PROBABILISTIC ANALYSIS OF SEISMIC SAFETY AGAINST LIQUEFACTION

J.E.A. Pires (I)
Y.K. Wen (II)
A. H-S. Ang (II)
Presenting Author: J.E.A. Pires

SUMMARY

Liquefaction of sand deposits during earthquakes is studied as a problem of random vibration of hysteretic systems in which the random nature of the seismic ground motions, and the nonlinear soil response behavior are properly accounted for. It is assumed that liquefaction will occur when the excess pore pressure becomes equal to the initial effective vertical stress, i.e. when the sand stiffness has deteriorated to zero. The random vibration results and the uncertainty analysis of the soil properties are used to calculate the reliability of sand deposits against liquefaction under a random seismic loading with a given intensity and duration.

INTRODUCTION

The excess pore pressure rise in saturated sand deposits under random seismic loadings is represented in terms of a continuous damage parameter. The damage parameter is a function of the shear-strain hysteretic energy dissipated and of the amplitude of the hysteretic restoring shear-stress. The random vibration analysis leads to the determination of the mean and standard deviation of the equivalent duration of an earthquake loading with a specified intensity capable of causing liquefaction. These quantities can be used to calculate factors of safety for the deposit as well as the associated reliability levels. Comparison with previously observed field behavior has showed that the method can be useful for a risk-based design against liquefaction.

PORE PRESSURE GENERATION

Uniform Cyclic Loading

For saturated undrained conditions, the work performed in rearranging the particles of sand, ΔW, when the excess pore pressure rises from zero to $\bar{u}$ is given by

$$\Delta W = \frac{\bar{\nu}\sigma'_{co}}{\eta_w} \frac{e_o}{g(e_o - e_m)} \int_o^{r_u} \frac{dr'_u}{f(1+r'_u)} \qquad (1)$$

(I) Department of Nuclear Energy, Brookhaven National Laboratory; formerly, Visiting Research Associate, University of Illinois at Urbana-Champaign.

(II) Professor of Civil Engineering, University of Illinois, Urbana, ILL 61801.

where $r_u = \bar{u}/\sigma'_{vo}$ is the excess pore pressure ratio, σ'_{co} is the initial effective confining pressure, e_o is the initial void ratio of the sand, e_m is the minimum void ratio for the sand, η_W is the bulk modulus of the water, and $\bar{\nu}$ is a parameter independent of the void ratio of the sand; $f(1+r_u)$ and $g(e_o-e_m)$ are nondecreasing functions (Nemat-Nasser and Shokooh, 1979). For liquefaction $r_u = 1$, and the work performed, $\Delta W(r_u = 1)$, is a constant for a given initial state of the sand. For the purposes of this study, the work done when the excess pore pressure rises from 0 to r_u, $\Delta W(r_u)$ is normalized with respect to $\Delta W(r_u = 1)$; in this manner the need to measure $\bar{\nu}$, σ'_{co}, η_W, e_o and $g(e_o-e_m)$ is eliminated. The normalized measure of work is then defined as $r_W = \Delta W(r_u)/\Delta W(1)$.

Let the energy dissipated by hysteresis in one cycle of amplitude $\bar{\tau} = \tau/\sigma'_{co}$ be denoted as $E_c(\bar{\tau})$. The value of ΔW after N cycles of constant amplitude loading may be considered proportional to the number of cycles of loading if the amplitude of shear-stress is large (Nemat-Nasser and Shokooh, 1979); thus,

$$\Delta W = h(\bar{\tau})NE_c(\bar{\tau}) \tag{2}$$

where $h(\bar{\tau})$ is a weight function that depends only on the amplitude of the shear-stress $\bar{\tau}$. For liquefaction to occur,

$$\Delta W(1) = N_\ell h(\bar{\tau})E_c(\bar{\tau}) \tag{3}$$

where N_ℓ is the number of cycles of constant stress amplitude, $\bar{\tau}$, causing liquefaction of the sand. Then, the energy ratio r_W is, according to Eqs. 2 and 3 given by $r_W = N/N_\ell$. The function $h(\bar{\tau})$ as well as $\Delta W(1)$ are obtained from the test results of undrained resistance to liquefaction under uniform cyclic stress loading.

It has been suggested (Seed, Martin and Lysmer, 1976) that N/N_ℓ and r_u may be related by

$$\frac{N}{N_\ell} = \sin^{2\theta}\left(\frac{\pi r_u}{2}\right) \tag{4}$$

where θ is an empirical constant. Substituting N/N_ℓ in Eq. 4 by r_W and differentiating, the differential equation relating the excess pore pressure rise to the energy ratio r_W, can be written as

$$\frac{dr_u}{dr_W} = \frac{1}{\theta\pi\cos\left(\frac{\pi r_u}{2}\right)\sin^{2\theta-1}\left(\frac{\pi r_u}{2}\right)} \tag{5}$$

Equation 5 is easily extended for calculating the excess pore pressure rise under random seismic loads.

Random Seismic Loading

Under random seismic loading, the quantity ΔW at any time t after the start of the excitation is given by

$$\Delta W = \int_{o}^{t} X(t')\dot{E}_T(t')dt' \tag{6}$$

where $\dot{E}_T(t')$ is the rate of shear-strain energy dissipated at time t', and X(t') is an equivalent weighing function defined by

$$X(t') = \frac{\int_{o}^{\bar{\tau}_{max}} p_{\bar{\tau}}(\sigma_{\bar{\tau}},\sigma_{\dot{\bar{\tau}}},\bar{\tau},t')h(\bar{\tau})E_c(\bar{\tau})d\bar{\tau}}{\int_{o}^{\bar{\tau}_{max}} p_{\bar{\tau}}(\sigma_{\bar{\tau}},\sigma_{\dot{\bar{\tau}}},\bar{\tau},t')E_c(\bar{\tau})d\bar{\tau}} \tag{7}$$

in which $\bar{\tau}_{max}$ is the normalized static shear strength of the sand, and $p_{\bar{\tau}}(\sigma_{\bar{\tau}},\sigma_{\dot{\bar{\tau}}},\bar{\tau}, t')$ is the probability density function of the peaks of $\bar{\tau}$. Using the chain rule of differentiation together with Eqs. 5 and 6, the differential equation describing the excess pore pressure rise can be written as

$$\frac{dr_u}{dt} = \frac{X(t)\dot{E}_T(t)}{\Delta W(1)\ \cos\left(\frac{\pi r_u}{2}\right)\sin^{2\theta-1}\left(\frac{\pi r_u}{2}\right)} \tag{8}$$

Equations 6 and 8 are appropriately modified for considering the deterioration of the sand stiffness as a result of the excess pore pressure rise (Pires, Wen and Ang, 1983).

SEISMIC RELIABILITY EVALUATION

Inelastic Response Analysis

The response of soil deposits under earthquake loadings is often in the inelastic range; in addition, the behavior of the material is hysteretic, and the stiffness or strength are likely to deteriorate with the number of oscillations. The model and the analytical technique recently proposed by Wen (1980), Baber and Wen (1981), are capable of yielding the statistics and probabilities of the seismic response of horizontally layered soil deposits (Pires, Wen and Ang, 1983). In this model, the hysteretic restoring shear stress is described by a first-order differential equation which allows for a simple linearization of the equations of motion (Wen, 1980). The solution for the response requires the determination of the covariance matrix [S] of the response variables satisfying the following matrix differential equation

$$\frac{d[S]}{dt} = [G][S] + [S][G]^T - [B] \tag{9}$$

where [G] and [B] are the matrices of the lumped-mass system and ground motion parameters, respectively.

The hysteretic shear-strain energy E_T at any depth in the deposit is important, because the excess pore pressure rise is a function of E_T (see Eqs. 6 thru 8). The above formulation allows the determination of the statistics (mean and standard deviation) of E_T, which are required in the reliability analysis. Details of the evaluation of these statistics can be found in Pires, Wen, and Ang (1983).

The ground motion, particularly the strong ground motion phase, of a given earthquake may be modeled by a stationary Gaussian random process. In this form, it can be characterized by a power spectral density function, such as the Kanai-Tajimi spectrum. The expected peak ground acceleration a_{max} is the product of the root-mean-square ground acceleration a_{rms} and the "peak factor" (PF); average values for (PF) have been suggested (Vanmarcke and Lai, 1980).

Reliability Evaluation

For an earthquake loading with a given amplitude and duration, the failure condition at any depth within the deposit is $[\Delta W(1) - \Delta W < 0]$, where $\Delta W(1)$ and ΔW are random variables. The statistics of ΔW are obtained from the random vibration analysis, including the uncertainties in ground motion parameters and some soil properties through sensitivity analysis and first-order-second-moment approximation (Pires, Wen and Ang, 1983). The statistics of $\Delta W(1)$ are obtained from the uncertainty analysis of the undrained resistance against liquefaction under uniform cyclic stress loading.

ILLUSTRATIVE APPLICATIONS

Homogeneous Saturated Sand Deposit

The deposit is idealized as consisting of several layers to model the changes in soil properties with depth as shown in Fig. 1. The undrained cyclic resistance curves against liquefaction for the sand are shown in Fig. 2. The statistics of the time until liquefaction at 12.5-foot depth, T_L, for several stationary loadings with different peak base accelerations a_{max} are shown in Fig. 3.

The statistics of the time till liquefaction were also calculated for a specific load with an expected peak ground acceleration of 0.10 g and the modulating function shown in Fig. 4. At a depth of 12.5 feet, the mean duration is $\mu_{T_L} = 3.6$ seconds and $\sigma_{T_L} = 1.8$ seconds, when the stiffness deterioration of the sand is considered; whereas these values become 3.2 and 1.7 seconds, respectively, when a total stress analysis is used. As expected the statistics of the time till liquefaction can be well predicted with the total stress analysis; the stiffness deterioration leads to slightly longer expected times for liquefaction.

Case Studies

The probabilities of liquefaction predicted with the proposed methodology are compared with the field performance of sand deposits during past earthquakes. Usually, the historical data for the in-situ resistance of the sand are plotted against the intensity of the earthquake load, and a boundary separating the cases of liquefaction and no-liquefaction is determined (Seed and Idriss, 1981). A convenient parameter to represent the intensity of an earthquake is the ratio of the average shear stress τ_{ave} developed on horizontal surfaces of the sand to the initial effective vertical stress σ'_{vo}. Values of these shear-stress ratios known to be associated with some evidence of liquefaction or no-liquefaction in the field are plotted as a function of the standard penetration resistance (SPT), N_1, corrected to a value of σ'_{vo} equal to 1 ton/sq-ft (Seed and Idriss, 1981).

A graphical representation of some of the available historical data is shown in Fig. 5, where the boundary separating the cases of liquefaction and no-liquefaction (Seed, Arango and Chan, 1975) is shown by a solid line; information about these data points can be found in Table 1.

The proposed methodology is used to calculate the probability of liquefaction for some of the data points in Fig. 5, namely: (1) at three locations in the city of Hachinohe (Japan) during the Tokachioki earthquake of May 16, 1968 (points 5, 6 and 9 in Fig. 5); (ii) at three locations in the city of Niigata (Japan) during the Niigata earthquake of 1964 (points 1, 3 and 4 in Fig. 5); and, (iii) at one location in the city of Niigata (Japan) for two historical earthquakes of magnitude 6.1 and 6.6 (points 10 and 11 in Fig. 5).

The probabilities of liquefaction were calculated for each case considering two different values of the c.o.v.'s of the shear stress ratio that causes liquefaction in a given number of uniform loading cycles; these probabilities are summarized in Fig. 5. The model by Fardis and Veneziano (1981), is used to characterize the undrained resistance to liquefaction under uniform cyclic stress loading. The statistics of the strong-motion duration shown in Table 1 were obtained with the correlations between magnitude and epicentral distance proposed by Lai (1980), and Shinozuka, Kameda and Koike (1983). Details concerning the modeling of the soil profiles for each case as well as the assumptions underlying the analysis can be found in Pires, Wen and Ang (1983).

The comparison with the observed historical data shows that the proposed methodology appears to be a viable procedure for predicting the seismic reliability of sand deposits against liquefaction, and for assessing the relative reliability of design alternatives.

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Table 1 Historical Data on Liquefaction (Partial Data).

Case History	Data	Site	M	Distance (Miles)	$\overline{T}_E$ (sec)	COV T_E	a_{max} (g)	Depth of Water Table (ft)	D (ft)	σ'_{vo} (psf)	N-SPT	N_1	D_r	$\frac{\tau_{ave}}{\sigma'_{vo}}$	Field Behavior	Reference
1	1964	Niigata	7.5	32	15	0.8	0.17	3	20	1200	6	8	53	0.195	Liq.	Seed and Idriss (1971)
2	1964	Niigata	7.5	32	15	0.8	0.17	3	25	1500	15	18	64	0.195	Liq.	Kishida (1966)
3	1964	Niigata	7.5	32	15	0.8	0.17	3	20	1200	12	16	64	0.195	No-Liq.	Seed and Idriss (1971)
4	1964	Niigata	7.5	32	15	0.8	0.17	12	25	2000	6	6	53	0.12	No-Liq.	Seed and Idriss (1971)
5	1968	Hachinohe	7.8	45-100	15	0.8	0.21	3	12	800	14	21	78	0.23	No-Liq.	Ohsaki (1970)
6	1968	Hachinohe	7.8	45-100	15	0.8	0.21	3	12	800	<4	<6	~45	0.23	Liq.	Ohsaki (1970)
7	1968	Hachinohe	7.8	45-100	15	0.8	0.21	5	10	800	15	23	80	0.185	No-Liq.	Ohsaki (1970)
8	1968	Hakodate	7.8	100	15	0.8	0.21	3	15	1000	6	9	55	0.205	Liq.	Kishida (1970)
9_B	1968	Hachinohe	7.8	45-100	15	0.8	0.21	7	47	2900	25	21	75	0.19	No-Liq.	Ohsaki (1970)
9_T	1968	Hachinohe	7.8	45-100	15	0.8	0.21	7	9	850	15	22	80	0.16	No-Liq.	Ohsaki (1970)
10	1802	Niigata	6.6	24	10	0.8	0.12	3	20	1200	12	16	64	0.135	No-Liq.	Seed and Idriss (1967)
11	1887	Niigata	6.1	29	8	0.8	0.08	3	20	1200	12	16	64	0.09	No-Liq.	Seed and Idriss (1967)

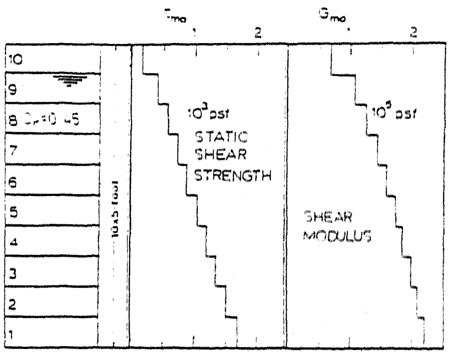


Fig. 1 Homogeneous Sand Deposit.

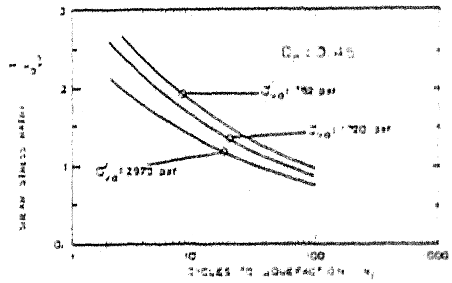


Fig. 2 Cyclic Resistance to Liquefaction.

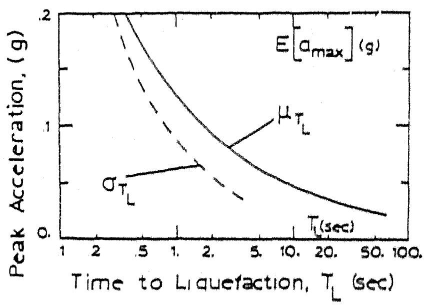


Fig. 3 Statistics of Time to Liquefaction.

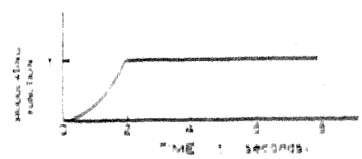
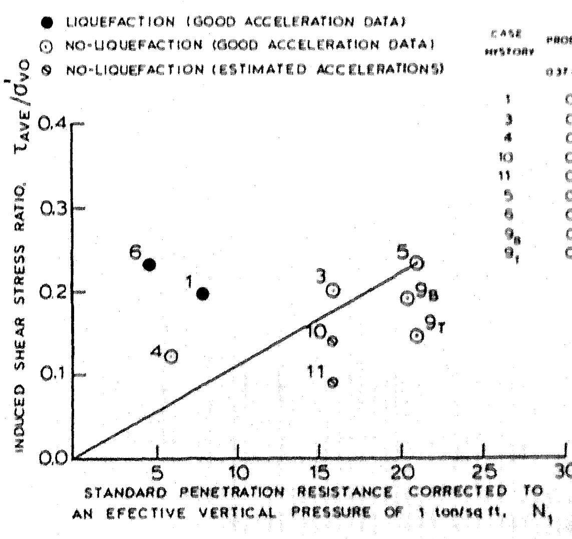


Fig. 4 Modulating Function of Base Excitation.



CASE HISTORY	PROBABILITIES OF LIQUEFACTION $0.37 < \Omega_T < 0.40$	$\Omega_T = 0.10$	$(\tau_{ave}/\sigma'_{vo})_{Required}$ / $(\tau_{ave}/\sigma'_{vo})_{Actual}$
1	0.611	0.64	[illegible]
3	0.34	0.32	0.9
4	0.42	0.41	0.6
10	0.14	0.09	1.1
11	0.0062	0.0023	2.0
5	0.33	0.30	1.0
6	0.92	0.94	[illegible]
9_B	0.26	0.23	1.2
9_T	0.11	0.08	1.5

$\tau = \frac{\tau}{\sigma'_{vo}}$

Fig. 5 Predicted Probabilities of Liquefaction for Some Historical Data.