A SIMPLIFIED METHOD FOR DYNAMIC ANALYSIS OF EMBANKMENT

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SUMMARY

Simplified analytical methods were proposed for estimating a seismic response of embankments constructed on horizontally multi-layered grounds. This response of embankments is obtained by multiplying three functions, a frequency response function of the embankment on a rigid base; a function expressing the effect of interaction between the soil layer and the embankment; and the frequency response function of free surface motion. Calculated results were compared with the solution obtained by the sub-structure method. Finally a new approach was developed to transfer the effect of multi-layered ground into the natural frequency and modal damping of embankment resting on the rigid base.

INTRODUCTION

For many types of embankments constructed on multi-layered ground, the response magnitude which is induced by earthquake shaking is strongly affected by the structure of soil layers, the shape and material characteristics of the embankment. The practical methods proposed so far have neglected the influence of the ground layer and assumed that the embankment is a triangle shear beam resting on the rigid base. We here propose simplified analytical procedures for estimating the response of embankments by considering the effect of ground rigidity. The first stage is to develop a hybrid analytical method combining the wave solution of a multi-layered ground with the finite element scheme of the embankment as an arbitrary configuration. The second stage is to propose the method to estimate equivalent natural frequencies and modal dampings to be used in the mode-superposition analysis. In these equivalent values, we consider the effects of the rigidity of the underlying soil layer and the energy dissipation caused by waves radiating into the half-space.

BASIC EQUATION FOR ELASTIC MEDIA

All the problems considered in this section are two dimensional. The matrix method developed by Thomson and corrected by Haskell (Ref.1) is used to express the response of the multi-layered ground. In this method, the ground is assumed to have horizontally homogeneous multi-layers overlaying a homogeneous half-space. The horizontal and vertical displacement components $\bar{u}_x$ and $\bar{u}_z$, and stress components $\bar{\tau}_{zx}$ and $\bar{\tau}_{zz}$, are written as follows:

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$$\bar{u}_x = r_1(k,z,\omega)\exp\{i(kx-\omega t)\},\ \bar{\tau}_{zx} = r_3(k,z,\omega)\exp\{i(kx-\omega t)\}$$
$$\bar{u}_z = r_2(k,z,\omega)\exp\{i(kx-\omega t)\},\ \bar{\tau}_{zz} = r_4(k,z,\omega)\exp\{i(kx-\omega t)\} \quad (1)$$

in which k is the wave number of x direction, and ω is the circular frequency of the propagating wave, and $i=\sqrt{-1}$.

Although the expression of Eq.(1) is in the domain of wave number, we can express the solution in a Cartesian coordinate (x,z) as a superposition of plane waves as follows:

$$f(x,z,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(k,z,\omega)\exp(ikx)\,dk \quad (2)$$

in which $\bar{f}$ is one of functions appearing in Eq.(1). Considering the continuity of displacements and stresses at an interface between two layers, the following expression is obtained

$$R_{j+1} = P_j R_j \quad (3)$$

in which subscripts j and $(j+1)$ mean the numbers of the layers counting from the ground surface, P_j is the transfer matrix of jth layer given by Aki (Ref.2), and R_j is the motion-stress vector at the top of the jth layer given by $(r_1,r_2,r_3,r_4)^T$.

The relation given by Eq.(3) is used to find a relation between the motion-stress vector R_1 and the displacement amplitude $w=(\grave{P},\grave{S},\acute{P},\acute{S})^T$ at the top of base layer (Nth layer)

$$Fw = P_{N-1}P_{N-2}\cdots P_j\cdots P_1R_1 \quad (4)$$

in which F is given by Aki (Ref.2), $\grave{P}$ and $\grave{S}$ are amplitudes of down going P and S waves, $\acute{P}$ and $\acute{S}$ those of incident waves.

If $\acute{P}$ and $\acute{S}$ are given, the following expression is obtained from Eq.(4) among displacement amplitudes $\bar{u}=(r_1,r_2)^T$ and stress amplitudes $\bar{\tau}=(r_3,r_4)^T$ at ground surface, and incident wave amplitude

$$\bar{u}(k,\omega) = A(k_0,\omega)g - S(k,\omega)\bar{\tau}(k,\omega) \quad (5)$$

in which k_0 is the wave number of incident wave motion.

Substituting Eq.(5) into Eq.(2), we can get

$$\bar{u}(x;\omega) = \bar{u}^*(x,\omega) - \frac{1}{2\pi}\int_{-\infty}^{\infty}S(k,\omega)\bar{\tau}(k,\omega)\exp(ikx)\,dk$$
$$\bar{u}^*(x,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}Ag\cdot\exp(ikx)\,dk \quad (6)$$

If the stress distribution $\tau(x,\omega)$ defined by the finite discrete values σ_n $(n=1,2,\cdots,m)$ and interpolating functions $\xi_n(x)$ as follows

$$\tau(x,\omega) = \sum_{n=1}^{m}\xi_n(x)\sigma_n \quad (7)$$

The transformation into wave number region of Eq.(7) is

$$\bar{\tau}(k,\omega) = \sum_{n=1}^{m}\int_{-\infty}^{\infty}\xi_n(x)\sigma_n\cdot\exp(-ikx)\,dx = \sum_{n=1}^{m}\bar{\xi}_n(k)\sigma_n \quad (8)$$

Substituting Eq.(8) into Eq.(6)

$$u(x,\omega) = u^*(x,\omega) - \sum_{n=1}^{m}g_n(x,\omega)\sigma_n,\quad g_n(x,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}S(k,\omega)\bar{\xi}_n(k)\exp(ikx)\,dk \quad (9)$$

HYBRID ANALYSIS OF SOIL STRUCTURE SYSTEMS

The response of soil structure with arbitrary configuration is usually calculated by transforming continuous media into a discrete system using such spatial and time discretization techniques as the finite element method. However, in most such analysis, it has been necessary to assume that the underlying soil layer is rigid. This makes it difficult to take into account the energy dissipation caused by waves radiating into underlying half-space. To overcome this difficulty, the analytical wave solution of multi-layered ground is combined with the discrete solution of soil structure obtained through the finite element method.

The equation of motion for a soil-structure in frequency domain is expressed by the finite element method as follows:

$$[Q]\{\delta\}=\{f\}\ ,\quad [Q]=-\omega^2[M]+i\omega[C]+[K] \tag{10}$$

in which δ and f are nodal displacement and force vectors, M , C and K are mass, damping and stiffness matrices. For the purpose of combining the wave solution of multi-layered ground given Eq.(9), it is necessary to express Eq.(11) in partitioned form.

$$\begin{bmatrix} Q_{BB} & Q_{BS} \\ Q_{SB} & Q_{SS} \end{bmatrix} \begin{Bmatrix} \delta_B \\ \delta_S \end{Bmatrix} = \begin{Bmatrix} f_B \\ f_S \end{Bmatrix} \tag{11}$$

in which subscript B shows the variables at nodal points on the base of the embankment and subscript S shows the variables of the inner part of the embankment. Putting the nodal points from 1 to m along the interface between the embankment and the ground, as shown in Fig.1, and assuming that the stress representations at these nodal points are σ_n, the nodal displacement vector δ_B is given as follows:

$$\delta_B=\{u(x_1,\omega),u(x_2,\omega),\cdots,u(x_m,\omega)\}^T \tag{12}$$

where x_n $(n=1,2,\cdots,m)$ is the coordinate of the nodal point along the ground surface. In order to eliminate f_B from Eq.(11) by the use of Eqs.(12) and (9), the relationship between nodal force f_B and the representation of the stress magnitude at nodal point σ must be given

$$\sigma=V^{-1}f_B \tag{13}$$

Substituting Eqs.(13) and (12) into Eq.(9), we can set

$$\delta_B=\delta_B^*-\Gamma f_B \tag{14}$$

in which

$$\delta_B=\{u^*(x_1,\omega),u^*(x_2,\omega),\cdots,u^*(x_m,\omega)\},\ \Gamma=GV^{-1},\ G_{nj}=g_n(x_j,\omega) \tag{15}$$

Eliminating f_B from Eq.(11) by using Eq.(14),

$$\begin{bmatrix} Q_{BB}+\Gamma^{-1} & Q_{BS} \\ Q_{SB} & Q_{SS} \end{bmatrix} \begin{Bmatrix} \delta_B \\ \delta_S \end{Bmatrix} = \begin{Bmatrix} \Gamma^{-1}\delta_B^* \\ f_S \end{Bmatrix} \tag{16}$$

SIMPLIFIED DYNAMIC ANALYSIS OF EMBANKMENT

We simplify the embankment as a trapezoid shear beam resting on the layered ground and restrict the vertical displacement of the soil layer to be zero. Inevitably the stress and displacement at the bottom of the embankment must have constant values which are independent of coordinate x. The scattering wave solution in the underlying layered ground , therefore, must satisfy unrealistic boundary conditions, both of constant displacement and stress, for a certain region of x coordinate along which the base of the embankment is connected to the ground. To avoid this difficulty, we consider only the constant stress condition at the interface between the ground and the embankment. The mean value of displacement for the region $-b<x<b$ is, then, assumed as the displacement at the bottom of the embankment. The following relation between the stress τ_B on the ground surface and the mean displacement u_B is obtained by carrying out the similar calculation introduced through the deduction from Eq.(1) to Eq.(6).

$$u_B=u^*-D(\omega)\tau_B \tag{17}$$

in which $D(\omega)$ is a complicated function of frequency. u^* is the displacement of the free surface for the case of no soil structures on the ground.

The displacement of the embankment is given as follows:

$$u=A\{J_0(k_\beta z)-\frac{J_1(k_\beta h)}{Y_1(k_\beta h)}Y_0(k_\beta z)\} \tag{18}$$

in which $k_\beta=\omega/\beta$, β is the shear wave velocity of the embankment, and $Y_n(\cdot)$ are the Bessel function of the first and second kind, $z=h$ and H mean the top and bottom of the embankment. From Eq.(18), we also get the relationship between u_B and τ_B

$$u_B=E(\omega)\tau_B \tag{19}$$

in which

$$E(\omega)=\mu k\frac{J_1(k_\beta H)Y_1(k_\beta h)-J_1(k_\beta h)Y_1(k_\beta H)}{J_0(k_\beta H)Y_1(k_\beta h)-J_1(k_\beta h)Y_0(k_\beta H)} \tag{20}$$

Substituting Eq.(19) into Eq.(17), and considering the frequency response function of the embankment $H(\omega)$, the response of the embankment u_T is given as follows:

$$u_T=u^*\Phi(\omega)H(\omega)\ ,\ \ \Phi(\omega)=1/\{1-E(\omega)D(\omega)\} \tag{21}$$

EQUIVALENT MODAL FREQUENCY AND DAMPING

For the design purpose, Eq.(21) is still too complicated to evaluate the response of the embankment. We here transform the effect of the interaction given by the function $D(\omega)$ in Eq.(17) into the equivalent modal frequency and damping of the embankment resting on the rigid base. For the case of no input motion ($u^*=0$), the following characteristic equation is deduced from Eq.(17) and Eq.(19),

$$\mu k \frac{J_1(k_\beta H)Y_1(k_\beta h)-J_1(k_\beta h)Y_1(k_\beta H)}{J_0(k_\beta H)Y_1(k_\beta h)-J_1(k_\beta h)Y_0(k_\beta H)} = -D(\omega) \tag{22}$$

Solving the Eq.(22) for ω, we get the complex natural frequency ω_n

$$\omega_n = a_n + ib_n \tag{23}$$

On the other hand, the natural frequency of the embankment with internal viscosity resting on the rigid base is given as follows:

$$\omega_n = ih_n\omega_{0n} \pm \omega_{0n}\sqrt{1-h_n^2} \tag{24}$$

in which ω_{0n} is the undamped natural frequency and h_n is the damping ratio for the nth mode. Equating both Eqs.(23) and (24), the equivalent undamped natural frequency ω_{neq} and damping ratio h_{neq} are given by

$$\omega_{neq} = \sqrt{a_n^2+b_n^2}\ ,\ h_{neq} = b_n/\sqrt{a_n^2+b_n^2} \tag{25}$$

APPLICATION

The dimension and material properties of the embankment used in the numerical examples are shown in Fig.2. We consider only the vertically propagating incident wave. In order to examine the validity of the simplified analytical method on the accuracy of the results, some numerical computations are performed for the embankment resting on the two layered ground as shown in Fig.3. The shear wave velocity of each layer is also given in the figure. The full line is the result of two dimensional analysis for both horizontal and vertical (with o sign) displacements at the crest of the embankment. The chain line is obtained by the analysis restricting the vertical displacement of the whole system to be zero. The second peak of the horizontal response in two dimensional analysis is not followed by the one dimensional analysis because this peak is caused by resonance for the vertical movement. Note that even the simplified method (broken line) yields a good result especially for lower frequency. This is a useful tool with which to make dynamic response analysis of embankment resting on the layered ground.

Discrete Fourier analysis and synthesis were used to examine the efficiency of the simplified method on the seismic response to the transient incident wave pattern. The incident motion used in the analysis is the NS component of the accelerogram recorded at El Centro (1940). The time history of the response at the crest is shown in Fig.4 for both the one dimensional and the simplified method. The acceleration wave forms obtained by both methods do not differ much although the amplitude calculated by the simplified method is a bit larger.

The rigidity of the base layer is one of the predominant parameters to control the response magnitude. The results are shown in Fig.5. The response magnitude at the crest of the embankment becomes larger as the rigidity of the base layer increases.

The effect of ground structures on the response is shown in Fig.6. This result is also calculated by the one dimensional analysis. The response of the embankment is strongly affected by ground structures, especially, by the position of the layer with the lowest shear wave velocity and its thickness.

The equivalent natural frequency and damping are shown in Figs.7 and 8 for the embankment resting on the two layered ground. In Fig.7 natural frequencies up to the 4th mode are shown as functions of shear wave velocity of the surface layer. The shear wave velocity of the base layer is four times that of the surface layer. The horizontal lines are natural frequencies of the embankment resting on the rigid base. The sloping lines are those of the ground. Around the region, in which the horizontal and sloping lines intersect, one of the equivalent natural frequencies appear in the frequency region lower than both of the lines and the other in the higher frequency region. As the rigidity of the ground increases, the equivalent natural frequency approaches that of the embankment resting on the rigid base. The equivalent damping dose not have the systematic tendency of the equivalent natural frequency. In the region where the equivalent natural frequency approaches that of the ground, the equivalent damping becomes very large. If the ground is the homogeneous half-space, the equivalent modal damping shows a proportional tendency to the impedance ratio between the embankment and the ground. The relation between the modal damping and the impedance ratio is shown in Fig.9.

Finally the modal superposition analysis was done to validate the equivalent natural frequency and damping for response analysis of the embankment. The results shown in Fig.10 assure the usefulness of the equivalent technique proposed above.

CONCLUSION

The simplified analytical method to calculate the response of the soil-structure resting on the layered ground were proposed and applied it some numerical examples. The principal results and conclusions of the present study are

(1) The hybrid analytical procedure combining the wave solution of the layered ground with discrete solution obtained by the finite element method were programed to get the exact numerical solution. And this was applied to checking the results obtained by the proposed simplified method.

(2) Several numerical results show that the response of the embankment is strongly affected by the ground structures. Especially, the position of the layer with the lowest shear velocity, its thickness, and the rigidity of the base layer are the most predominant parameters which control the response magnitude.

(3) To use the modal superposition method for estimating the response magnitude, the effect of the interaction between soil-structure and layered ground was transformed into the internal damping and rigidity of the structure by proposing the concept of the equivalent natural frequency and damping.

REFERENCES

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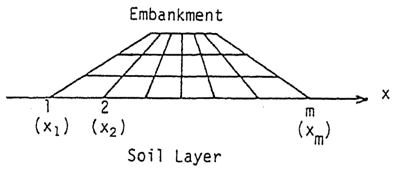

Fig.1 Finite element idealization of model embankment

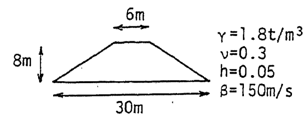

Fig.2 The dimension and material properties of embankment

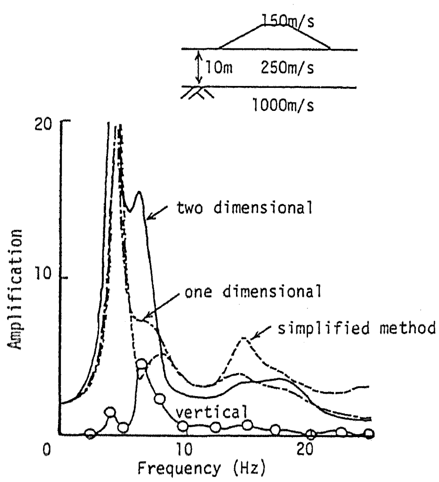

Fig.3 Frequency response function at the crest of embankment

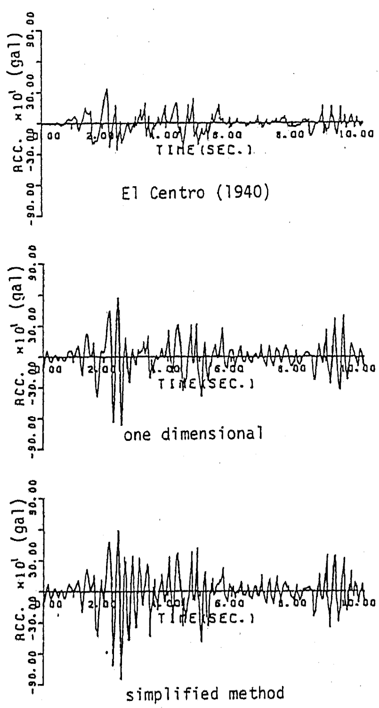

Fig.4 Time history of response acceleration

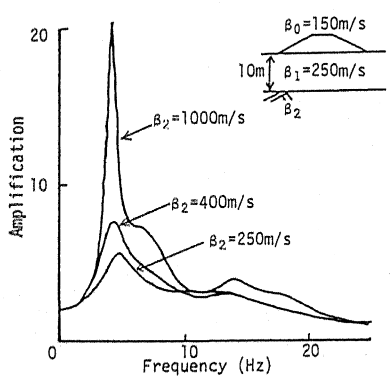

Fig.5 Frequency response function at the crest of embankment

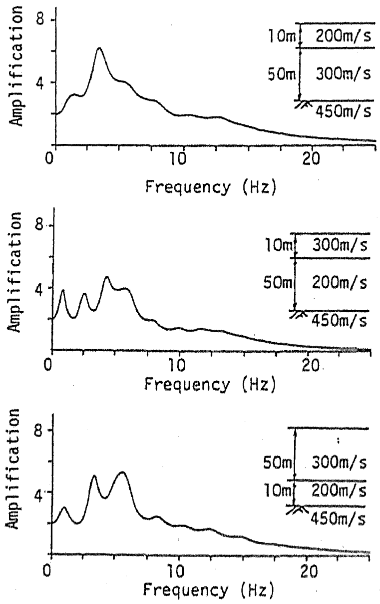

Fig.6 Effect of the layer structure on the frequency responce function

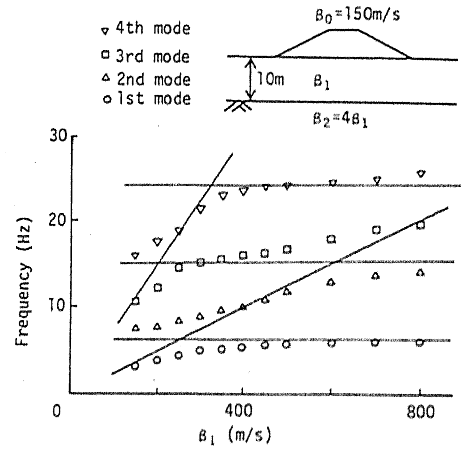

Fig.7 Change of natural frequencies with shear wave velocity of the surface layer

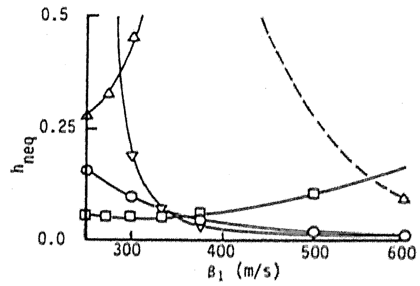

Fig.8 Change of modal dampings with shear wave velocity of the surface layer

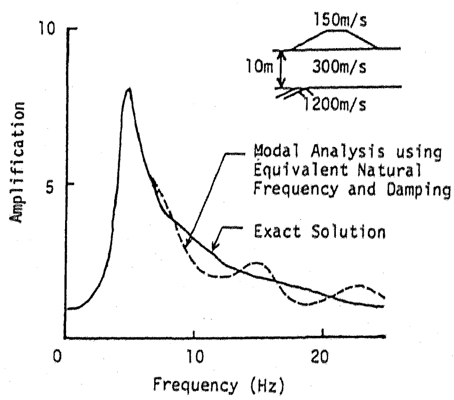

Fig.10 Comparison of the results obtained by the modal analysis with that of exact solution

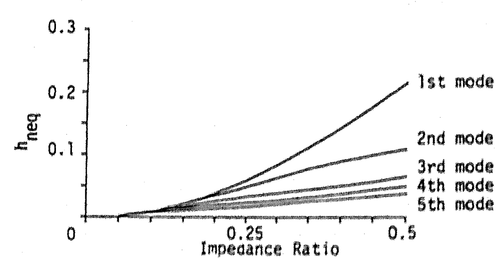

Fig.9 Change of modal dampings