THE EFFECT OF HYDRODYNAMIC PRESSURES ON THE DYNAMIC RESPONSE OF EMBEDDED STRUCTURES

G.A. Harstead (I)
N.F. Morris (II)
A.I. Unsal (III)
Presenting Author: G.A. Harstead

SUMMARY

A structure embedded in saturated soil which is subjected to dynamic pressures in the surrounding soil. These hydrodynamic pressures can significantly affect the response of the structure to the imposed dynamic loads. Under static conditions, there is no movement of the ground water; however, under dynamic conditions the relative movement of the structure and the soil causes flow of the ground water. The dynamic pressure, ground water flow, and relative movement of the structure and soil are related. An equation of motion can, therefore, be developed and solved.

INTRODUCTION

In nuclear power plants it has often been necessary to have concrete structures deeply embedded into the soil or rock where the water table is very high. Since practically all nuclear power plants are designed for high earthquake effects, overall stability must be considered (Figure 1).

In general, the static seismic overturning moment is compared to the resisting moment. The resisting moment is calculated based upon a rigid body tipping of the structure, such that a net downward force is equal to the gravity load less both the vertical seismic force and the buoyant force. This resultant often indicates instability. However, if the soil hydrodynamic forces were considered using time history analysis, the results in most cases would indicate stability.

(I) President, Harstead Engineering Associates Inc., Park Ridge, NJ

(II) Senior Engineer, Harstead Engineering Associates Inc.

(III) Associate, Harstead Engineering Associates Inc.

THEORY

Another situation in which hydrodynamic effects can be important, arise from the blowdown loads in BWR Mark I reactor systems (Figure 2). The torus of the system is subjected to a time history loading which oscillates rapidly with large peak loads acting vertically upward. The vertical loads are transferred by the torus support structure into the mat. An idealized mathematical model of this situation is presented in Figure 3. The basic assumptions are:

1. Water flows in granular material as the piston is moved.
2. The granular material is uniformly graded such that its density remains unchanged during the flow of water.
3. The rock is impermeable.
4. The water flows freely under the porous sand and the piston.

Dary's Law (Reference 1) can be stated as:

$$v = K\ s$$

where:

v = velocity of flow
s = slope of hydraulic gradient
K = coefficient (units of velocity)

The discharge is:

$$q = K_p\ A_g\ S$$

where:

K_p = coefficient of permeability
A_g = gross area
s = change in head per length

This can be written as:

$$q = K_p\ A_g\ w/nl \qquad (1)$$

where:

w = hydrodynamic pressure
n = unit weight of water

Flow under the piston is:

$$q = A_p\ v_p \qquad (2)$$

where:

A_p = area of piston
v_p = velocity of piston

Equate (1) & (2):

$$K_p\ A_g\ w/nl = A_p\ v_p$$

Solve for w, hydrodynamic pressure:

$$w = \frac{v_p \ A_p \ n l}{K_p \ A_g} \tag{3}$$

The dynamic force is F_d, where:

$$F_d = w \ A_p = \frac{A_p^2 \ \ l \ n}{A_g \ K_p} \ \ v_{mat}$$

Relationship between force and velocity is analogous to viscous damping:

$$C = A_p^2 \ l \ n \ / A_g \ K_p \tag{4}$$

While the hydrodynamic effects are not due to viscosity, the mathematical result is analogous to a damping effect. Therefore, it is possible to include the hydrodynamic effects into the classical equations of motion. For a single degree of freedom system:

$$M \ddot{x} + C \dot{x} + K x = F(t)$$

where:

M = mass of piston head
K = spring stiffness representing stiffness of mat
$F(t)$ = forcing function on piston head
C = hydrodynamic coefficient
Equation (4)

EXAMPLES

For gravel $K_p = 0.1557$ ft/sec (Reference 1)

then:

$C = 1738$ kip-sec/in

Assume the following parameters which approximate a simplification of the actual configuration (Figure 3).

$C = 1738$ kip-sec/in
$K = 20833$ kip/in
$M = 2$ kip-sec^2/in

then:

$w = 102$ rad/sec

Critical damping:

$C_{cr} = \sqrt{K \ M}$ (Reference 2)

$C_{cr} = 408$ k-sec/in

Since C $>$ 408, the system is overdamped.

$\beta = C/2M = 434.5$

Assume the forcing function is F sin Ω t and assume that we have resonance, then:

$\Omega = w$

a) Now assume system is critically damped, then:

x (dynamic) = -F/K (M w/C) cos Ω t

x (static) = - F/K

The dynamic load factor is:

DLF= x(dynamic)/x(static) = Mw/C cos Ω t

DLF = 0.5

If F = 900 kips, then:

x(max)= 0.022 in

$\dot{x}$(max)= 2.2 in/sec

$\ddot{x}$(max)= 230 in/sec/sec

Maximum forces in the system are:

Inertial Force = M$\ddot{x}$ = 458 kips

Hydrodynamic Force = M$\dot{x}$ = 900 kips

Spring Force = Kx = 458 kips

b) Now assume the system is overdamped and resonant.

$M \ddot{x} + C \dot{x} + K x = P \sin \Omega t$

where:

Ω = 160 rad/sec

C = 1500 kip-sec/in

K = 50,000 kip/inch

M = 2 k sec^2/in

P = 900 kip

P/CΩ = 0.00375

x = P/CΩ cosΩ

x(max) = 0.00375

x(static) = P/K = 0.018

DLF = K/CΩ = 0.208

$\dot{x}$(max) = P/C = 0.6

$\ddot{x}$(max) = PΩ/C = 96

Maximum forces in the system are:

Inertial Force = M$\ddot{x}$ = 190 kips

Hydrodynamic Force = C$\dot{x}$ = 900 kips

Spring Force = Kx = 190 kips

CONCLUSION

The simple examples are representative of possible situations occuring in nuclear plants; however, the results indicate dynamic effects will be greatly mitigated by the hydrodynamic effects. While not specifically discussed, a limiting hydrodynamic pressure would be the creation of a vacuum under the piston. Inasmuch as a time history analysis must be performed, the inclusion of this would not pose a serious problem.

Several simplifying assumptions were made in order to demonstrate the significance of hydrodynamic effects even with small displacements. It should be pointed out however, that the assumed incompressibility of the porous sand would be an unconservatism particularly for backfills which are not properly consolidated.

REFERENCES

1) 'Hydrology for Engineers', Linsley, Kohler and Paulus, McGraw-Hill
2) 'Introduction to Structural Dynamics', J.M. Biggs, McGraw-Hill

FIGURES

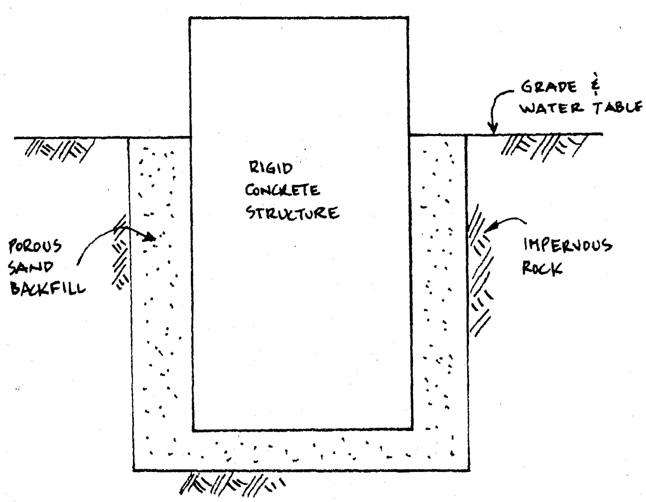

FIGURE 1

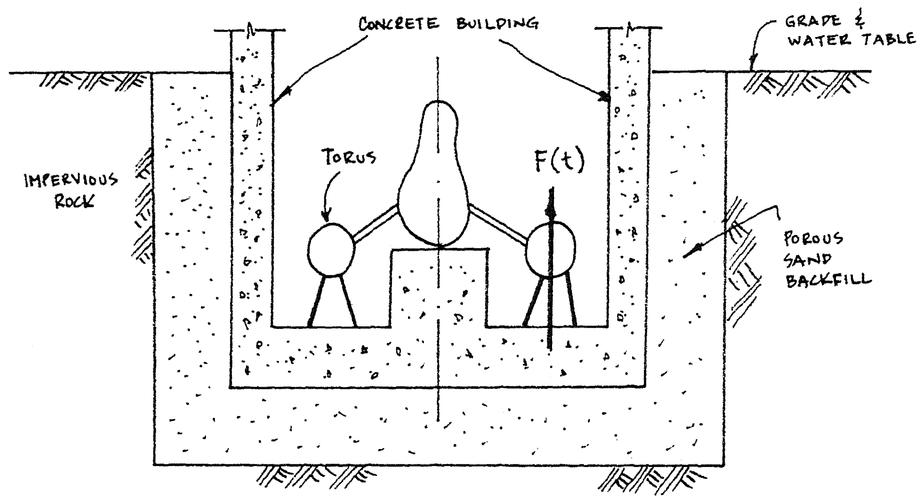

FIGURE 2

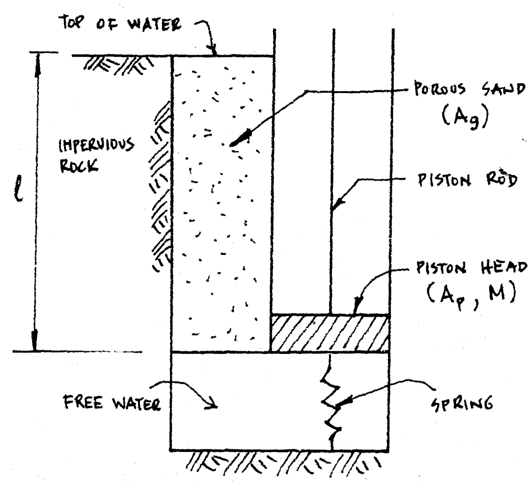

FIGURE 3