

SEISMIC RESPONSE OF EARTH AND ROCKFILL DAMS  
WITH NON-DETERMINISTIC PROPERTIES

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SUMMARY

This paper presents a finite element procedure to evaluate the seismic response of earth dams assuming the material properties as random. The approach is based on the complex response method and makes use of the theory of perturbations to implicitly account for the uncertainties in the shear moduli and damping ratios. Through comparisons with Monte Carlo-type simulations, it is shown that the proposed method is reliable and relatively inexpensive for practical applications.

INTRODUCTION

The seismic analysis of earth and rockfill dams by means of the finite element method has been thoroughly investigated in the past. However, these analyses have considered the dynamic characteristics of the materials as deterministic, using the mean value of the parameters in the computations.

There exists a large amount of results of laboratory and field evaluations of moduli and dampings of cohesive and granular materials showing a significant scatter. Furthermore, during the construction of an earth dam the material placement and environmental conditions may vary and, consequently, affect the dynamic (as well as static) characteristics of the constitutive materials. Additionally, time effects under sustained loading cannot be accurately quantified. All this leads to the conclusion that the commonly used deterministic assumption for dynamic soil properties is hardly tenable. A much better hypothesis would be to treat them as random variables.

A method based on the theory of perturbations which takes into account the uncertainties in the shear modulus and damping ratio of the constitutive materials is presented in this paper. Calculations by this method are compared with Monte Carlo-type simulations; the results clearly show the suitability of the proposed method.

METHOD OF ANALYSIS

The handling of the material uncertainties is not straightforward. One possible way of solving the problem would be to use Monte Carlo methods. For example, if the shear moduli,  $G$ , of the constitutive materials of the dam are known to have a wide scatter and if their probability distributions are known, the analysis problem may be solved by generating samples of  $G$  values consistent

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with their probability laws, and the analysis performed for each group of values of  $G$  generated. The results can then be analyzed to produce the desired statistical response quantities. This method involves a great number of numerical computations and hence its practical use is limited. An alternate procedure based on perturbation techniques (Ref. 1) coupled with numerical simulations is presented in this section. This method retains the formalism of the Monte Carlo method but eliminates the burden of solving the full problem for each set of soil parameter values.

The equation of motion for undamped systems can be written in the frequency domain as

$$(K - \omega_r^2 M) \{U\}_r = - \{m\} \ddot{Y}_r \quad (1)$$

where  $K$  and  $M$  are the stiffness and mass matrices, respectively;  $\{U\}_r$  are the amplitudes of the relative displacements of the dam with respect to the rigid base, for each frequency  $\omega_r$ ,  $r = 0, 1, 2, \dots$ , of the input motion  $\ddot{Y}_r$ ; and  $\{m\}$  is a vector related to  $M$  and the input motion direction. The material damping,  $\beta$ , is introduced in  $K$  using complex shear moduli:  $G^* = G \exp(2i\beta)$ , where  $G$  is the shear modulus and  $i = \sqrt{-1}$ .

The material properties involved in the equation of motion are the shear modulus, damping ratio, and unit weight. Statistical evaluation (Ref. 2) of laboratory and field determinations of shear moduli and damping ratios have shown that both parameters should be considered as random variables. On the other hand, the unit weight may be considered deterministic because its scatter is negligible compared to that of the modulus and damping. Thus only the  $K$  matrix was considered random.

Using perturbation techniques in Eq 1, and grouping the terms of equal order the following equations are obtained (Refs 2 and 3):

$$\text{Zero order:} \quad (K - \omega_r^2 M) \{U_0\}_r = - \{m\} \ddot{Y}_r \quad (2)$$

$$\text{First order:} \quad (K - \omega_r^2 M) \{U_1\}_r = - Q \{U_0\}_r \quad (3)$$

where  $\{U_0\}$  and  $\{U_1\}$  are the unperturbed and perturbed solutions, respectively; and  $Q$  is the first order perturbation stiffness matrix.

It is very interesting to point out that Eqs 2 and 3 are identical except for the loading vector. Henceforth the perturbed solution of the problem can be obtained in a straightforward manner by solving Eq 2 for unitary loading and then multiplying Eqs 2 and 3 by their corresponding loading vectors, the values of  $\{U_0\}_r$  and  $\{U_1\}_r$  are computed. Assuming small perturbations, the total solution is

$$\{U\}_r = \{U_0\}_r + \{U_1\}_r = (-I + L_r^{-1} Q) L_r^{-1} \{m\} \ddot{Y}_r \quad (4)$$

where  $I$  is the unitary matrix; and  $L_r^{-1} = (K - \omega_r^2 M)^{-1}$

In order to achieve the probabilistic solution, Eq 4 is used repeatedly in a simulation-type approach: a sample of Q matrices consistent with its probability distribution is generated and by means of Eq 4 a sample of responses is calculated. This procedure is equivalent to the Monte Carlo method; however, it does not require solving the equation of motion for each value of Q because Eq 4 involves only products between the unitary deterministic solution,  $L^{-1}$ , and the probabilistic stiffness matrix, Q. Accordingly, the numerical simulation can be performed advantageously. Once the response sample is obtained it is analyzed to produce the desired statistical response quantities.

The matrix Q is made up with complex shear moduli computed by means of the following expression (Ref. 2)

$$G^* = G_o^* + \Delta G^* \quad (5)$$

where

$$\Delta G^* = G_o^* (R_G \cdot CV_G + 2i\beta_o R_\beta \cdot CV_\beta)$$

here  $G_o^*$  and  $\beta_o$  are the unperturbed modulus and damping ratio, respectively, and are usually obtained from a laboratory test program;  $R_G$  and  $R_\beta$  are normally  $(N(0, 1))$  distributed random numbers; and  $CV_G$  and  $CV_\beta$  are the modulus and damping coefficients of variation, respectively. The values of the variation coefficients may be calculated from statistical evaluation of the results yielded by soil testing programs.

From the statistical studies of the response sample uncertainty bands may be computed. Thus the probabilistic response,  $\{U\}_r$ , will be within the following limits:

$$\{U_o\}_r - \alpha S \leq \{U\}_r \leq \{U_o\}_r + \alpha S \quad (6)$$

here S is the standard deviation, and  $\alpha$  is a parameter which controls the width of the response uncertainty band (i. e.  $\alpha = 1$  corresponds to 68% probability interval).

The above theory was incorporated into a new computer code, DARE (Ref. 4), which is a plane-strain finite element program. The strain-dependent nature of the material characteristics is considered by means of an iterative procedure.

#### EVALUATION OF THE ANALYSIS METHOD

In order to validate the proposed probabilistic method of analysis, parallel computations were performed by the probabilistic approach and the Monte Carlo method, for the wedge-shaped dam shown in fig. 1. The dam was assumed homogeneous and the dynamic characteristics of the soil non-linear as shown in Fig 2. The uncertainty bands for G and  $\beta$  used in the analyses are depicted in Fig 2 and the response spectrum of the input motion is presented in Fig 3.

To generate the seismic response sample, 20 sets of material properties (G and  $\beta$ ) were produced assuming that the scatter around the mean values of G and  $\beta$  included in Fig 2a conforms with a normal probability distribution having

coefficients of variation as indicated in Fig 2b. Then 20 deterministic analyses (Monte Carlo simulation) were carried out using program DARE. The results were then evaluated statistically to produce mean values and confidence limits. Similarly, using the theory proposed above and computer program DARE, a set of 20 seismic responses was generated and the corresponding mean values and confidence limits evaluated. Comparisons between the acceleration response spectra calculated with both approaches at points A and B (see Fig 1) are shown in Figs 4 to 7. The accuracy of the probabilistic method is remarkable throughout the frequency range for both the mean spectral values and their corresponding uncertainty limits.

#### EFFECT OF $CV_G$ AND $CV_\beta$

The width of the confidence bands of the response depends mainly upon the degree of uncertainty in the material properties of the dam. From Eq 5 it may be concluded that the effect of  $CV_\beta$  is about one order of magnitude smaller than that of  $CV_G$ , for typical values of soil damping ratios (8 to 15%). Hence, from the practical point of view it would suffice to consider the mean of the damping-shear strain curve.

To study the effect of  $CV_G$  the response of the triangular dam was computed at its crest for a number of constant values of  $CV_G$  and keeping  $CV_\beta$  equal to zero. The mean response spectrum as well as the upper limit ( $\alpha = 1$ ) of the response spectra corresponding to  $CV_G = 20$  and 40% are shown in Fig 8. As it was expected, the width of the response uncertainty band increases with  $CV_G$ . It seems that at least for the studied case, the effect of  $CV_G$  is to amplify the response by approximately a constant all spectral ordinates. However, this may not be the case for zoned dams or for other geometries.

#### NUMBER OF EQUIVALENT CYCLES

In the seismic analysis of earth dams to evaluate liquefaction potential or earthquake-induced deformations it is required to compute the number of equivalent series of uniform stress cycles (Ref. 5). The conventional method (Ref. 6) which is based on a weighing procedure has the disadvantage that it requires a considerable number of steps to calculate the number of uniform stress cycles,  $N$ . Instead, an expedite method which permits computation of  $N$  in each element of the finite element mesh is presented below. The approach is based on energy concepts. The energy,  $E$ , contained in a shear stress-time history,  $\tau(t)$ , of duration  $T$  can be computed by means of

$$E = \sum_{t=0}^T |\tau(t)|^2 \Delta t \quad (7)$$

Similarly, the energy,  $e$ , contained in  $N$  uniform stress cycles of amplitude  $\tau_e$  is

$$e = \frac{N (\tau_e)^2}{2} \quad (8)$$

From Eqs 7 and 8 it is obtained the relation to calculate the N stress cycles of amplitude  $\tau_e$

$$N = \frac{2 \sum_{t=0}^T |\tau(t)|^2 \Delta t}{(\tau_e)^2} \quad (9)$$

This approximation to evaluate N is very attractive because it can be used in the probabilistic approach to evaluate the effect of  $CV_G$  on N and obtain the corresponding uncertainty bands. This type of information is very helpful in evaluating liquefaction potential of dams as was proved in a recent study (Ref. 7).

#### CONCLUSIONS

It is widely acknowledged that considerable judgement is required to determine representative soil characteristics and that it is usually necessary to consider ranges of properties for analyses purposes. A new procedure, which includes such uncertainties, has been developed for evaluating the seismic response of earth dams by combining elements from the theory of perturbations, the finite element method, and the complex response method.

Results obtained by the probabilistic approach are in excellent agreement with corresponding results by the Monte Carlo method and have the added advantage that the computer costs are a small fraction of those generated by the Monte Carlo method. Since the probabilistic method provides confidence limits on all results it is potentially more useful for the design of earth and rock-fill dams.

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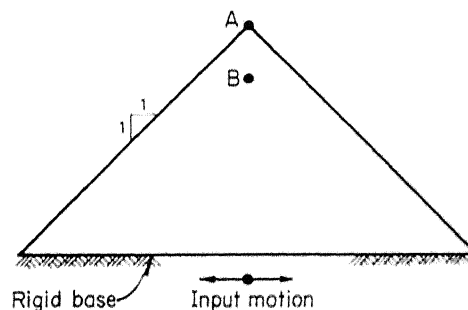


Fig 1. Triangular Dam Used in the analyses

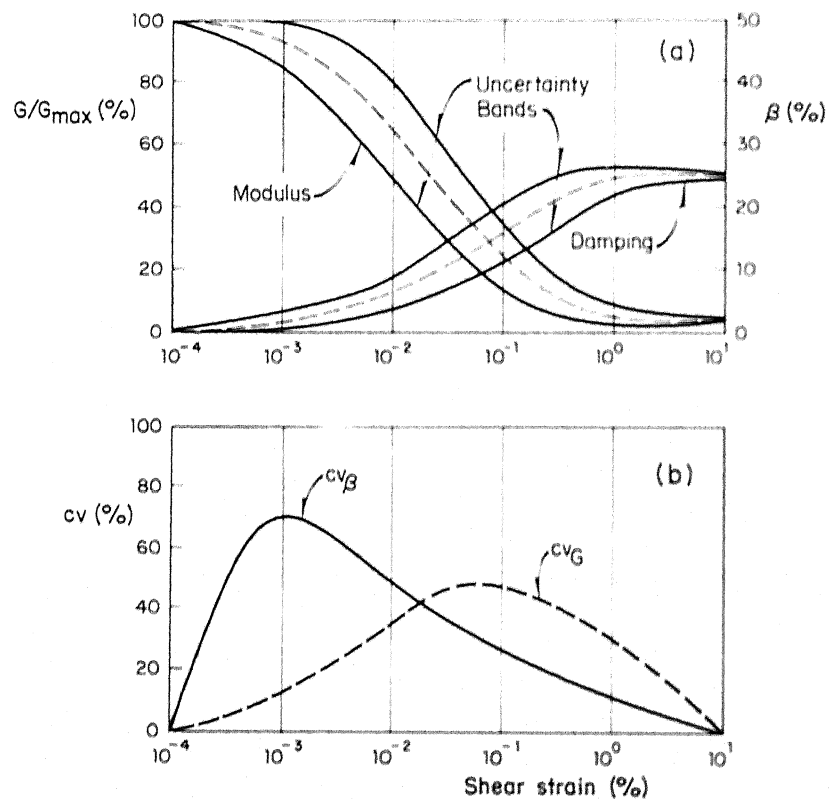


Fig 2. Shear Modulus, Damping and Coefficients of Variation for Granular Soils

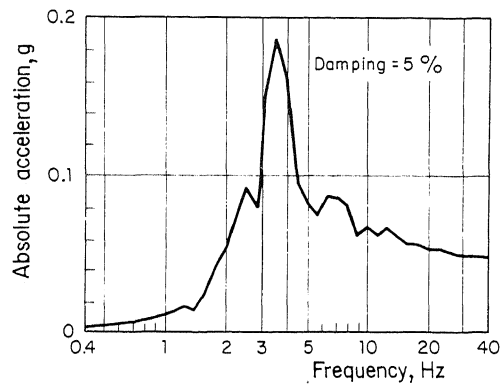


Fig 3. Response spectrum of input motion

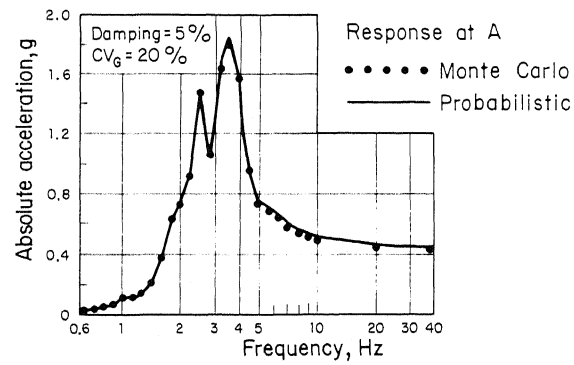


Fig 4. Comparison between Monte Carlo and probabilistic methods.  
Mean response spectra

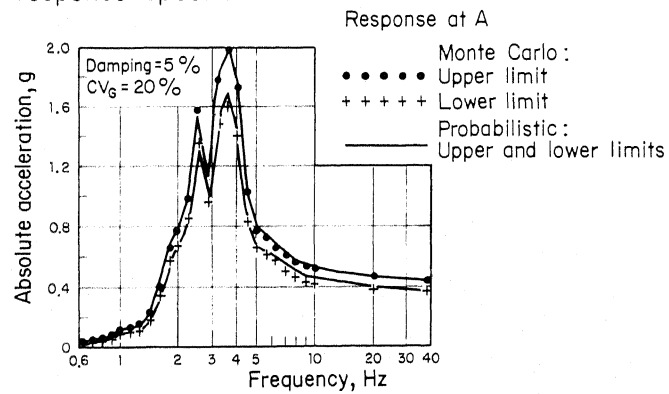


Fig 5. Comparison between Monte Carlo and probabilistic methods.  
68% confidence band

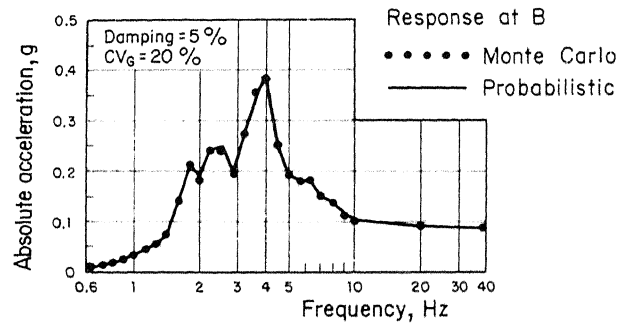


Fig 6. Comparison between Monte Carlo and probabilistic methods.  
Mean response spectra

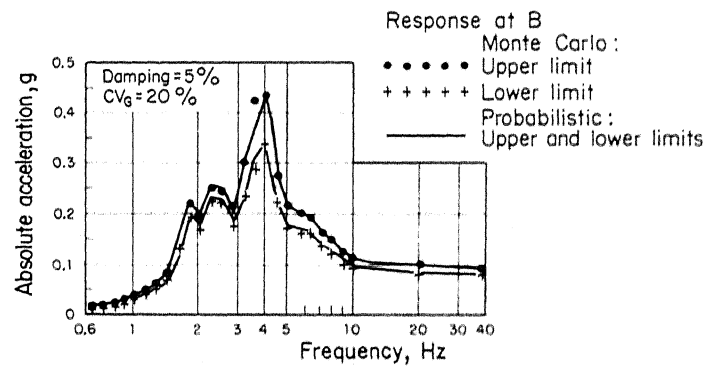


Fig 7. Comparison between Monte Carlo and probabilistic methods.  
68% confidence band

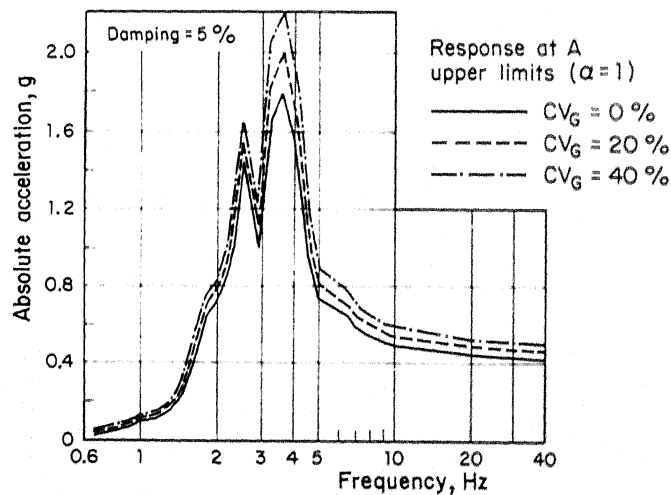


Fig 8. Effect of  $CV_G$  on the seismic response