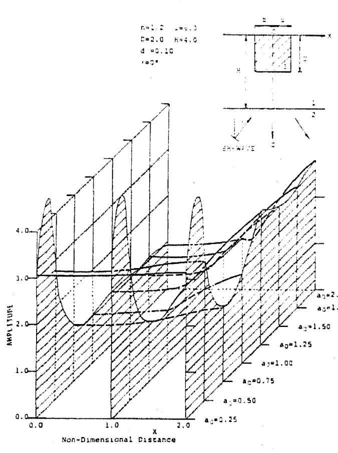
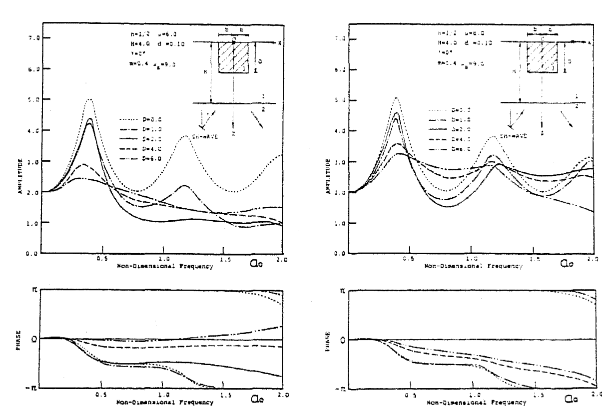
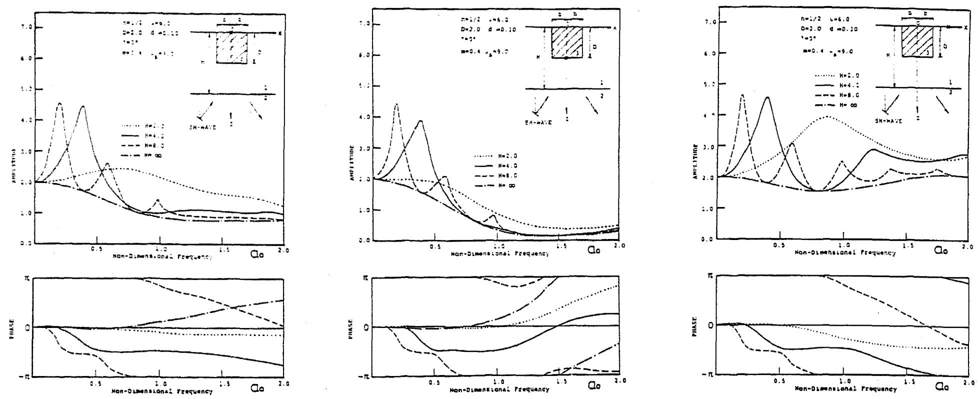


Fig. 6 Displacement of Free Surface



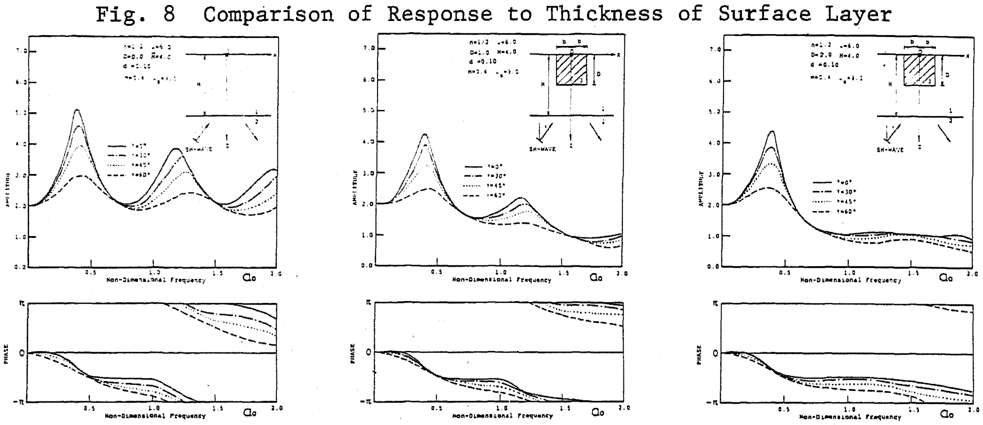
(a) (b)

Fig. 7 Displacement at O versus Frequency



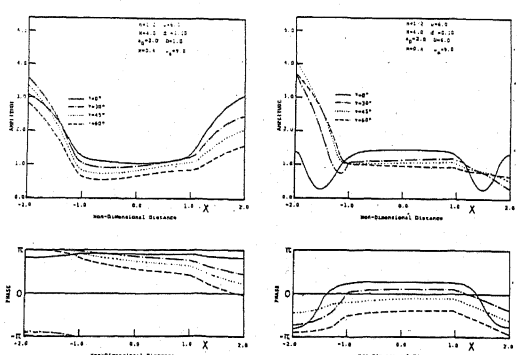
(a) (b) (c)

Fig. 8 Comparison of Response to Thickness of Surface Layer



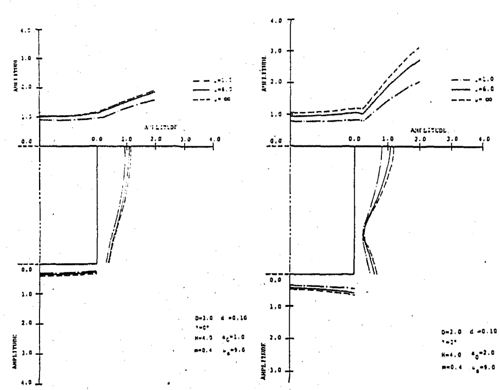
(a) (b) (c)

Fig. 9 Comparison of Response to Incident Angle



(a) (b)

Fig. 10 Response Distribution of Free Surface



(a) (b)

Fig. 11 Response Distribution of Surface of Foundation