HYDRODYNAMIC SPECTRA

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SUMMARY

Damped and undamped Bessel spectra of earthquake accelerograms are defined and their application to the evaluation of water pressure and earth pressure against retaining structures under seismic excitation is explained. A representative sample of undamped Bessel spectra is presented. A conjecture due to Rosenblueth according to which the undamped Bessel spectrum is approximately equal to the absolute acceleration response spectrum of a linear oscillator with 15% critical (viscous) damping appears to be confirmed.

INTRODUCTION

The integrals

$$S(t, \Omega_n) = \Omega_n \int_o^t a(\tau)\ J_o\left[\Omega_n(t-\tau)\right] d\tau, \quad (n=1,2,\dots) \qquad (1)$$

appear for the first time in earthquake engineering in connection with the problem of evaluating hydrodynamic pressures on a rigid, vertical dam due to seismic excitation (Refs 1, 2). More recently, the same integrals have been found to be of use in the calculation of the earth thrust on the vertical backface of a rigid retaining wall under seismic action (Refs 3,4). The meaning of the symbols in Eq. 1 is as follows:

$a(.)$	:	horizontal component of ground acceleration perpendicular to the back face of the retaining structure;
$J_o(.)$	:	Bessel function of the first kind and zero order;
t	:	time;
τ	:	dummy integration variable;
Ω_n	:	natural circular frecuency of the n^{th} mode (Eqs. 7, 9)

The right hand side of Eq. 1 bears some ressemblance with the familiar Duhamel integral for the seismic response of a simple harmonic (undamped)

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oscillator. Therefore, by analogy, an undamped Bessel spectrum or hydrodynamic spectrum can be defined by the relation

$$B(\Omega, o) = \sup_t |S(t,\Omega)| \quad (2)$$

In the calculation of hydrodynamic pressures, damping due to viscosity of the fluid can be disregarded altogether. This is not strictly the case for the dynamic earth pressure problem; in fact, we may presume that the contrary is true; i.e., internal damping in the backfill material (as well as radiation of energy through the base of the wall-backfill system) has a non-negligible effect on the seismic response of earth-retaining structures. If radiation damping is disregarded, but internal damping in the backfill material is taken into account through the introduction of a damping term of the Voigt type, then, instead of the integrals of Eq. 1, the following integrals appear in the solution of the seismic earth thrust problem (Refs. 3, 4):

$$S(t, \Omega_n;\zeta) = \frac{2\Omega_n}{\pi\sqrt{1-\zeta^2}} \int_o^t a(\tau) \, \text{Im}\, K_o \left[\Omega_n(t-\tau)e^{-i \,\text{arc cos}\zeta}\right] d\tau \quad (3)$$

$$(n = 1, 2, \ldots)$$

Here $K_o[.]$ denotes the modified Bessel function of imaginary argument known as MacDonald's function of order zero, Im stands for "imaginary part", and ζ denotes damping expressed as a fraction of its critical value.

A damped Bessel spectrum can then be defined by

$$B(\Omega,\zeta) = \sup_t |S(t,\Omega;\zeta)| \quad (4)$$

It can be shown that $S(t,\Omega_n)$ and $S(t, \Omega_n;\zeta)$, as defined by Eqs. 1 and 3, respectively, satisfy the relation

$$S(t, \Omega_n) = S(t, \Omega_n; o) \quad (5)$$

Therefore, for $\zeta = o$, Eq. 4 reduces to Eq. 2.

As far as we know, only a few hydrodynamic spectra (undamped Bessel spectra) had been computed until recently: two by Kotsubo (Ref 1), and one by Flores (Ref. 5). All three are reproduced by Newmark and Rosenblueth (Ref. 6). The spectra computed by Kotsubo correspond to records for which the maximum acceleration does not exceed 3 gal, and the duration of motion is approximately 5 sec. Thus, these two hydrodynamic spectra may be considered as not significant for engineering applications. The one computed by Flores belongs to a 12 second segment of the NS component of the 1940, El Centro accelerogram.

The interest of the senior author of the present paper on hydrodynamic spectra was aroused in 1981, when working on the problem of the seismic thrust of the backfill against a rigid, vertical retaining wall. An approximate solution of this problem was propounded by him through the introduction of a simplified elastic model for the backfill material (Ref. 3). Subsequently, additional refinements were introduced to take into account the possibility of wall rotation due to foundation compliance; a discrete linear model was elaborated in which the loss of energy through radiation by elastic waves propagating horizontally along the backfill and away from the wall, was approximately taken into account through the introduction of an "equivalent" linear dashpot (Ref. 4). However, the application of these models to engineering practice was bound to remain limited while adequate information on Bessel spectra of real accelerograms continued to be lacking. It was known that the calculation of these spectra, with the algorithms that were at that time available, offered special computational difficulties (Ref. 5). It was then decided to develop a new algorithm. This was done by Rivera and Sierra (Ref. 7) under the joint guidance of F.J. Sánchez-Sesma and the senior author of the present paper. Since then a total of 24 hydrodynamic spectra have been computed using the new algorithm. A representative sample of the results is reported here for the first time. A full report will be published elsewhere (Ref. 8).

UNDAMPED BESSEL SPECTRA

Water impounded in a reservoir such as the one depicted in Fig. 1 is suposed to be at rest up to the instant $t = o$, and occupies the region defined by the inequalities $x > o$, $o < y < H$, $-\infty < z < \infty$. Except for the free surface, the boundaries of the fluid are assumed to be rigid and to start moving at the instant $t = o$ in the x-direction with given acceleration $a(t)$. Furthermore the following assumptions are made: water is a continuous inviscid, and compressible fluid; its velocity field is continuous and differentiable with respect to everyone of its arguments; there is no cavitation, and the fluid remains in contact with the rigid boundaries of the reservoir; displacements are small; surface waves are negligible.

Under the assumptions stated above the pressure distribution on the face of the dam is found to be (Refs. 1, 2):

$$p(0,y,t) = \frac{8\rho H}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} S(t,\Omega_n) \cos\left(\frac{\Omega_n y}{c}\right) \qquad (6)$$

where $S(t,\Omega_n)$ is defined in Eq. 1 and Ω_n is the n^{th} natural circular frequency of the sheet of water for compressional - dilatational waves propagating vertically:

$$\Omega_n = (2n-1)\pi c/2H \qquad (7)$$

The problem for the earth-retaining wall can be formulated in a similar way when the backfill is represented by a simplified elastic model introduced by the senior author of the present paper (Ref. 3). For the case of zero material damping in the backfill it is found that the dynamic thrust on the retaining wall has the following distribution

$$p(0,y,t) = \frac{8\rho H\alpha}{\pi^2\beta} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} S(t,\Omega_n) \sin\left(\frac{\Omega_n y}{\beta}\right) \tag{8}$$

Here ρ is the mass density of the backfill; α is the velocity of compressional waves propagating horizontally in the backfill material; β is the velocity of shear waves propagating vertically and

$$\Omega_n = (2n-1)\pi\beta/2H \tag{9}$$

is the circular frequency of the n^{th} shear mode of an infinite stratum having the same depth and the same elastic properties as the backfill, free at the top and fixed at the bottom. When damping is considered, Eq. 8 remains valid with $S(t, \Omega_n; \zeta)$ instead of $S(t,\Omega_n)$.

The algorithm employed by Flores for the computation of hydrodynamic spectra was based in a representation of the accelerogram as a sequence of rectangular pulses of short duration. A substantial improvement in accuracy and efficiency is accomplished if this representation is replaced by a sequence of triangular pulses, or equivalently, by a sequence of ramp functions. Through this approach, any continuos accelerogram composed of a succesion of linear segments can be reproduced exactly, with a substantial saving in the processing time of its Bessel spectrum.

COMPARISON OF UNDAMPED BESSEL SPECTRA WITH ORDINARY RESPONSE SPECTRA

It can be shown that undamped Bessel spectra have the following asymptotic properties

$$\lim_{\Omega \to \infty} B(\Omega,0) = \sup_t |a(t)| \tag{10}$$

$$\lim_{\Omega \to 0} \frac{B(\Omega,0)}{\Omega} = \sup |v(t)| \tag{11}$$

These suggest that $B(\Omega,0)$ should be compared with ordinary absolute acceleration response spectra. Now, both in the hydrodynamic problem as in the seismic earth pressure problem, energy is lost to the system through waves propagating horizontally in the retained medium which acts as a wave guide. Therefore undamped Bessel spectra should be compared with damped absolute acceleration spectra. Rosenblueth (Ref. 9; see also Ref. 6, p. 367) suggests 0.15 as a suitable value of damping expressed as a fraction of its critical value. This conjecture appears to be confirmed by the results we have obtained. Thus, in spite of the fact that radiation damping cannot be represented exactly by an "equivalent" viscous damping, absolute acceleration response spectra for 15% critical damping can be employed as a substitute for undamped Bessel spectra with satisfactory accuracy as far as practical applications are concerned. Here we offer a few examples of this comparison in Figs. 2a-7b. (See Table 1)

FINAL REMARKS

As yet no Bessel spectra for non zero damping have been computed. Computational difficulties are perhaps harder than in the case of zero damping and on the other hand the field of application of prospective results seems to be rather restricted. A promising line for future developments that deserves to be explored is the use of transform techniques (as the one propounded by Linz in Ref. 10) as a computational aid in the evaluation of Bessel spectra.

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TABLE 1. Figure captions and data for identification of accelerograms used in the computation of spectra

Recording station	Date	Component	Duration (sec)	Figure
Sicartsa T	March, 14, 1979	N90W	22.08	2a
Sicartsa RM	March, 14, 1979	N90W	21.98	2b
Oaxaca	Nov., 29, 1978	N00E	14.93	3a
		N90E	12.61	3b
La Villita Dam (Base)	March, 14, 1979	Long	13.88	4a
		Transv.	14.32	4b
Texcoco Centro Lago, A+B	March, 14, 1979	N90W	57.81	5
Infiernillo Dam N-180	March, 14, 1979	Long	64.92	6a
		Transv.	67.82	6b
Oaxaca	Oct., 24, 1980	N00E	33.80	7a
Medicina		N90W	33.84	7b

N.B. Dashed curves are undamped Bessel spectra $B(T,0)$ plotted as functions of $T = 2\pi/\Omega$. Continuous curves are ordinary absolute acceleration response spectra for $\zeta = 0.15$. (Except in Fig. 5 where two response spectra are plotted with damping as shown). The ordinates of Figs. 2a-7b are in g's.

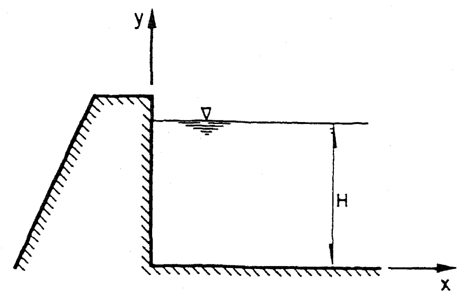

Fig 1

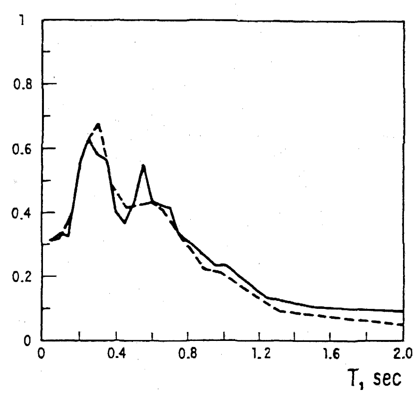

Fig 2a

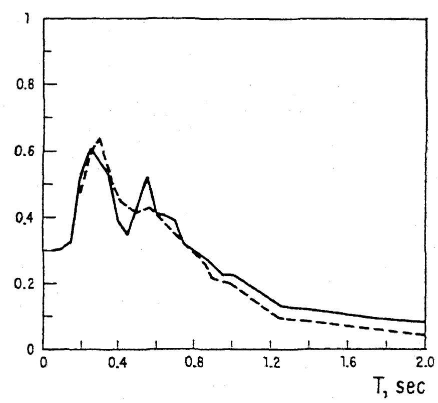

Fig 2b

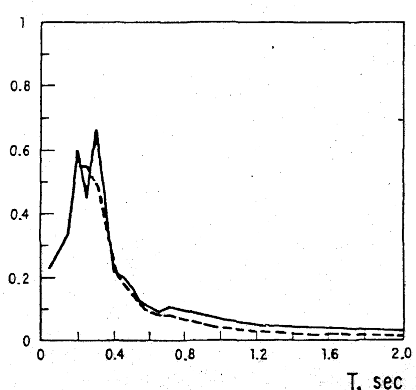

Fig 3a

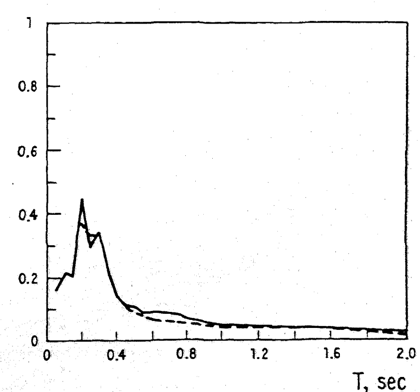

Fig 3b

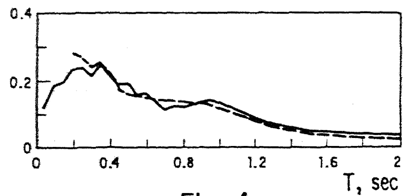

Fig 4a

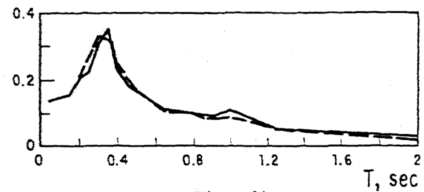

Fig 4b

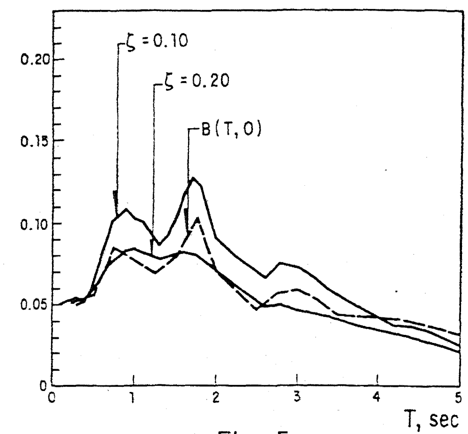

Fig 5

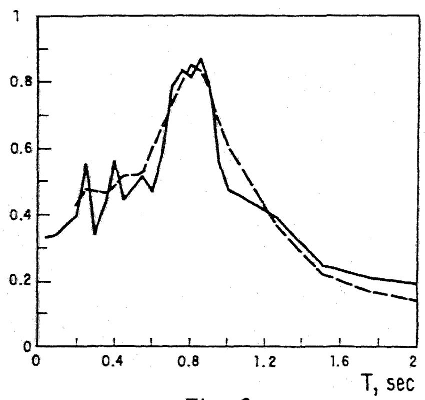

Fig 6a

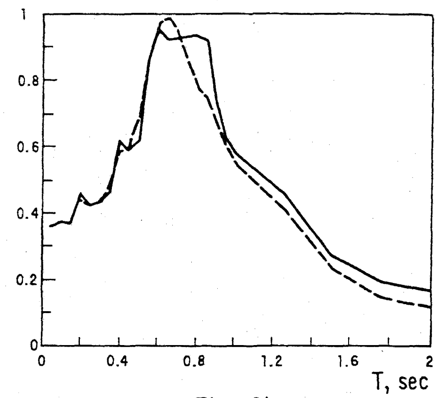

Fig 6b

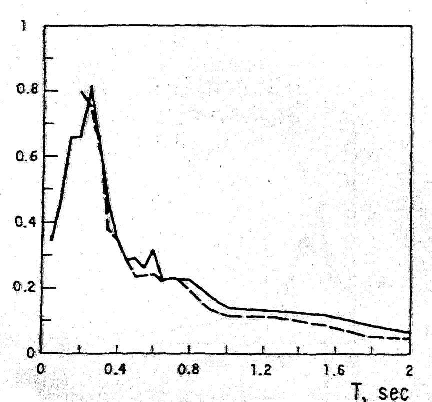

Fig 7a

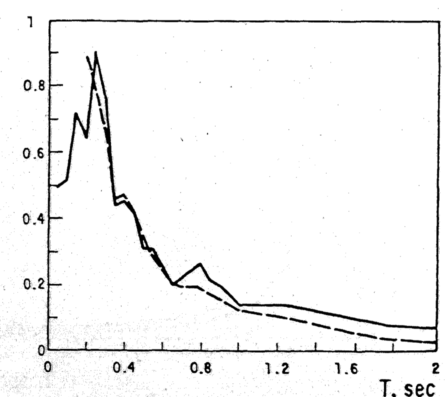

Fig 7b