TWO-DIMENSIONAL EFFECTIVE STRESS LIQUEFACTION ANALYSIS OF LAYERED SAND DEPOSITS

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SUMMARY

This paper is concerned with two-dimensional dynamic analysis of saturated sand deposits considering a liquefaction phenomenon. An elasto-plastic constitutive equation of sand and the Biot's theory of two-phase mixture are used. Two simplified methods of liquefaction analysis are proposed in order to introduce the effective stress concept in soil mechanics. As a numerical application, two-dimensional behavior of saturated sand deposits are analyzed under the plane strain condition.

INTRODUCTION

Recently, many methods of effective stress liquefaction analysis have been proposed. But, almost all of these methods deal with one-dimensional behavior of ground during earthquakes. On the other hand, it becomes clarified that it is necessary to develop a method of two-dimensional liquefaction analysis, in relation to prediction of the behavior of embankment near river and/or the interaction between soil and pipeline under the ground. This paper is concerned with the two-dimensional dynamic analysis procedure of ground considering a liquefaction phenomenon.

According to the reports on Miyagi-ken-oki earthquake 1978 in Japan, the liquefaction seems to greatly influences the behavior of embankement near river. As to the constitutive equation of sand, one of the authors has already proposed the elasto-plastic constitutive equation, which is effective under cyclic loads, based the elasto-plasticity theory and the concept of bounding surface.

The proposed equations coincide with the modified Christian's method of consolidation when the acceleration term is neglected. Therefore, it includes the generation and dissipation of pore water pressure due to seepage flow.

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EQUATION OF MOTION OF TWO-PHASE MIXTURE

Equation of motion for a two-phase mixture [4),5)] can be given by

$$\frac{\partial \sigma^s_{ij}}{\partial x_j} = \bar{\rho}^s \frac{dv^s_i}{dt} + \Pi_i - \bar{\rho}^s b^s_i \qquad (1)$$

$$\frac{\partial \sigma^f_{ij}}{\partial x_j} = \bar{\rho}^f \frac{dv^f_i}{dt} - \Pi_i - \bar{\rho}^f b^f_i \qquad (2)$$

where σ^α_{ij} is the partial stress tensor, v^α_i is a component of velocity vector, b^α_i is a component of body force vector, Π_i is a component of interaction force vector, $\bar{\rho}^\alpha$ is a partial mass density, x_i is a coordinate and t is a time.

Total stress is defined by

$$\sigma_{ij} = \sigma^f_{ij} + \sigma^s_{ij} \qquad (3)$$

The following definition of effective stress tensor σ^e_{ij} is valid only when soil particle (not the soil skelton) and pore fluid are incompressible. [2)]

$$\sigma^e_{ij} = \sigma^s_{ij} - (1-n)u_w\delta_{ij} \qquad (4) \qquad \sigma^f_{ij} = nu_w\delta_{ij} \qquad (5)$$

$$= \sigma_{ij} - u_w\delta_{ij}$$

where n is a porosity and u_w is a pore fluid pressure.

Acording to Ishihara,[6)] Π_i is given by

$$\Pi_i = -d(v^f_i - v^s_i) \qquad (6) \qquad d = \rho^f g n^2/k \qquad (7)$$

where k is a permeability coefficient and $\rho^f g$ is the weight of water per unit volume.

From an economic and convienient point of view, it is not so effective to directly apply Eqs.(1) and (2) to liquefaction analysis. In this paper, we will present the following approximated and simplified procedures.

FORMULATION I

when we can assume that the difference between the acceleration of solid phase and that of fluid phase is small, Eqs.(1) and (2) becomes

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \bar{\rho}\,\frac{dv_i^s}{dt} - b_i\bar{\rho} \tag{8}$$

$$\frac{\partial \sigma^f_{ij}}{\partial x_j} = \bar{\rho}^f\frac{dv_i^s}{dt} - \Pi_i - b_i\bar{\rho}^f \tag{9}$$

where $\bar{\rho} = \bar{\rho}^s + \bar{\rho}^f$ and $b_i = b_i^s = b_i^f$.

Formulation I coincides with the U(displacement of solid phase)-P(pore water pressure) form proposed by Zienkiewicz et al.[7] only when water compressibility is zero. Zienkiewicz et al. applied the effective stress concept to the mixture constituted by compressible fluid and solid. They considered that the effective stress concept and compressibility of constituents is independent. On the contrary, the authors consider that the effective stress concept in the soil mechanics is equivalent to the incompressibility of constituents of mixtures.

Next, we will derive the more simplified form Formulation II.

FORMULATION II

It is assumed that the Eq.(8) governs the balance of linear momentum of soil as a whole. By neglecting the acceleration term of fluid phase, Eq.(2) becomes

$$\frac{\partial \sigma^f_{ij}}{\partial x_j} = -\,\Pi_i \tag{10}$$

Eq.(10) is a simple extention of one dimensional equation of seepage flow(Darcy's law). In this formulation, Eqs.(8) and (10) must be solved simultaneously. Almost all of the methods of one-dimensional liquefaction analysis that have been proposed until now are corresponding to Formulation II.

LIQUEFACTION ANALYSIS BY FINITE ELEMENT METHOD

Following the above Formulation II, we will discretize the governing equations by finite element method and finite difference method.

The next relation is obtained.

$$[M]\{\Delta\ddot{\bar{u}}\} + [K]\{\Delta\bar{u}\} + \{K_v\}\ \Delta u_w = \{\Delta F_e\} \tag{11}$$

$$[M] = \int_V [N]^T \bar{\rho} [N]\, dv \quad (12) \qquad [K] = \int_V [B]^T [D][B]\, dv \tag{13}$$

$$\{K_v\} = \int_V \{B_v\}\, dv \quad (14) \qquad \{\Delta F_e\} = \int_V [N]^T \{\Delta F_b\} dv + \int_S [N]^T \{\Delta T_s\} ds \tag{15}$$

Next, we will set out to obtain the formulation of Eq.(10). When soil particle and pore fluid are incompressible, the following relation can be reduced from the equation of balance of mass[5].

$$v^s_{i,i} = \{n(v^s_i - v^f_i)\}_{,i} \tag{16}$$

Substituting eq.(16) into Eq.(10), the following equation is obtained.

$$\frac{d\varepsilon_{kk}}{dt} = -\frac{k}{\rho^f g}\frac{\partial^2 u_w}{\partial x_i} \tag{17}$$

As $d\varepsilon_{kk}=\Delta v$, Eq.(17) becomes

$$\{K_v\}^T\{\Delta\bar{u}\} = -\frac{k}{\rho^f g}\frac{\partial^2 u_w}{\partial x_i^2} \qquad (=\alpha) \tag{18}$$

In the formulation of the right hand side of Eq.(18), forward finite difference scheme is used.

Combining Eqs.(11) and (18), and considering the viscous boundary and the energy transmitting through the base layer, the generalized equation of motion for the element is given by

$$[M]\begin{bmatrix}\{\Delta\ddot{\bar{u}}\}\\ \{0\}\end{bmatrix} + [C]\begin{bmatrix}\{\Delta\dot{\bar{u}}\}\\ \{0\}\end{bmatrix} + \begin{bmatrix}[K] & \{K_v\}\\ \{K_v\}^T & \{0\}\end{bmatrix}\begin{bmatrix}\{\Delta\bar{u}\}\\ \{\Delta u_w\}\end{bmatrix} = \begin{bmatrix}\{\Delta F_e\}\\ \alpha\end{bmatrix} \tag{19}$$

where matrix [C] is determined by the viscous boundary condition and the elastic constants of base rock.

CONSTITUTIVE EQUATION OF SAND[1),8)]

The boundary surface defines the boundary between the normally consolidated region($f_b \geqq 0$) and the overconsolidated region($f_b < 0$) is given by

$$f_b = \bar{\eta}^*_{(0)} + M^*_m \ln(\sigma'_m/\sigma'_{me}) = 0 \tag{20}$$

where σ'_m is mean effective stress, σ'_{me} is preconsolidation pressure and $\bar{\eta}^*_{(0)}$ is a stress parameter representing the effective of anisotropic consolidation introduced by Sekiguchi and Ohta[9)], and is defined by

$$\bar{\eta}^*_{(0)} = \{[\eta^*_{ij} - \eta^*_{ij(0)}][\eta^*_{ij} - \eta^*_{ij(0)}]\}^{\frac{1}{2}} \tag{21}$$

in which

$$\eta^*_{ij} = s_{ij}/\sigma'_m \quad , \qquad \eta^*_{ij(0)} = s_{ij(0)}/\sigma'_{m(0)} \tag{22}$$

where $\eta^*_{ij(0)}$ is the value of η^*_{ij} at the end of anisotropic consolidation and s_{ij} is a component of deviatoric stress tensor. M^*_m in Eq.(20) is the value of $(\eta^*_{ij}\eta^*_{ij})^{\frac{1}{2}}$ when the maximum volumetric compression takes place due to shear.

The plastic strain increment $d\varepsilon^p_{ij}$ is determined by the following non-associated flow rule.

$$d\varepsilon^p_{ij} = \Lambda \frac{\partial f_p}{\partial \sigma_{ij}} df \tag{23}$$

where f_p is a plastic potential function and f is a plastic yield function.

The plastic potential function f_p is assumed to be given by

$$f_p = \bar{\eta}^* + \tilde{M}^* \ln(\sigma'_m / \sigma'_{m(n)}) = 0 \tag{24}$$

In this equation, $\bar{\eta}^*$ is a relative stress parameter and is given by

$$\bar{\eta}^* = [(\eta^*_{ij} - \eta^*_{ij(n)})(\eta^*_{ij} - \eta^*_{ij(n)})]^{\frac{1}{2}} \tag{25}$$

$\sigma'_{m(n)}$ in Eq.(24) and $\eta^*_{ij(n)}$ in Eq.(25) are the values of σ'_m and η^*_{ij} at the n-th time turning over state of loading direction during cyclic loading. While the parameter $\tilde{M}^*$ in Eq.(24) is a variable which is defined as

$$\tilde{M}^* = -\eta^* / \ln(\sigma'_m/\sigma'_{me}) \tag{26}$$

in which $\eta^* = (\eta^*_{ij}\eta^*_{ij})^{\frac{1}{2}}$. (27)

On the other hand, the yield function is assumed to be given by

$$f = \bar{\eta}^* \tag{28}$$

The hardening rule is given by the following hyperbolic curve.

$$\bar{\gamma}^* = \frac{\eta^*(M^*_f + \eta^*_{(n)})}{G'(M^*_f + \eta^*_{(n)} - \bar{\eta}^*)} \tag{29}$$

in which $\bar{\gamma}^*$ is a strain hardening parameter corresponding to $\bar{\eta}^*$, and is defined by

$$\bar{\gamma}^* = [(e^p_{ij} - e^p_{ij(n)})(e^p_{ij} - e^p_{ij(n)})]^{\frac{1}{2}} \tag{30}$$

$\eta^*_{(n)}$ in Eq.(29) is the value of η^* at the n-th time turning point during the cyclic loading.

Total strain increment is obtained, taking into account of elastic components of strain increment as follows.

$$d\varepsilon_{ij} = d\varepsilon^p_{ij} + d\varepsilon^e_{ij} \tag{31}$$

$$d\varepsilon^e_{ij} = \frac{1}{2G} ds_{ij} + \frac{\kappa}{(1+e)\sigma'_m} d\sigma'_m \frac{1}{3} \delta_{ij} \tag{32}$$

NUMERICAL EXAMPLES

Finite element mesh is shown in Fig.1. The 4-node isoparametric element is used. Consistent mass matrix is employed in the formulation of mass matrix. Calculation is carried out under the plane strain condition.

The upward incident wave is assumed to be sinosoidal and is given by

$$u_1 = A_0 \sin(2\pi f_1) \sin(2\pi f_2) \tag{33}$$

u_1 : acceleration, $A_0 = 0.1$ (m/sec), $f_1 = 0.5$, $f_2 = 0.05$

Eq.(11) is directly integrated by Wilson's implicit method with Θ=1.4, and Δt(time interval) is 0.0025 sec. The approximation of the right hand side of Eq.(17) in detail is explained by Akai and Tamura[3].

The parameters used in calculation are listed in Table 1.

Figs.2 and 4 show the pore water pressure-time profile. The excess pore water pressure increases during the cyclic loading. Shown in Fig.3 that the mean effective stress decreases. Comparing Figs.2 and 4, it is seen that the developed excess pore water pressure is greatly influenced by the coefficient of permeability.

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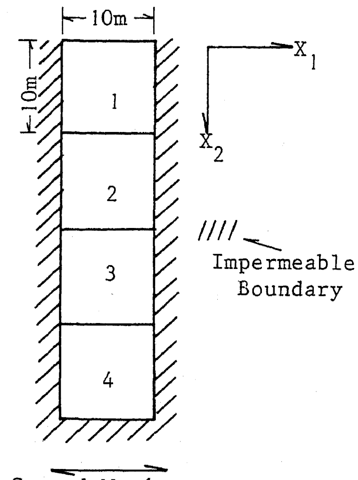

Fig.1

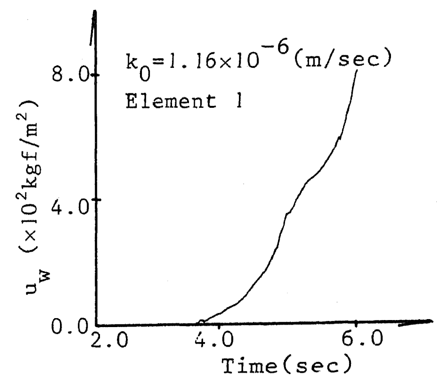

Fig.2

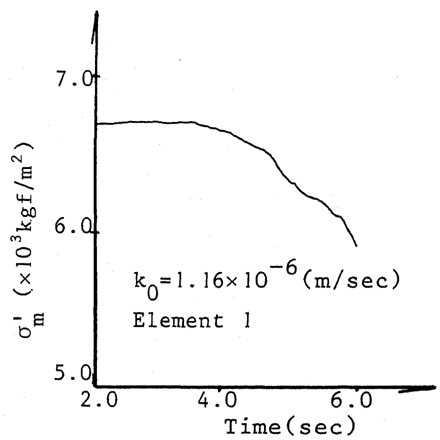

Fig.3

Table 1
Parameters

$\varkappa$	0.003
M_m^*	1.11
M_f^*	1.28
G_0	2.8×10^6 kgf/m^2
G'	800
e_0	1.0
K_0	0.5
ρ	185 kg/m^4/sec^2 (Density)
ρ_B	185 kg/m^4/sec^2 (Density of base rock)

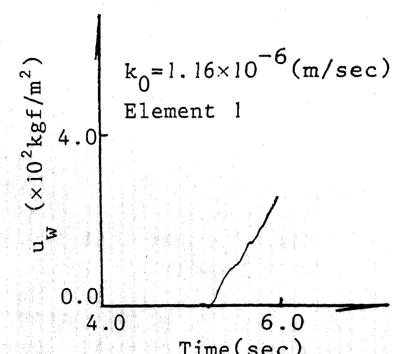

Fig.4