A SIMPLE MODEL TO ILLUSTRATE THE SIGNIFICANCE OF FOUNDATION FLEXIBILITY IN SOIL-STRUCTURE INTERACTION PROBLEMS

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SUMMARY

In some recent work on the effect of foundation flexibility the expected multiple resonances were not obtained because authors either neglected the foundation mass or limited the range of excitation frequencies. Also damping in the foundation was neglected. In this paper a simplified model, which consists of a simple structure and a flexible foundation on an elastic layer, is investigated to demonstrate the dependence of natural frequencies on the stiffness and mass parameters and show that damping in the structure, foundation and layer must be included to obtain meaningful resonant responses.

INTRODUCTION

In classical work on soil-structure interaction (SSI) the foundation is represented as a rigid mass of circular or rectangular plan form. Its steady-state response to harmonic excitation in a single mode (e.g. vertical translation) is similar to that of a single degree-of-freedom system. When a n-degree-of-freedom model of a structure is added, the combined system has $(n+1)$ resonances. Recently several authors have considered foundation flexibility, treating the foundation analytically or by the finite element method as a plate. Even in the absence of a structure an infinite spectrum of resonances would be expected; however, as some authors have either neglected the foundation mass or limited the range of excitation frequencies, multiple resonances have been reported infrequently. The author (Ref.1) has given a survey of this recent work. He also investigated a simplified model, which consisted of a flexible beam and an elastic layer, in order to demonstrate the effects of various parameters on resonant frequencies and the significant effects of both foundation (beam) and layer damping on resonant response. In Ref.1 no attempt was made to model a structure. The purpose of this paper is to add a single degree-of-freedom system, which represents a simple structure, to the foundation model of Ref.1 and investigate the effects of structural and foundation parameters on resonant frequencies and amplitudes.

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THEORY

Figure 1 shows the simplified model. The mass m and spring of stiffness k (with associated hysteretic damping ζ) comprise a single degree-of-freedom representation of the structure. The uniform beam of length L, mass per unit length ρA and flexural rigidity EI (with associated hysteretic damping μ) is an idealization of the flexible foundation. The elastic or Winkler layer of stiffness C per unit length (with associated hysteretic damping η) idealizes the soil or elastic half-space. The excitation is the harmonic force $Pe^{i\omega t}$, which is applied to the mass m. This form of excitation has applications outside earthquake engineering. For the latter a transverse acceleration, which is applied to the base HJ in Fig.1, would be more representative; however, the method of this paper cannot deal directly with this type of input.

The following non-dimensional parameters are used

stiffness ratio (beam to layer), $\beta = EI/CL^4$

stiffness ratio (structure to layer) $\varkappa = k/LC$

mass ratio, $\phi = m/\rho AL$

frequency factor, $\Omega = \rho A\omega^2/C$

dynamic magnification factor (DMF) for the relative displacement of the mass m (i.e. the elastic distortion of the structure), $R = (k/P)|y-V(o)|$

Using the symmetry and considering only $0 < x < \frac{1}{2}L$ and putting $v(x,t) = V(x)e^{i\omega t}$, the beam equation is satisfied if

$$V(x) = B_1 \sin \lambda^+ x + B_2 \cos \lambda^+ x + B_3 \sinh \lambda^+ x + B_4 \cosh \lambda^+ x \qquad (1)$$

where, due to the presence of the damping, λ^+ is complex and is related to the frequency factor Ω by

$$\beta(1 + i\mu)(\lambda^+ L)^4 = \Omega - (1 + i\eta) \qquad (2)$$

For small damping this is approximated as

$$\lambda^+ L = \lambda L \left[1 - \tfrac{1}{4} i \left\{\frac{\eta}{\beta(\lambda L)^4} + \mu\right\}\right] \qquad (3)$$

where λ is real and $\beta(\lambda L)^4 = \Omega - 1$. These equations apply for $\Omega > 1$, i.e. for frequencies greater than the natural frequency of a rigid beam on the elastic layer. (Solutions for $\Omega < 1$ will be given later.) Using the equation of motion for mass m, which relates y to $V(o)e^{i\omega t}$, applying the boundary conditions at $x = o$ and $x = \frac{1}{2}L$ and approximating trigonometric and hyperbolic functions of complex agreement in the manner of Refs.2 and 3 [i.e., if $\frac{1}{2}\lambda^+ L = \frac{1}{2}\lambda L - i\epsilon\lambda L$, where $\epsilon \ll 1$, $\sin(\frac{1}{2}\lambda^+ L) = \sin(\frac{1}{2}\lambda L) - i\epsilon\lambda L \cos(\frac{1}{2}\lambda L)$ etc], the following results are obtained.

Natural frequencies of the system are obtained from the roots of

$$16\beta\alpha^3 (1 - \Omega\phi/\varkappa) + \Omega\phi F_1/F_2 = o \qquad (4)$$

where $F_1 = 1 + \cos\alpha \cosh\alpha$, $F_2 = \cos\alpha \sinh\alpha + \sin\alpha \cosh\alpha$, $\alpha = \frac{1}{2}\lambda L$.

At resonance, given by a root of equation (4), the DMF for the relative displacement of the mass m

$$R = (1 - \Omega\psi/\varkappa)^{-1}|1 - 4iZ| \tag{5}$$

where $$Z^{-1} = (1-G)\mu - \frac{(3+G)\eta}{\Omega-1} + \frac{4\Omega\psi/\varkappa}{1-\Omega\psi/\varkappa}\zeta \tag{6}$$

$G = \alpha F_3{}^2/F_1F_2$ and $F_3 = \cos\alpha + \cosh\alpha$

For all values of the parameters β, $\varkappa$ and ψ there is one natural frequency for which $\Omega < 1$; the resonant response at this frequency is also of interest. In this case the solution of the beam equation with damping terms included is

$$V(x) = B_1 \sin\theta^+x \sinh\theta^+x + B_2 \sin\theta^+x \cosh\theta^+x + B_3 \cos\theta^+x \sinh\theta^+x + B_4 \cos\theta^+x \cosh\theta^+x \tag{7}$$

where θ^+ is complex and is related to the frequency factor Ω by

$$4\beta\ (1+i\mu)\ (\theta^+L)^4 = 1 + i\eta - \Omega \tag{8}$$

Using a small damping approximation, similar to equation (3), and proceeding as in the earlier case, natural frequencies are obtained from the roots of

$$64\beta\phi^3(1 - \Omega\psi/\varkappa) - \Omega\psi\ f_1/f_2 = 0$$

where $\phi = \frac{1}{2}\theta L$, $64\beta\ \phi^4 = 1-\Omega$, $f_1 = \cos^2\phi + \cosh^2\phi$,
$f_2 = \sin\phi\cos\phi + \sinh\phi\cosh\phi$.

The DMF at resonance ($\Omega=\Omega_1$, where Ω_1 is the root of equation (9)) for the relative displacement of m is given by equation (5), provided that G is redefined in terms of ϕ as $G = \phi\ f_3{}^2/f_1f_2$, where $f_3 = 2\cos\phi\cosh\phi$. In order to use equation (5) for resonant response, the damping terms η, ζ and μ must be small in magnitude and also small compared to $|1-\Omega\psi/\varkappa|$ and $|1-\Omega|$.

RESULTS

In Figures 2 and 3 the natural frequency parameter $\Omega_r (=\rho A\omega_r{}^2/C)$ is plotted against the stiffness ratio β for two sets of values of the structural parameters ψ and $\varkappa$. For stiff beams (large values of β) the system reduces to a two degree-of-freedom system and in all cases the curves for Ω_1 and Ω_2 are asymptotic to these values.

Considering the symmetric modes of the beam on the elastic layer (without the structure), the natural frequencies are given by $\Omega_1' = 1$ (rigid body mode), $\Omega_2' = 500.5\beta + 1$, $\Omega_3' = 14620\beta + 1$. These frequency factors are shown in Figure 2 ($\psi = 0.1, \varkappa = 1$) by broken lines. Considering the frequency factors of the complete system (full lines), Ω_2 lies slightly below Ω_2' for low values of β, but for high values Ω_2 approaches the higher of the two frequency factors of the limiting 2 DOF system. Frequency factor Ω_3 lies slightly below Ω_3' for low values of β, but is asymptotic to the curve for Ω_2' for high values. In Figure 3 for a heavier, stiffer structure ($\psi = 1$, $\varkappa = 10$) the curves for Ω_r' would be as in Figure 2, but are not shown. For this larger structure the natural frequency factor curves Ω_r are not close to curves for Ω_r', except that for large values of β Ω_3 is again asymptotic to the curve for Ω_2'.

In Figures 4 and 5 the DMF at resonance for the relative displacement of the structural mass m is plotted against β; the integer r indicates whether the first, second or third resonance is being considered. The four different types of line indicate the resonant response when only layer damping exists (η = 0.01), only structural damping exists (ζ = 0.01), only beam damping exists (μ = 0.01), and all three dampings co-exist ($\eta=\zeta=\mu=0.01$). In this manner the relative importance of the different damping mechanisms is illustrated. For ψ = 0.1, $\varkappa$ = 0.1 (Figure 4) layer and structural damping are important at Ω_1 and Ω_2 for moderate and large values of β; this is to be expected as the corresponding modes have rigid body characteristics. As β increases, the third resonance is dominated by beam damping, as this is a predominantly flexural mode with the layer and structure having little effect. For ψ = 1, $\varkappa$ = 10 (Figure 5) the behaviour of the resonant amplitudes is more complex, particularly in the intermediate range of β from approximately 7.10^{-3} to 5.10^{-2}, where Figure 3 shows coupling phenomena in the natural frequency curves for Ω_2 and Ω_3. The first resonance is dominated by the layer damping; the second resonance is dominated by layer, beam and structure damping for low, intermediate and high values of β respectively; the third resonance is dominated by beam damping for low and high values of β and by structure damping for intermediate values.

Although all the plotted results relate to damping values of 0.01, resonant responses for other <u>small</u> values of damping can be obtained by scaling, provided that the imaginary term in equation (5) >>1, as Z is inversely proportional to damping.

CONCLUSIONS

Allowance for the foundation (beam) flexibility (i.e. β finite) introduces additional natural frequencies (Ω_r for $r \geqslant 3$). If the beam is assumed to be rigid ($\beta \to \infty$), the system has two asymptotic natural frequencies, which would be represented by horizontal lines on such plots as Figures 2 and 3. If the upper of these lines intersects the curves for Ω_2', (the natural frequency factor for the fundamental symmetric flexural mode of the beam on the layer), coupling effects occur as demonstrated in Figures 2 and 3.

The response of the mass relative to the beam at resonance is the most important response parameter. By plotting resonant response for each of the three damping mechanisms (structure, beam and layer) acting separately and for all three acting simultaneously, it is demonstrated that ranges of the stiffness and mass ratios exist for which each damping mechanism is dominant. When coupling effects exist in the frequency spectra, there will be similar complex behaviour in the interchange of dominance between damping mechanisms.

Although the results cannot be applied directly to practical problems, it may be concluded that for structure - flexible foundation - soil problems all flexibility and mass effects must be considered to give complete frequency spectra; in addition all damping mechanisms must be included to give proper values of resonant response. In this context, as pointed out in Ref.1, authors who neglect the damping in the plate, when considering the interaction of a flexible foundation and an elastic half-space or stratum, may obtain erroneous resonant values.

REFERENCES

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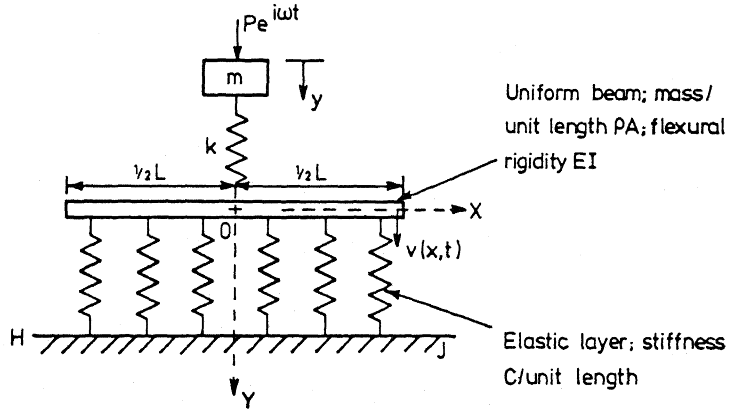

Fig.1. Model system. Hysteretic damping constants: spring, ζ; beam, μ; and layer, η.

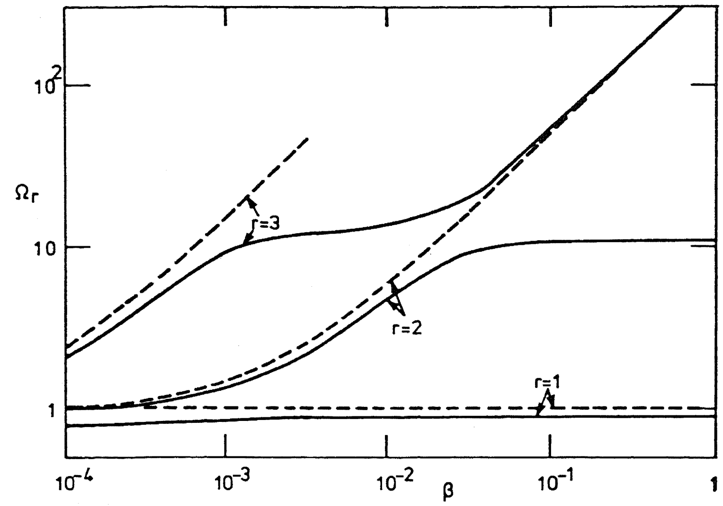

Fig.2. Natural frequency factors Ω_r versus stiffness ratio β for ψ = 0.1, $\varkappa$ = 1, ——— Ω_r; ----Ω_r'.

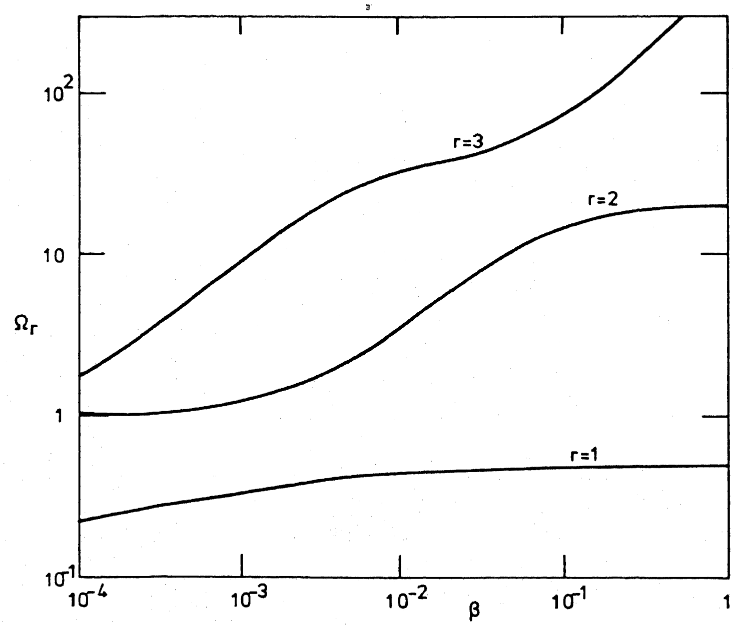

Fig.3. Natural frequency factors Ω_r versus stiffness ratio β for ψ = 1, $\varkappa$ = 10.

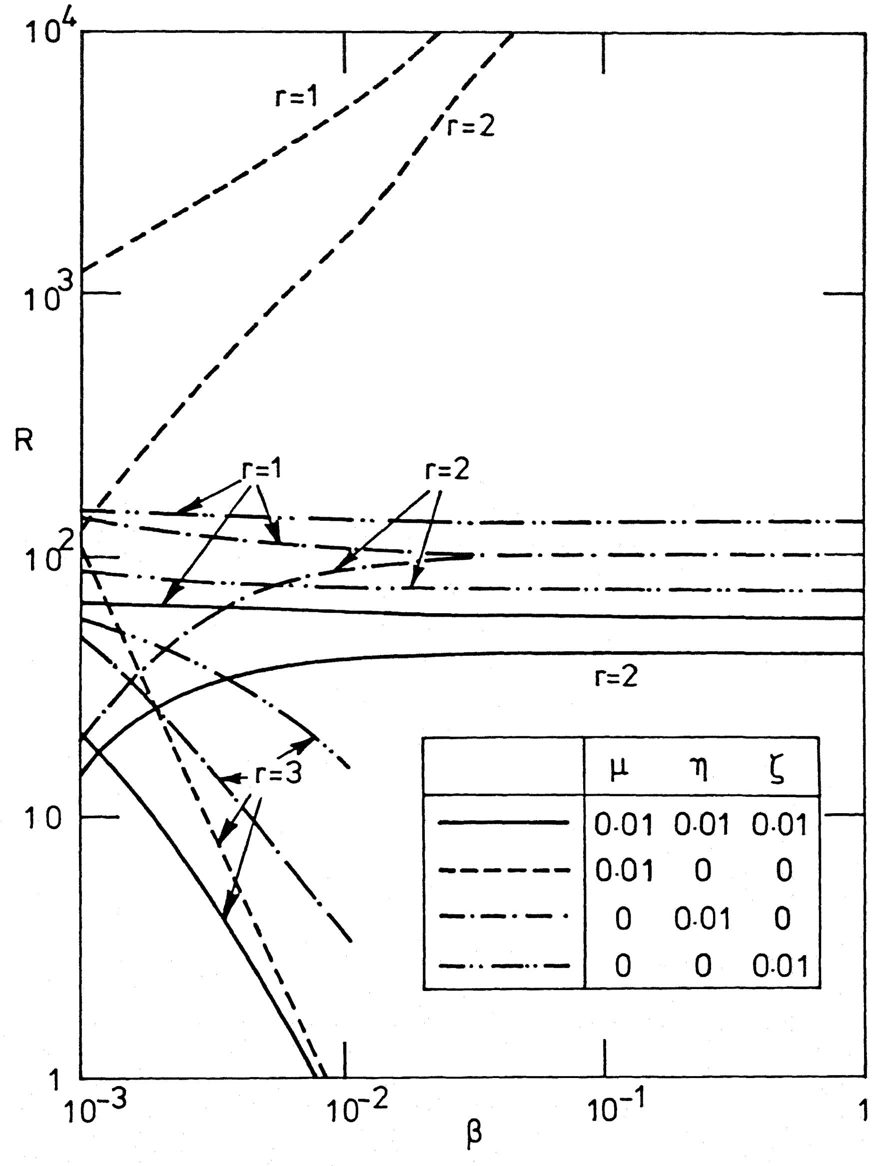

Fig.4. DMF for the relative displacement of mass m at the r^{th} resonance, R, versus stiffness ratio β for ψ = 0.1, $\varkappa$ = 0.1.

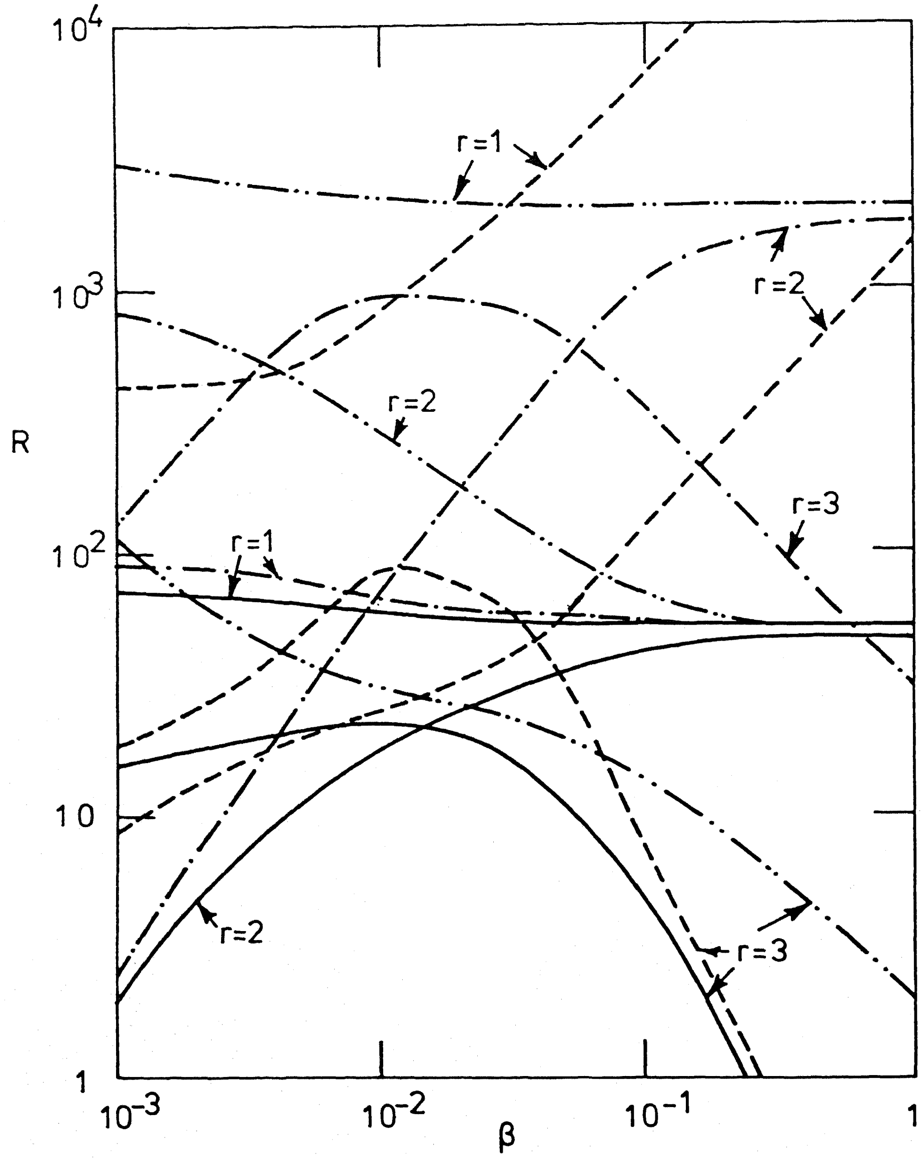

Fig.5. DMF for the relative displacement of mass m at the r^{th} resonance, R, versus stiffness ratio β for ϕ = 1, $\varkappa$ = 10. (For key see Fig.4).