EFFECTS OF PILE GROUP FOUNDATION ON SEISMIC RESPONSE OF STRUCTURES

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SUMMARY

The equation for the responses of pile groups subjected to ground excitation is formulated idealizing the soil medium as a "Winkler model for pile group." Using the solution of this equation, the expression of the responses of the pile-supported structure subjected to the ground excitation is obtained. The computed results indicate that the piling at the base can alter the responses of the structure subjected to the ground excitation.

FORMULATION OF RESPONSE OF PILE-SUPPORTED STRUCTURE

Response of Pile Group

The grouped piles and soil system is divided into a number of layers and pile segments as shown in Fig. 1. The soil around the pile shafts will be treated as a "Winkler model for a pile group" as shown in Fig. 1 (1-4). Assuming a harmonic ground excitation and omitting the time factor, the equation of the piles within a segment is approximately written by

$$[EI]\ \{\frac{d^4u}{dz^4}\} - \omega^2 [m]\ \{u\} = -[k_x]\{\{u\} - U(z)\{1\}\} \tag{1}$$

where [EI] and [m] = flexural rigidity and mass per unit length of the pile, respectively; $[k_x]$ = complex stiffness of a "Winkler model for pile group" for lateral pile motion (3,4); {u} = lateral pile displacements; and U(z) = free-field soil displacement under harmonic ground motion induced by vertically propagating one-dimensional shear waves. Assuming $U(z) = Ae^{i\beta z} + Be^{-i\beta z}$ in which $\beta = \omega/v_s^*$, $v_s^* = v_s \times \sqrt{1 + 2Di}$, v_s = shear wave velocity, and D = soil material damping ratio, the solution of Eq. 1 is

$$\{u\} = [\phi(z)]\ \{C\} + U(z)\{\delta\} \tag{2}$$

where {C} = unknown constants; $[\phi(z)]\{C\} = \sum_{n=1}^{N} (C_{1n}e^{\lambda nz} + C_{2n}e^{-\lambda nz} + C_{3n}e^{i\lambda nz} + C_{4n}e^{-i\lambda nz})\{\eta\}_n$; λ_n^4 and $\{\eta\}_n$ = n-th eigenvalue and eigenvector obtained from the equation, $[(\lambda^4)[EI] - \omega^2[m] + [k_x]]\ \{C\} = \{0\}$; $\{\delta\} = [[f_x][\beta^4[EI] - \omega^2[m] + [I]]^{-1}\ \{1\}$ where $[f_x] = [k_x]^{-1}$.

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Eq. 2 leads to the displacement and force responses of the piles in a following matrix form

$$(\underline{u}\ \underline{\psi}\ \underline{P}\ \underline{M})^T = [\Phi(z)]\ \{C\} + (\underline{u}^*\ \underline{\psi}^*\ \underline{P}^*\ \underline{M}^*)^T \quad (3)$$

where $\underline{u}$ and $\underline{\psi}$ = vectors containing lateral and rotational displacements of N piles, respectively; $\underline{P}$ and $\underline{M}$ = vectors containing shear and moment forces of N piles; and

$$[\Phi(z)] = \begin{bmatrix} [\phi(z)] \\ [\phi'(z)] \\ EI\,[\phi'''(z)] \\ -EI\,[\phi''(z)] \end{bmatrix} \text{ and } (\underline{u}^*\ \underline{\psi}^*\ \underline{P}^*\ \underline{M}^*)^T = \begin{Bmatrix} U(z)\{\delta\} \\ U'(z)\{\delta\} \\ EI\ U'''(z)\{\delta\} \\ -EI\ U''(z)\{\delta\} \end{Bmatrix} . \quad (4)$$

The responses $\underline{u}^*$, $\underline{\Psi}$, $\underline{P}^*$ and $\underline{M}^*$ correspond to those of the infinitely long pile embedded in an infinite medium due to the free-field soil displacement U(z). Assuming the directions of the pile forces and the displacements as shown in Fig. 1, the following relationship is obtained from Eq. 3;

$$\{(\underline{u}\ \underline{\psi}\ \underline{P}\ \underline{M})^T-(\underline{u}^*\ \underline{\psi}^*\ \underline{P}^*\ \underline{M}^*)^T\}_{top} = [t]\ \{(\underline{u}\ \underline{\psi}\ \underline{P}\ \underline{M})^T-(\underline{u}^*\ \underline{\psi}^*\ \underline{P}^*\ \underline{M}^*)^T\}_{bottom} \quad (5)$$

where $[t] = [\Phi(0)][\Phi(\ell)]^{-1}$; "top" and "bottom" = locations at z = 0 and z = ℓ, respectively.

Applying the transfer matrix approach (1-4), Eq. 5 leads to the following relationship;

$$\{(\underline{u}\ \underline{\psi}\ \underline{P}\ \underline{M})_0^T-(\underline{u}^*\ \underline{\psi}^*\ \underline{P}^*\ \underline{M}^*)_0^T\} = [T]\{(\underline{u}\ \underline{\psi}\ \underline{P}\ \underline{M})_M^T-(\underline{u}^*\ \underline{\psi}^*\ \underline{P}^*\ \underline{M}^*)_M^T\} \quad (6)$$

where $[T] = [t_1]\ [t_2]\ \text{---}\ [t_M]$. Assuming $(\underline{P}\ \underline{M})_M^T = [k_b]\ (\underline{u}-U\ \underline{\psi})_M^T$, the following expression of the pile-head forces can be obtained from Eq. 6;

$$\begin{Bmatrix} \underline{P} \\ \underline{M} \end{Bmatrix}_0 = [K_H]\begin{Bmatrix} \underline{u} \\ \underline{\psi} \end{Bmatrix}_0 - [K_h]\begin{Bmatrix} \underline{u}^* \\ \underline{\psi}^* \end{Bmatrix}_0 + \begin{Bmatrix} \underline{P}^* \\ \underline{M}^* \end{Bmatrix}_0 - [A]\begin{Bmatrix} \underline{u}^* \\ \underline{\psi}^* \end{Bmatrix}_M - [B]\begin{Bmatrix} \underline{P}^* \\ \underline{M}^* \end{Bmatrix}_M - [C]\begin{Bmatrix} \underline{1} \\ \underline{0} \end{Bmatrix} U_M$$

$$\approx [K_H]\begin{Bmatrix} \underline{u} \\ \underline{\psi} \end{Bmatrix}_0 - [K_H]\begin{Bmatrix} \underline{u}^* \\ \underline{\psi}^* \end{Bmatrix}_0 + \begin{Bmatrix} \underline{P}^* \\ \underline{M}^* \end{Bmatrix}_0 \quad (7)$$

where $[K_h]$ = flexural pile-head stiffness = $[[T_{11}] + [T_{12}][k_b]]\ [[T_{21}] + [T_{22}]\ [k_b]]^{-1}$; $[A] = [T_{21}] - [K_h][T_{11}]$; $[B] = [T_{22}] - [K_h][T_{12}]$; $[C] = [T_{22}][k_b] - [K_h]\ [T_{12}]\ [k_b]$; and $[T_{11}]$, $[T_{12}]$, $[T_{21}]$ and $[T_{22}]$ are submatrices defined by

$$[T] = \begin{bmatrix} [T_{11}] & [T_{12}] \\ [T_{21}] & [T_{22}] \end{bmatrix} .$$

Following similar procedures described above, the axial pile-head forces can be also expressed by (1,3,4)

$$\{P\}_0 = [K_V]\{w\}_0 \quad (8)$$

where $[K_v]$ = axial pile-head stiffness; and $\{w\}_0$ and $\{P\}_0$ = axial pile displacement and force, respectively.

Response of Structure Supported by Pile Foundation

The equations of the motion of the structure subjected to ground excitation as shown in Fig. 2 can be written by

$$[m_{st}]\{\ddot{u}(t)\} + [c_{st}]\{\dot{x}(t)\} + [k_{st}]\{x(t)\} = \{0\}$$

$$\begin{Bmatrix} \{1\}^T[m_{st}]\{\ddot{u}(t)\} \\ \{h\}^T[m_{st}]\{\ddot{u}(t)\} + I_0\,\ddot{\psi}(t) \end{Bmatrix} = -\begin{Bmatrix} P(t) \\ M(t) \end{Bmatrix} \tag{9}$$

where $[m_{st}]$, $[c_{st}]$ and $[k_{st}]$ = mass, damping and stiffness matrices of the structure, respectively; $\{h\}$ = heights of the masses of the structure; $(P(t)\ M(t))^T$ = forces acting at the base of the structure; $\{u\}$ and $\{x\}$ = displacements defined in Fig. 2; and I_0 = mass moment of inertia of the base. The forces acting at the piled base are the summation between those through the piles and the contact between the base and soil. Assuming a harmonic ground excitation and omitting the time factor, the forces acting at the rigid base are expressed by

$$\begin{Bmatrix} P \\ M \end{Bmatrix} = [K]\begin{Bmatrix} u_0 \\ \psi \end{Bmatrix} + \begin{Bmatrix} \Delta P \\ \Delta M \end{Bmatrix} \tag{10}$$

where, denoting the stiffness matrix of massless rigid basemat without piles as $[K_b]$, $[K]$ and $(\Delta P\ \Delta M)^T$ are

$$[K] = \begin{bmatrix} \{1\}^T[K_h(P,u)]\{1\} & \{1\}^T[K_h(P,\psi)]\{1\} \\ \{1\}^T[K_h(M,u)]\{1\} & \{1\}^T[K_h(M,\psi)]\{1\} + \{d\}^T[K_v]\{d\} \end{bmatrix} + [K_b]$$

$$\Delta P = -\{1\}^T[K_h(P,u)]\{u^*\} - \{1\}^T[K_h(P,\psi)]\{\psi^*\} + \{1\}^T\{P^*\} - K_b(P,u)U_0$$

$$\Delta M = -\{1\}^T[K_h(M,u)]\{u^*\} - \{1\}^T[K_h(M,\psi)]\{\psi^*\} + \{1\}^T\{M^*\} \tag{11}$$

where U_0 = free-field soil response at the ground surface; $\{d\}$ = distances between the center of the rotation over the piles; and the notations related to the stiffnesses are defined by

$$[K_h] = \begin{bmatrix} [K_h(P,u)] & [K_h(P,\psi)] \\ [K_h(M,u)] & [K_h(M,\psi)] \end{bmatrix} \text{ and } [K_b] = \begin{bmatrix} K_b(P,u) & K_b(P,\psi) \\ K_b(M,u) & K_b(M,\psi) \end{bmatrix} . \tag{12}$$

COMPUTED RESULTS

Eq. 10 can be broken into the following two equations:

$$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = [K]\begin{Bmatrix} u_a \\ \psi_a \end{Bmatrix} + \begin{Bmatrix} \Delta P \\ \Delta M \end{Bmatrix} \quad \text{(13a)} \quad \text{and} \quad \begin{Bmatrix} P \\ M \end{Bmatrix} = [K]\begin{Bmatrix} u_b \\ \psi_b \end{Bmatrix} \quad \text{(13b)}$$

where $u_0 = u_a + u_b$ and $\psi = \psi_a + \psi_b$. Then, the absolute acceleration of the masses in Eq. 9 is expressed by $\{\ddot{u}(t)\} = \{\ddot{u}_a(t)\{1\} + \ddot{\psi}_a(t)\{h\}\} + \{\ddot{u}_b(t)\{1\} + \ddot{\psi}_b(t)\{h\} + \{\ddot{x}(t)\}\}$ in which $\ddot{u}_a$ and $\ddot{\psi}_a$ are known accelerations from Eq. 13a. In this case, $(P\ M)^T$ in Eq. 13b is $(P\ M)^T$ at the right hand side of Eq. 9, and $\ddot{u}_a$ and $\ddot{\psi}_a$ are the input motions in

Eq. 9. Thus, u_a and ψ_a correspond to the transmitted motions to the structure from the foundation system, and u_b and ψ_b correspond to the feed-back motions to the foundation system. Eq. 13 indicates that the conditions of the foundation affect the seismic response structure through the transmitted motions and the mechanical properties of the foundation.

When the structure is directly placed on the ground surface and is not supported by the piles, the transmitted motion to the structure is identical to a free-field ground motion at the surface (U_0). When the piles are used in the foundation system of the structure, the transmitted motions are modified by the piles as shown in Fig. 3. It is interesting to note that the ratio between the transmitted motion and the free-field ground surface motion is independent of the thickness of the soil deposit, when the pile tips are not fixed at the bedrock. The interesting features observed in the figure are; 1) the piles reduce the high frequency components of the lateral transmitted motion; 2) the amount of this reduction is larger for the stiffer piles; 3) the piles produce the rotational transmitted motion; 4) the rotational transmitted motion is larger for the stiffer and shorter piles; and 5) the group effects in the pile foundtion tend to enhance the above mentioned features. Fig. 4 shows the effects of piling on the transmitted motions due to 4x4 piles, where all dimensions are identical to Case A shown in Fig. 3 and the basemat area is four times of that for 2x2 piles previously considered. The results show that, the increases of the basemat area and numbers of piles reduce the rotational transmitted motion significantly, and reduce the high frequency components of the lateral transmitted motions further when the piles are stiff and basemat is in full contact with ground surface.

Fig. 5, which is obtained from the stiffness matrix [K] for Case A, illustrates that piles modify the mechanical properties of the foundation. It is seen that piling increases the values of those parameters, particularly those for the rotational motion. When the piles are used in a group, the variation of those parameters with frequency is complex due to the group effect. The group effect increases or decreases the values from those without group effect. Whether it decreases or increases is governed by the interference effects of the horizontally propagating waves generated from the piles in the soil medium (2,3,4).

The piles affect the responses of the structure subjected to ground excitation through affecting the transmitted motions and the mechanical properties of the structure and foundation system. Fig. 6 shows the effects of the piles on the responses of the structures subjected to ground excitation. The structure is idealized as a single degree of freedom system characterized by the natural frequency, ω_n, and the damping ratio, ξ, of the structure fixed at the base. The foundation "Case A" with 2x2 stiff piles was considered in the study. The transmitted motions and the spring and dashpot constants of this foundation under various conditions are shown in Figs. 3 and 5, respectively. The base was assumed to be massless. The figure indicates that; 1) the piles increase the natural frequency of the total system; 2) this trend is more significant for the structure with a large height-width ratio; 3) the piles increase the base shear at the frequencies above the resonant frequency of the piled-structure when the structure has a large height-width ratio, but can reduce it in this frequency range when the structure has a small height-width ratio; and 4) the piles generally reduce the base shear at the frequencies below the resonant frequency of the unpiled structure; this reduction is more significant for the structures with a larger height-width ratio.

CONCLUSIONS

Idealizing the soil as a "Winkler model for pile group" can greatly simplify the expression of the behavior of pile groups subjected to ground motion. When a group of piles are used as a part of the foundation system, they modify not only the mechanical properties of the foundation system but also the transmitted motions to the structure. This results in decreasing or increasing the base shear induced in the structure. The trend of this change in the response of the structure depends on the height-width ratio of the structure and the frequency contents of the motions, and is more pronounced for the structures with a larger height-width ratio.

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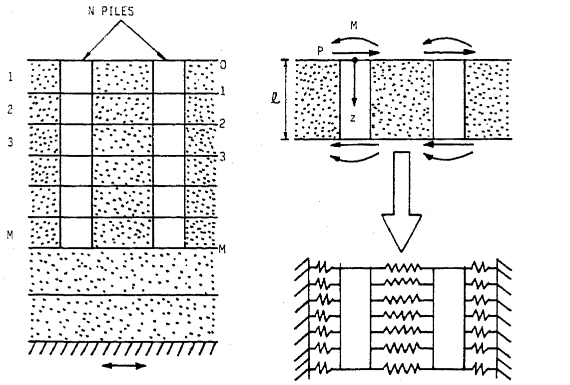


(a) Divided Soil-Pile System (b) Soil Layer Idealized as Winkler Model

Fig. 1 Idealization of Pile Group

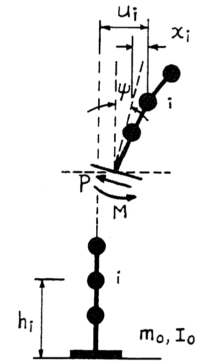
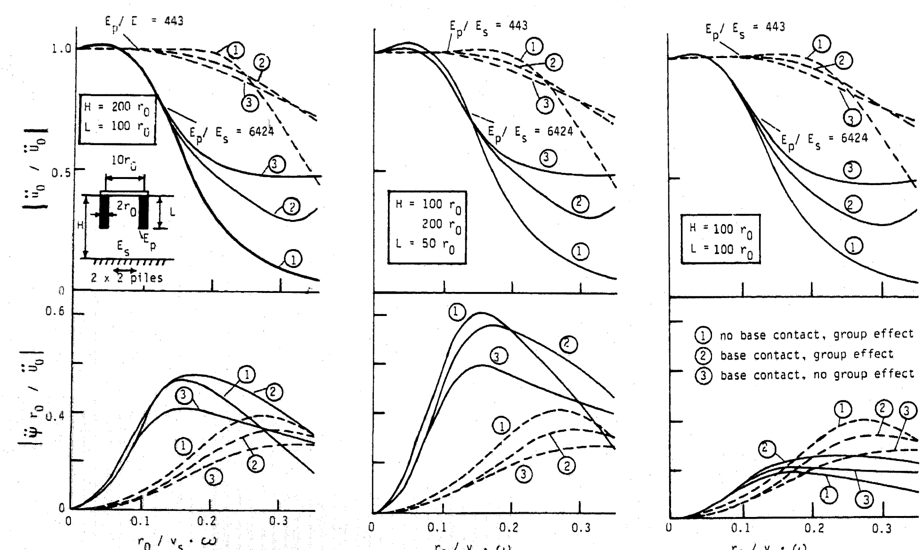


Fig. 2 Displacements of Structure Subjected to Ground Excitation



(a) Case A (b) Case B and Case C (c) Case D

Fig. 3a Transmitted Motions from Foundation made of 2x2 Piles (Amplitudes)

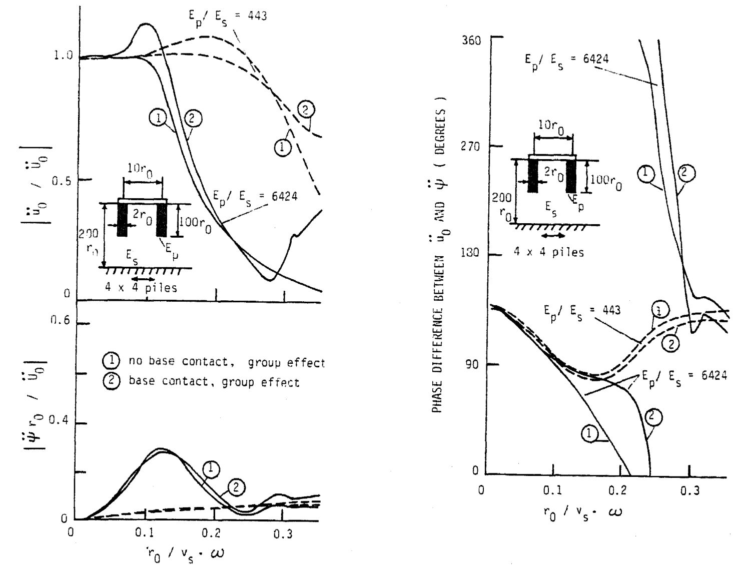


Fig. 4 Transmitted Motions from Foundation made of 4x4 Piles

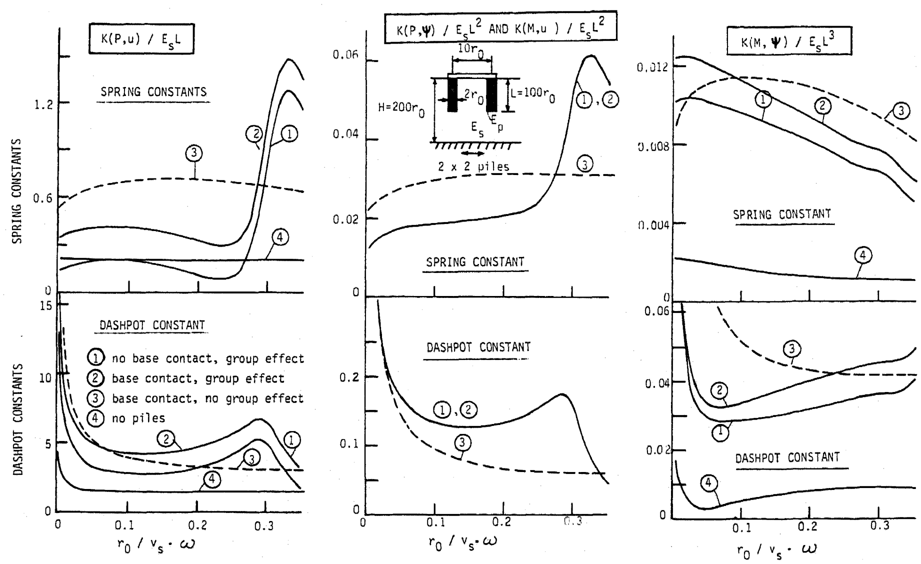


Fig. 5 Equivalent Spring and Dashpot Constant of Foundations

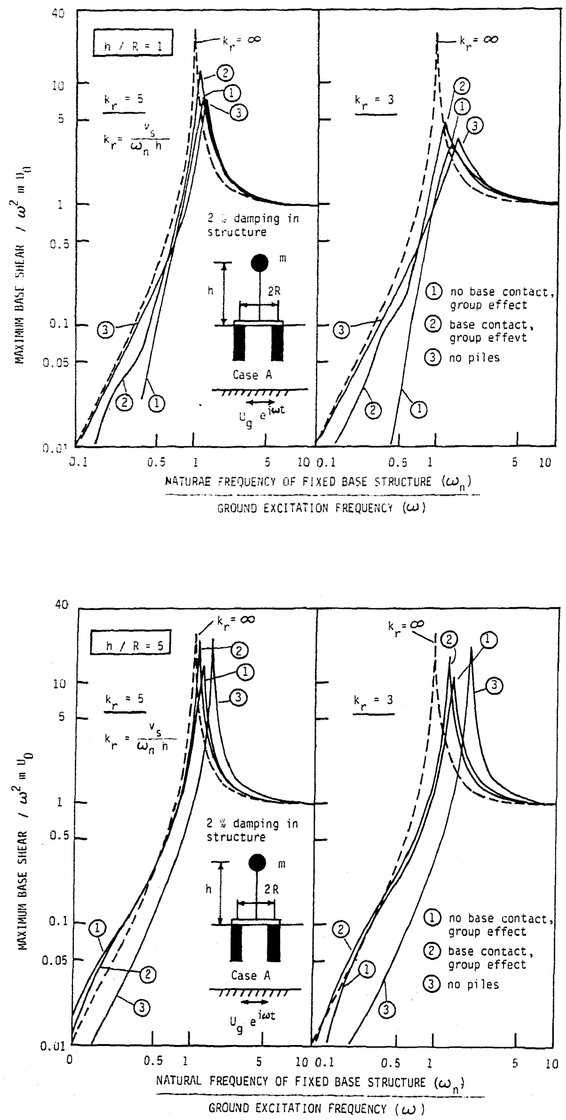


Fig. 6 Base Shear of Structure Induced by Ground Excitation