IN SITU SHEAR WAVE VELOCITIES FROM SPECTRAL ANALYSIS OF SURFACE WAVES

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SUMMARY

The Spectral-Analysis-of-Surface-Waves method is a new seismic method for quick and economical determination of shear wave velocity in situ. A vertical impact applied to the ground surface is used to generate Rayleigh waves with different frequencies. Propagation of the waves is monitored with two surface receivers located a known distance apart. By employing a fast-Fourier transform and spectral analyses, Rayleigh wave velocity for each frequency is calculated. With inversion, shear wave velocity, shear modulus and layering of the media are determined. Several case studies illustrate the utility of the method.

INTRODUCTION

Shear modulus and its variation with shearing strain are essential in characterizing soil behavior during earthquake shaking. The need for accurate and economical measurement of this variable has increased greatly over the last decade, primarily due to the increased complexity of analytical procedures. One of the key elements in characterizing shear moduli in situ is measurement of shear wave velocity in the field at small strains, shearing strains less than 0.001 percent. The state-of-practice in such measurements employs seismic techniques such as the crosshole and downhole tests. However, these in situ tests are under-utilized because of economic, time and personnel considerations.

A new seismic method for in situ measurement of low-amplitude shear wave velocities of soil deposits and thicknesses of soil layers is presented herein. This method is called the Spectral-Analysis-of-Surface-Waves (SASW) method. The SASW method is based upon generation and measurement of surface waves, Rayleigh waves. The method is fast, economical, nondestructive and requires no boreholes. In addition, the method has the potential of nearly full automation; hence, the need for specialized personnel to perform the test and analyze the data is minimized.

The testing technique and data reduction procedures associated with SASW testing are first discussed herein after which several case studies are presented. In the writers' experience at more than fifteen sites, shear wave velocities from crosshole and SASW tests are usually within 10 percent, and layering determined from the SASW profile is usually within a few inches of that determined from boring logs.

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METHOD OF SPECTRAL-ANALYSIS-OF-SURFACE-WAVES

The Spectral-Analysis-of-Surface-Waves method is a seismic method for determination of shear wave velocity and shear modulus profiles and layer thicknesses of soil, rock or pavement sites. The key to successful SASW testing is generation and measurement of Rayleigh waves. The test is conducted by placing two receivers on the ground surface at a preselected spacing. A vertical impulse is then applied to the surface which generates a transient signal containing Rayleigh waves over some range of frequencies. This group of waves is monitored by the receivers and captured in the time domain by a recording device. By utilizing a discrete-Fourier transform (DFT) algorithm, waves monitored by the receivers are digitized and transformed to the frequency domain. Spectral analysis techniques are then used to obtain the coherence function and phase information from the crosspower spectrum. The coherence function is a quantitive means of assessing the quality of signals captured by the receivers and the degree of distortion from noise. Coherence has a value between zero and unity, with a value of unity representing perfect correlation between the signals.

Phase information of the crosspower spectrum represents the phase difference between the two receivers at each frequency based on a rotating vector concept. By knowing the distance between receivers and the phase shift for each frequency, Rayleigh wave velocity and wavelength for each frequency are calculated. With this information a dispersion curve is constructed. A dispersion curve is a plot of (apparent) Rayleigh wave velocity versus wavelength. By applying an inversion process, an analytical technique for reconstructing the profile from the dispersion curve, the actual shear wave velocity profile is developed from which the shear modulus profile is calculated and layer thicknesses are determined.

In short, SASW testing and analysis procedures consist of three steps: first, field testing; second, construction of the dispersion curve; and finally inversion of the dispersion curve. Each of these three steps is discussed in detail in the following paragraphs.

Field Testing

The testing configuration is shown in Fig. 1a. To start the test, an imaginary centerline for the receiver array is selected. Two receivers are then placed on the ground surface an equal distance apart from the centerline. A vertical impulse is applied to the ground by means of a hammer. The impulse generates transient Rayleigh waves of various frequencies. Impulses are delivered several times, and the signals are averaged together. Heisey et al (Ref. 1) recommend that the optimum distance between the source and near receiver be approximately equal to the distance between receivers.

After testing in one direction is completed, the receivers are kept in their original position, but the source is moved to the opposite side of the imaginary centerline, and the test is repeated. By performing forward and reversed tests and by averaging the records from the two directions, the effect of any internal phase in the monitoring and recording systems is minimized.

Forward and reversed tests are performed at several receiver spacings as shown in Fig. 1b. The spacing between receivers is usually doubled from one spacing to the next. In each case symmetry of the receivers about the imaginary centerline is maintained. This arrangement is known as common receivers midpoint (CRMP) geometry (Ref. 2) and is preferred because the area covered in the previous test is included in the subsequent test. Close receiver spacings are used to sample near-surface material. As the distance between receivers increases, deeper materials are sampled.

Different sources are used for different receiver spacings. For close receiver spacings (4 ft or less) a small hammer (such as a claw hammer) capable of generating high frequencies (above 250 Hz) is used. As receiver spacing increases, heavier sources capable of generating lower frequencies are required. For the largest spacing of 128 ft used to date, a 1000-pound hammer has been used.

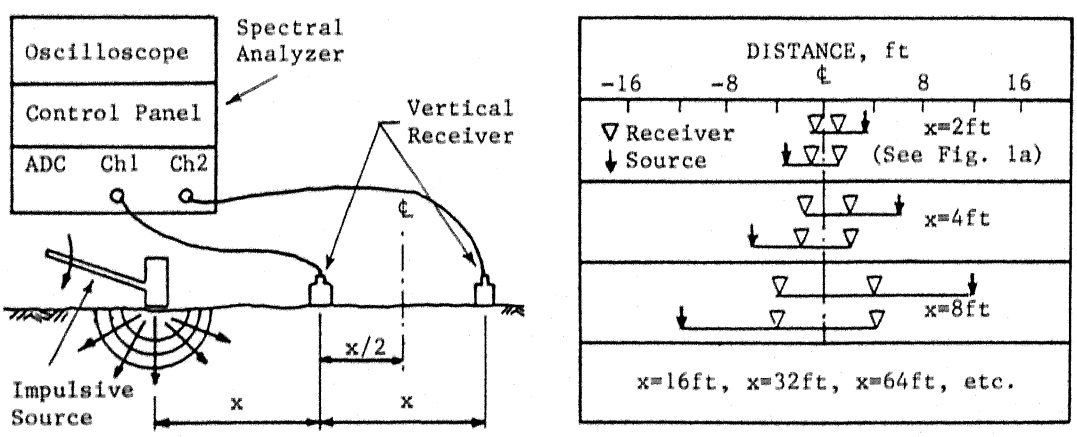
The recording device used in SASW testing is a spectral analyzer. A spectral analyzer is a digital oscilloscope that by means of a micro-computer attached to it has the capability of functioning in both the time and frequency domains. Spectral analysis functions can be performed rapidly with the equipment and can be saved on magnetic tape.

Construction of Dispersion Curve

A typical coherence function and phase information from the cross-power spectrum are shown in Fig. 2. These results were determined from one set of forward tests with a receiver spacing of 8 ft. The range of the frequencies excited is selected from the coherence function which for this record is from 15 Hz to 160 Hz. For each frequency (f) in this range, the phase difference (Φ) is determined, and the travel time between receivers (t) is calculated by

$$t = \frac{\Phi \cdot T}{360} \tag{1}$$

where the period (T) is the reciprocal of the frequency. Because the



a) General Configuration

b) Source-Receiver Array

Fig. 1 - Schematic of Experimental Arrangement for SASW Tests

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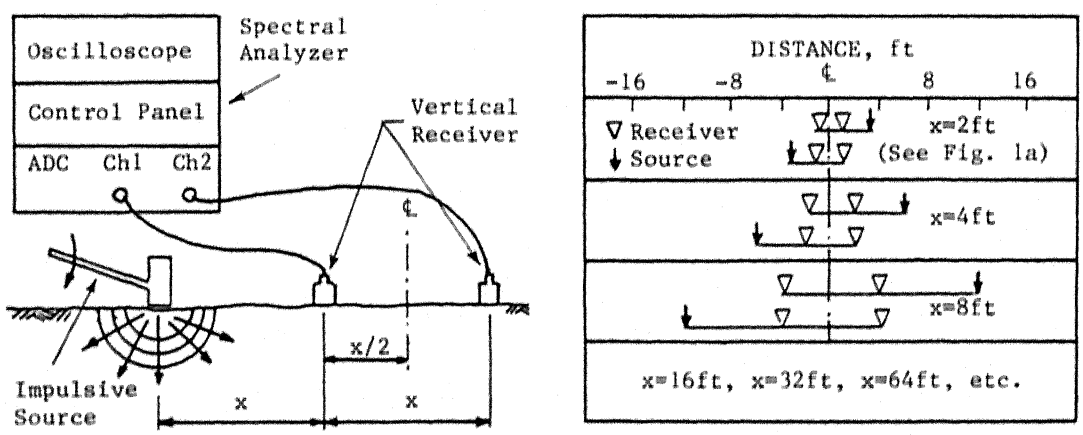

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b) Source-Receiver Array

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distance between receivers (X) is a known variable, Rayleigh wave velocity (V_R) at each frequency is simply calculated by:

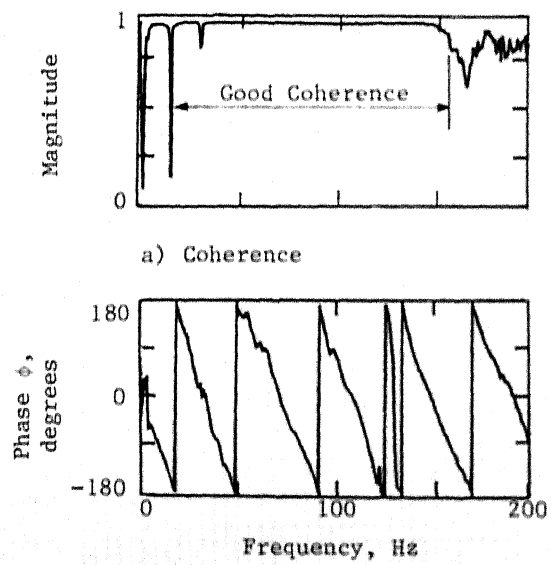
$$V_R = X/t \qquad (2)$$

and the corresponding Rayleigh wavelength (L_R) is calculated from

$$L_R = V_R/f \qquad (3)$$

By repeating this procedure for every frequency, the Rayleigh wave velocity corresponding to each wavelength is calculated.

If the stiffness of a soil site varies with depth, then the velocity of the Rayleigh (R) wave will vary with frequency or wavelength. The variation of R-wave velocity with wavelength is known as dispersion, and a plot of velocity versus wavelength is known as a dispersion curve. Based on tests at several soil sites, Heisey et al (Ref. 1) recommend that any data point with a wavelength greater than three times or less than one-half the distance between receivers should not be used to construct dispersion curves. These filtering criteria (which are based on equipment resolution and near-source effects) are applied to the dispersion curve for each receiver spacing. As the distance between geophones is doubled each time, dispersion curves from different tests over lap, and the final outcome consisting of overlapping dispersion curves is continuous. For example, the resulting continuous curve for the first case study (Wildlife site) is shown in Fig. 3.



b) Crosspower Spectrum

Fig. 2 - Typical Spectral Functions for a Receiver Spacing of 8 ft

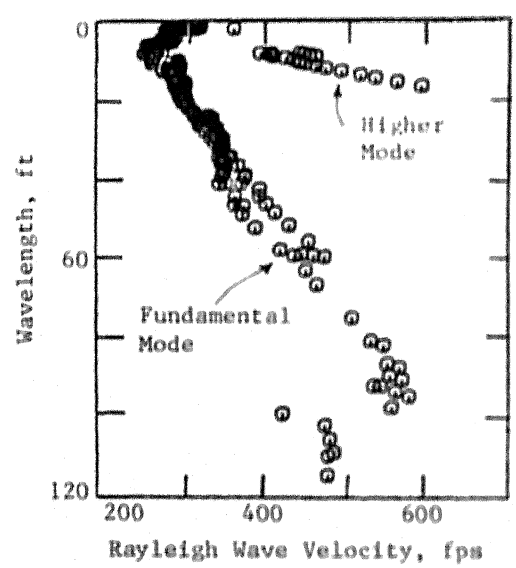


Fig. 3 - Dispersion Curve from Tests Performed at Wildlife Site

Inversion of Dispersion Curve

Inversion of the dispersion curve is the process of determining actual propagation velocities (especially shear wave velocities) from the dispersion curve. The crudest and simplest way of obtaining the shear wave velocity profile using the dispersion curve is to assume that shear wave velocity is approximately equal to 110 percent of Rayleigh wave velocity and the depth of sampling is equal to 0.50 to 0.33 of the wavelength (Refs. 1 and 3). Existence of a layer with relatively high or low velocity close to surface will result in a general shift of the R-wave velocities on the dispersion curve towards higher or lower velocities, respectively. As such, use of this crude method of inversion will cause some degree of error. However, for many soil deposits in which the properties do not vary greatly, utilization of this process may cause only a small error. For media in which there exists a significant variation in material properties with depth, the final shear wave velocity profile is significantly in error.

The inversion process used in this study is based on Haskell-Thomson's (Refs. 4 and 5) matrix for multilayered media with some modifications. The profile is divided into several layers. Generally, 15 to 20 layers are selected. For a soil site, uniform layer thicknesses are normally assumed. (For a pavement site, thin layers are selected close to the surface, with the thickness progressively increasing with depth.) A shear wave velocity profile using the crude inversion method discussed in the paragraph above is then selected as a first guess for each layer, and a theoretical dispersion curve is obtained from a computer program coded for this purpose. The theoretical curve and experimental curve (constructed from field data) are compared. If the two curves match within a reasonable tolerance the shear wave velocity profile is obtained. Otherwise, assumed shear wave

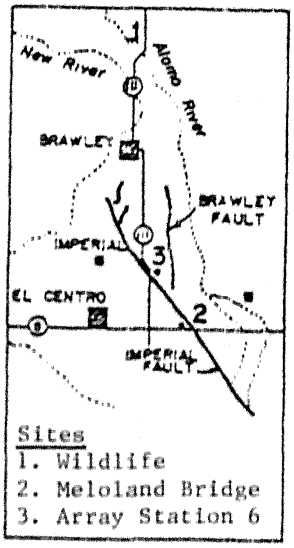


Fig. 4 - Plan View of Sites Tested in Imperial Valley

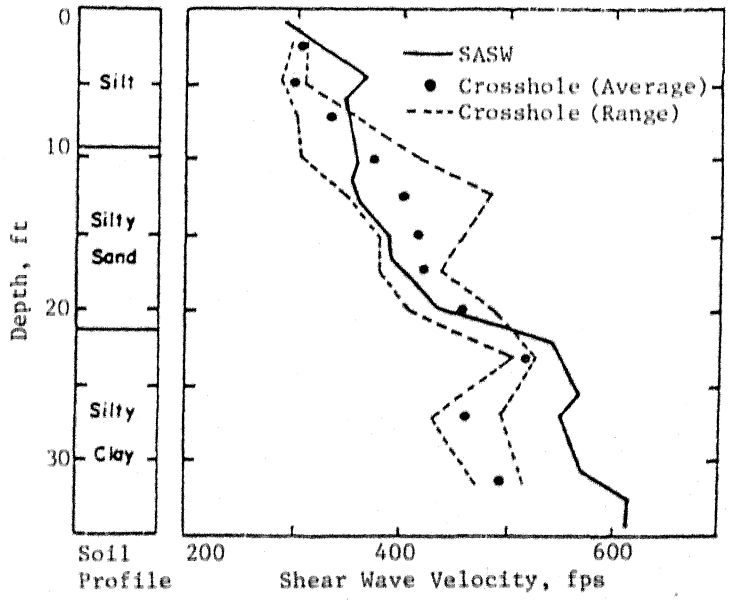


Fig. 5 - Shear Wave Velocity Profiles at Wildlife Site

velocities are changed, and the theoretical dispersion curve is recalculated. This trial and error process continues until the experimental and theoretical curves match reasonably well.

In this process the mass density and Poisson's ratio of different layers are assumed. However, the effect of these parameters on the final outcome is small (Ref. 6), especially for geotechnical engineering projects in which Poisson's ratio and mass density of different materials fall into fairly narrow ranges.

Once the shear wave velocity profile is determined, low-amplitude shear moduli (G_{max}) can be calculated by:

$$G_{max} = \rho V_s^2 \tag{4}$$

where ρ and V_s are mass density and shear wave velocity, respectively.

CASE STUDIES

In January 1983, SASW tests were performed at several soil sites in Imperial Valley, California. These sites were subjected to two recent earthquakes; Imperial Valley, 1979 and Westmorland, 1981. The location of the sites and their proximity to two major faults are shown in Fig. 4. The Wildlife site, which is located next to the Alamo River, exhibited extensive liquifaction during the 1981 earthquake.

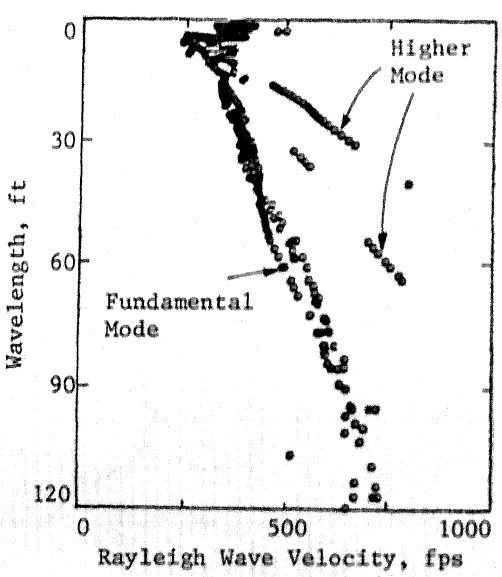


Fig. 6 - Dispersion Curve from Tests Performed at Array Station 6

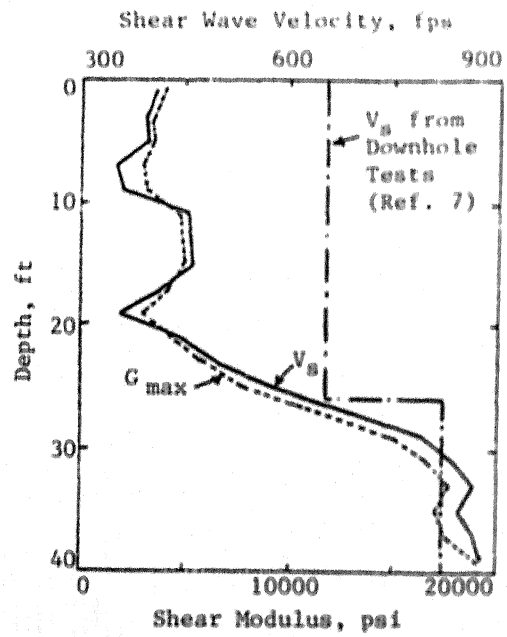


Fig. 7 - Shear Wave Velocity and Modulus Profiles at Array Station 6

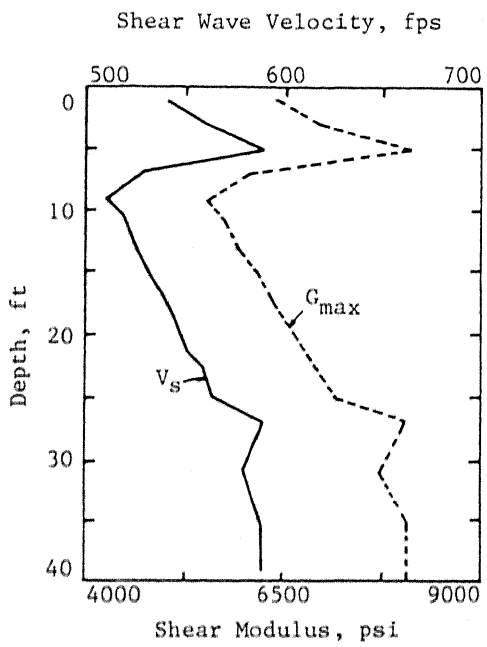


Fig. 8 - Shear Wave Velocity and Modulus Profiles at Meloland Bridge Overpass

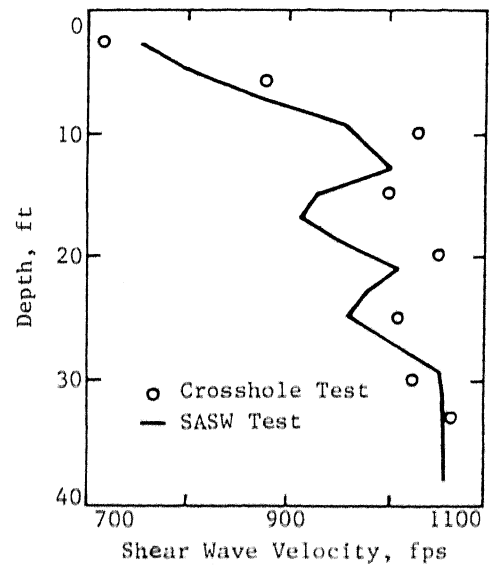


Fig. 9 - Shear Wave Velocity Profiles at Crest of Earth Dam

The shear wave velocity profile obtained at the Wildlife site by the SASW method is shown in Fig. 5. To construct the profile, 20 layers were used in the inversion process. The resulting profile is quite detailed, illustrating a significant benefit of the method. Also shown in Fig. 5 is the shear wave velocity profile determined by crosshole tests. The two seismic profiles compare very favorably with velocities from the two independent methods generally within about 10 percent.

Two other sites investigated in Imperial Valley are at the locations of the seismographs at Array Station 6 and the Meloland Bridge Overpass. The dispersion curve determined at Array Station 6 is shown in Fig. 6, and the V_s profiles at the two sites are shown in Figs. 7 and 8. Unfortunately, crosshole tests were not performed at either site. However, shear wave velocities determined from downhole tests performed by USGS personnel at Array Station 6 (Ref. 7) are shown in Fig. 7. A good comparison between wave velocities by the two independent methods is found below a depth of about 25 ft. At shallower depths, the SASW tests are felt to be more accurate.

The fourth site is an earth dam in South Carolina. The dam was being analyzed for performance during a design earthquake, and thus in situ shear wave velocity profiles were necessary. Testing was performed at several crest and toe sites, and the results from one crest site are shown in Fig. 9. Both SASW and crosshole tests were carried out at this site. The

comparison between the independent methods is very favorable, with the maximum difference in V_s values less than about 10 percent. However, the profile obtained by the SASW method illustrates the shear wave velocity profile in much more detail.

Field testing at each site took less than four hours, and in-house data reduction time for each site was approximately five hours. It is expected that with development of new hardware and software complete evaluation of a site can be done in the field in less than two hours. Also, development of a source capable of generating low frequencies, in the range of two to five Hertz, will permit evaluation of soils to depths of at least 150 ft. For testing presented in these studies, the lowest frequency was on the order of five Hertz.

CONCLUSIONS

The Spectral-Analysis-of-Surface-Waves (SASW) method for in situ determination of low-amplitude shear wave velocity, shear modulus, and layer thicknesses shows great potential. The method is based upon generation and measurement of transient Rayleigh waves and, as such, is fast and requires no boreholes. By employing elastic wave theory, fast-Fourier transforms and spectral analyses, Rayleigh wave velocity, shear wave velocity and shear modulus at numerous points in a material profile are obtained.

Four case studies illustrating the SASW method are presented. In two cases the SASW results are compared with shear wave velocities independently determined by crosshole seismic tests. The profiles from the two methods compare favorably, indicating the accuracy of the SASW method. In addition, a more comprehensive V_s profile is obtained by the SASW method in a fraction of the time required by other methods.

REFERENCES

1. Heisey, J.S., K.H. Stokoe, II, and A.H. Meyer, "Moduli of Pavement Systems from Spectral Analysis of Surface Waves," Transportation Research Record, No. 852, Washington, D.C., pp. 22-31, 1982.
2. Nazarian, S., K.H. Stokoe, II and W.R. Hudson, "Use of Spectral Analysis of Surface Waves for Determination of Moduli and Thicknesses of Pavement Systems," Transportation Research Record, No. 945, Washington D.C., 1983.
3. Ballard, R.F., Jr., "Determination of Soil Shear Moduli at Depth by In Situ Vibratory Techniques," U.S. Army Waterways Experiment Station, 1964.
4. Haskell, N.A., "The Dispersion of Surface Waves in Multilayered Media," Bulletin of Seismological Society of America, Vol. 43, No. 1, pp. 17-34, 1953.
5. Thomson, W.T., "Transmission of Elastic Waves through a Stratified Solid Medium," Journal of Applied Physics, Vol. 21, pp. 89-93, February, 1950.
6. Grant, F.S., and G.F. West, Interpretation Theory in Applied Geophysics, McGraw-Hill Book Co., Inc., New York, NY, 1965.
7. Chen, A.T.F., and M.J. Bennett, "Site Characterization for Stations 6 and 7, El Centro Strong Motion Array, Imperial Valley, CA" USGS Open File Report 82-1640, August, 1982.