EVALUATION OF COMPLIANCE FUNCTIONS OF SOIL FOUNDATION SYSTEMS BY BOUNDARY ELEMENT METHOD

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SUMMARY

This paper presents compliance functions of strip foundations evaluated by boundary element methods. The solutions are based on the free Green's function of the Fourier frequency transformed wave equation. The method is applicable to imbedded foundations, layered soil, and topographically irregular sites because boundary and interface conditions are enforced by the discrete boundary integral equations. Results presented here refer to rigid foundations with rough footings. They include surface and embedded foundations on the halfspace and on a layer on top of a rigid base rock.

BOUNDARY ELEMENT ANALYSIS

Cruse and Rizzo (Ref. 1) gave the fundamental solution for the transformed wave equation. It allows, together with Betti's theorem, one to rewrite the boundary value problem of elasto-dynamics in an integral equation form, where all field values may be expressed by the data on the boundary. Since the fundamental solution (or free Green's function) vanishes at infinity the part of the boundary which lies at infinity does not contribute to the boundary integral. This fact makes the boundary integral formulation especially attractive for boundary value problems with unbounded domains, as in the case of a foundation in the halfspace. If Green's function can be found which satisfies homogeneous boundary conditions for the considered problem, then the boundary integral formulation becomes even more advantages (Ref. 2). In the case of soil-foundation interaction the boundary integral equation leads to a system of algebraic equations, relating displacements $\underline{u}$ and tractions $\underline{t}$ of the soil in the form

$$\left(\frac{1}{2}\,\underline{I} + \underline{T}\right)\underline{u} = \underline{U}\,\underline{t} \tag{1}$$

In equation (1) $\underline{I}$ is the unit matrix, $\underline{T}$ and $\underline{U}$ are fully populated coefficient matrices depending on the fundamental solution, the shape functions of the boundary element approximation, and the domain. Condensation of the displacements to the kinematic degrees of freedom of the soil-foundation interface leads to the complex, frequency dependent dynamic stiffness matrix (impedance)

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of a flexible (limp) foundation. Its inverse is the dynamic flexibility matrix (compliance). In the case of a rigid foundation kinematic constraints in the foundation degree of freedom transform the stiffness matrix into the stiffness matrix related to the rigid body degrees of freedom.

NUMERICAL RESULTS

In the following examples results obtained by the boundary element method are compared with those found in the literature. They refer to a rigid strip foundation with a rough footing. Constant displacement/traction elements are used in all calculations.

Surface Foundation on the Halfspace

For surface foundations the coupling between swaying and rocking motion is usually neglected. This allows a considerable simplification in the numerical effort: only the soil-structure interface has to be discretized by boundary elements (Ref. 3).The complex non-dimensional compliance functions reduce to the diagonal terms of the compliance matrix

$$f_{xx}(a_o) = \frac{u_x G}{P_x} \quad , \quad f_{zz}(a_o) = \frac{u_z G}{P_z} \quad , \quad f_{mm}(a_o) = \frac{\phi G B^2}{M} \qquad (2)$$

for swaying, heaving and rocking motion. They are denoted in Fig. 1 as relaxed boundary conditions (R.B.C.). In equation (2) u_x, u_z represent horizontal and vertical motion, ϕ is the rotation of the center of the foundations; P_x, P_z, M denote the corresponding line loads. G is the shear modulus of the soil with Poisson's number $\nu = 0.3$. The dimensionless frequency is defined as

$$a_o = \frac{\omega B}{c_s}$$

where c_s is the velocity of a shear wave in the soil and ω the frequency. B is the half width of the strip-foundation. In the numerical analysis 10 boundary elements have been used within the foundation. The element discretization outside the foundation influences the diagonal elements of the compliance only slightly and may therefore be omitted. This has already been demonstrated in (Ref. 2) where it was shown that the difference between the boundary element solution (R.B.C.) and the halfspace solution is small. This fact is supported in Fig. 1 where impedance functions with non-relaxed boundary condition (NR.B.C.) are shown. Here the discretization is continued by 5 elements for a length B on each side of the foundation. If cross coupling between swaying and rocking is of interest the discretization has to extend further on both sides of the foundation by a factor of 4 to 5 (Ref. 2). Otherwise the off diagonal elements f_{mx} and f_{xm}, which reflect the coupling between swaying and rocking, will not be computed accurately enough, showing a pronounced asymmetry.

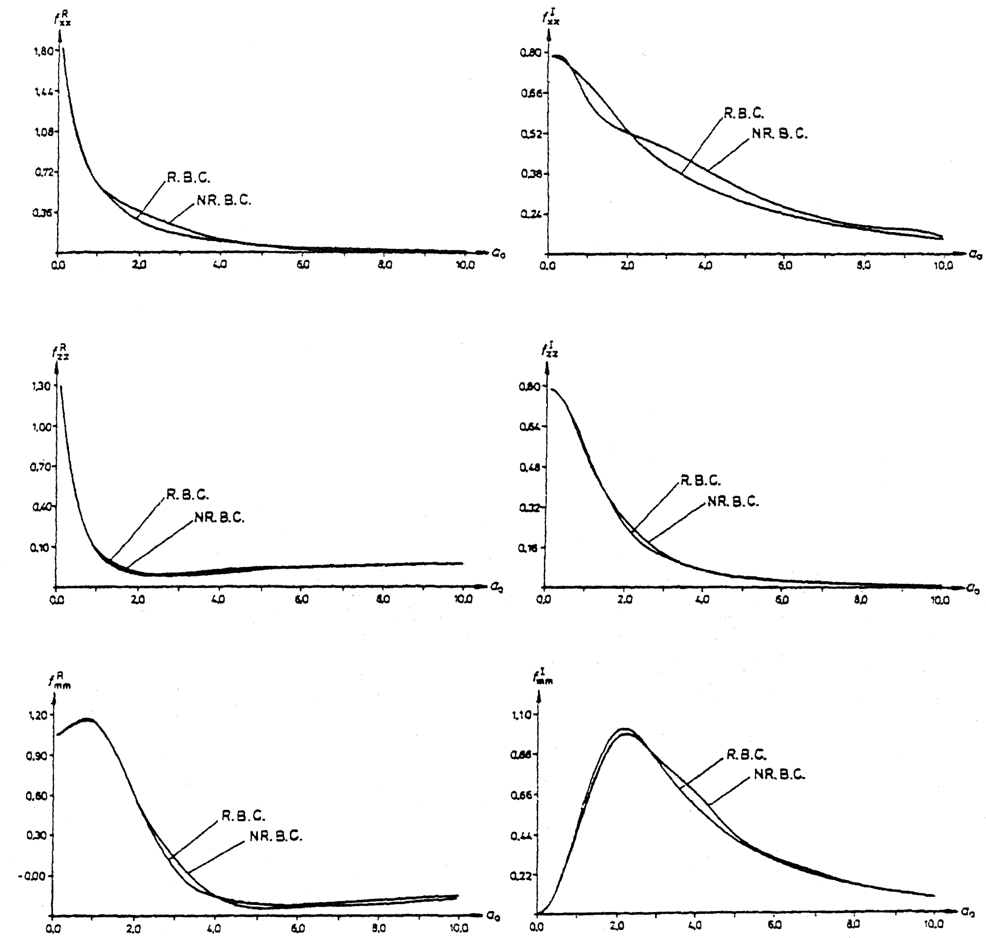

Fig. 1 Compliance functions of a surface foundation on the halfspace (ν = 0.3). Comparison of relaxed and non-relaxed boundary conditions.

Surface Foundation on a Soil Layer

The boundary element method is equally applicable to infinite and bounded domains as well as to exterior or interior regions. To demonstrate this a surface foundation on a soil layer with hysteretic damping, having the same geometrical and material data as used by Tassoulas (Ref. 4) and Gazetas (Ref. 5), is analysed. Geometrical and material properties as well as the discretization are shown in Fig. 2. Note that boundary elements are only necessary at the upper and lower boundary of the layer.

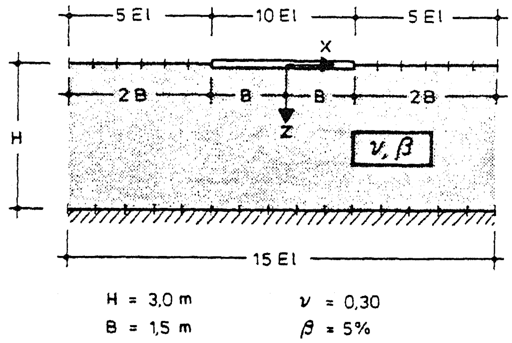

Fig. 2 Surface foundation on a soil layer.
Geometrical and material properties.

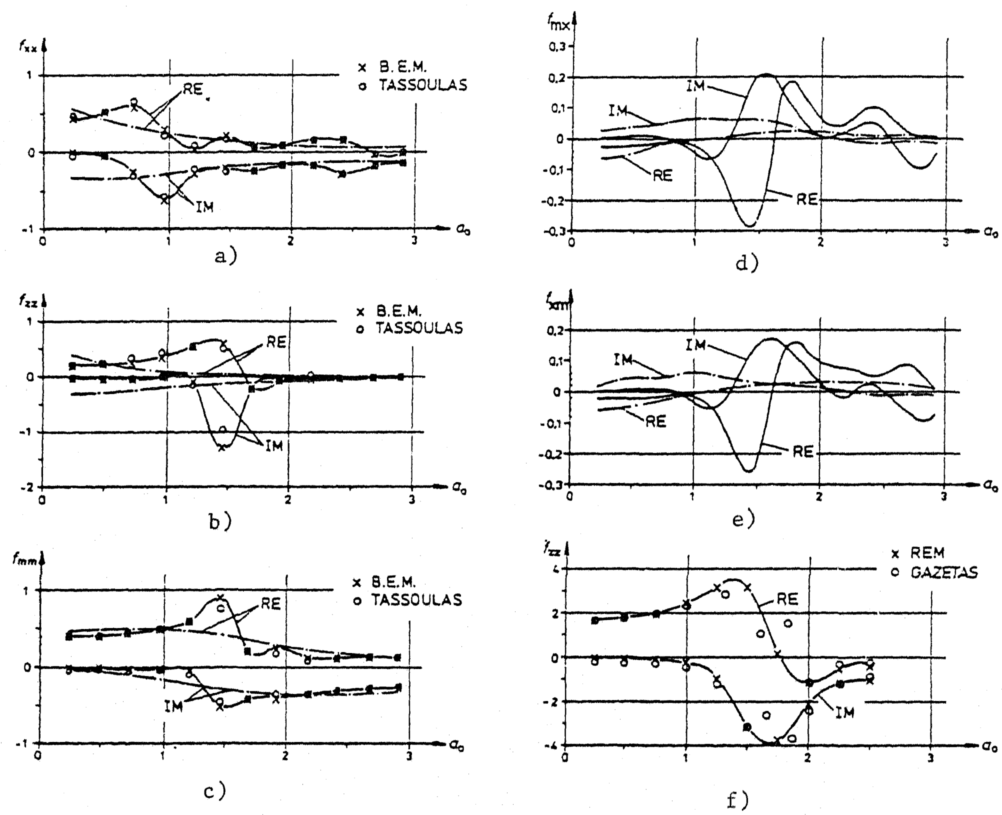

Fig. 3 Compliance functions of a surface foundation on a soil layer.
a) – e): Poisson's ration ν = 0,3; f) Poisson's ratio ν = 0.4.
–·– Halfspace Solution (B.E.M.)

Fig. 3 a) – c) compare the boundary element solution with results obtained by a semi-discrete finite element method (Ref. 4). The diagonal elements of the compliance show only small deviations at the resonant frequency of the layer. It is of interest to note here that for the soil layer only 5 elements on each side of the foundation are sufficient to obtain symmetric coupling coefficients, scaled as

$$f_{xm} = \frac{u_x Gb}{M} \quad , \quad f_{mx} = \frac{\phi Gb}{P_x} \tag{3}$$

Unfortunately the coupling coefficients are not given by Tassoulas (Ref. 4). The compliance functions for a deep soil layer ($H \rightarrow \infty$) are also indicated in Fig. 3 a) – e). The compliance function f_{zz} of the same foundation/soil layer configuration is shown in Fig. 3 f) but here with Poisson's ratio $\nu = 0.4$. In contrast to our results, which show the same qualitative behavior as in Fig. 3 b), the functions obtained by Gazetas (Ref. 5) by a semi-analytical method show two peaks in the frequency range $1 < a_o < 2$.

Embedded Foundation on a Soil Layer

Embedded foundations always have to be calculated with the complete equation (1) ($\underline{T} \neq 0$). The assumption, that horizontal and vertical motions are decoupled do not lead to the simplifications mentioned for surface foundation. Otherwise the boundary element analysis follows the same steps as for surface foundations. Any physical boundary conditions at the foundation, rough or smooth (relaxed or non-relaxed) may be considered. In Fig. 4 geometrical and material data for an embedded foundation on a soil layer are shown. The compliance functions are referred to the center of the rigid foundation. Again hysteretic damping is assumed. The somewhat finer discretization along the vertical sides of the foundation is suggested by numerical tests. In Fig. 5 all coefficients of the compliance matrix are shown and compared with results obtained by Chang Liang (Ref. 6) by a semi-discrete finite element method.

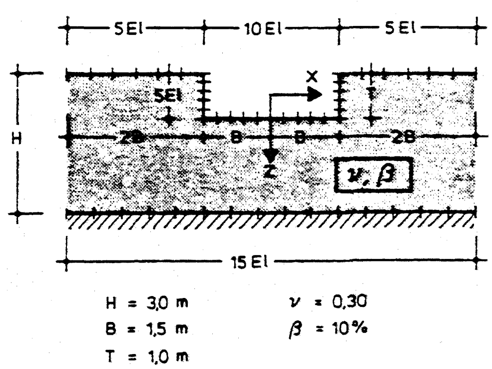

Fig. 4 Embedded foundation on a soil layer

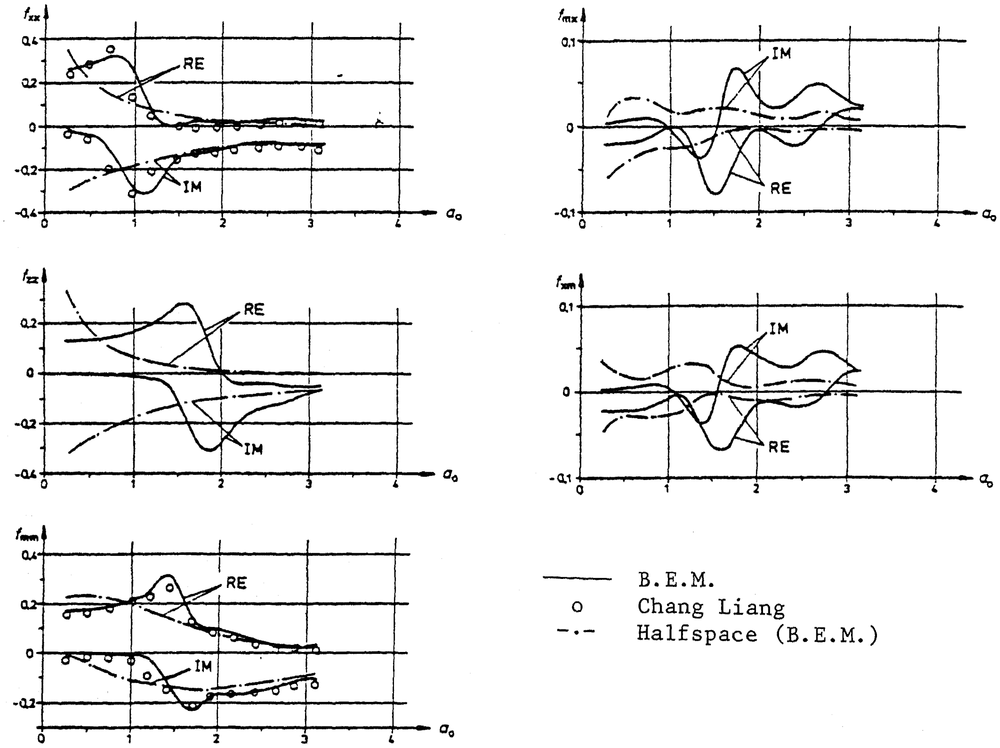

Fig. 5 Compliance functions of an embedded foundation on a soil layer

Whereas the diagonal elements show good agreement, the off-diagonal elements differ significantly. It should also be noted that the coupling coefficients of the boundary element solution still show some asymmetry. This should, according to our experience, improve as the region of discretization outside the foundation is enlarged.

CONCLUSION

The boundary element method is well suited for soil-structure interaction problems. Good agreement with other methods is found for surface and embedded foundations in a halfspace and on a soil layer on top of a rigid base rock. Applications to layered soils fall within range of the method. One of the biggest advantages over other methods may be its generality. Its application is not restricted to horizontally layered systems; the discretization may well follow the topographical disturbances of the site.

ACKNOWLEDGEMENT

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