STRONG GROUND MOTION DURATION AND EFFECTIVE CYCLIC ACCELERATION

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SUMMARY

The duration of strong ground motion is defined as the time interval during which the total energy supplied to all single degree of freedom structures is imparted with a uniform cyclic peak level. This peak level is defined as the effective cyclic acceleration. It is shown that its value is equal to $\sqrt{2}$ times the root mean square acceleration evaluated over the defined strong motion duration. The strong ground motion durations for 60 horizontal components of 30 earthquake recordings are estimated and compared with their corresponding values as given by Trifunac-Brady and McCann-Shah. It is found that the estimated durations are in general agreement with the Trifunac-Brady estimates and are longer than the McCann-Shah estimates.

INTRODUCTION

The duration of strong ground motion, its effective level of shaking and its frequency content are the main parameters that control the damaging effects of an earthquake. While the representation of the frequency content of the ground motion in terms of a response spectrum or a Fourier spectrum has found very wide acceptance (Ref. 1), as yet there are no single definitions of duration of strong motion and the level of shaking which are similarly accepted. Estimates of duration of strong shaking are of the utmost importance in problems related to liquefaction of soil deposits, seismic settlement, damage to yielding structures, analysis of nonlinear systems, response of lightly damped linear systems, selection of representative records in response studies of soils and structures, and generation of artificial accelerograms. Due to the recognition of its importance, there have been many attempts at its quantification. Bolt (Ref. 2) defined a "bracketed duration" as the time elapsed between the first and the last excursion levels greater than a certain acceleration amplitude. Trifunac and Brady (Ref. 3) defined the duration as the time elapsed between 5 and 95% of the quantity $s(t) = \int_0^t f^2(\tau)d\tau$, where $f(t)$ corresponds either to acceleration, velocity or displacement. Housner (Ref. 4) defined the duration as the time length of the segment with uniformly strong shaking. McCann and Shah (Ref. 5) defined the duration of the strong motion phase such that it would be consistent with the root mean square acceleration of the ground motion. Vanmarcke and Lai (Ref. 6) proposed that the strong motion duration is

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proportional to the quantity I/a^2 where I is the total motion "energy" or Arias intensity (Ref. 7) and a is the peak ground acceleration.

Similarly, as summarized by Housner (Ref. 4), there are various definitions for the level of shaking, such as peak ground acceleration, spectral intensity, and root mean square acceleration.

It has been recognized (Refs. 8 and 9) that the peak ground acceleration generally does not provide a reliable measure of strong ground shaking. Also, as pointed out by Housner, the spectrum intensity is more a measure of the effects of ground motion on the elastic response of structures than of the damaging effects of the ground motion. Root mean square acceleration, as is pointed out by McCann and Shah (Ref. 6), is the square root of the rate at which energy is input to all single degree of freedom systems and because of its integral property, it is less susceptible to large fluctuations due to extreme peaks. The main problem in defining a proper value for an effective RMS acceleration lies in the choice of an appropriate interval of time, i.e., duration of strong shaking phase, over which it is evaluated.

Based on the assumption that the significant duration of the ground motion and its effective level are related through the total energy delivered to all single degree of freedom structures, the duration of the strong motion phase is defined. It is shown that this definition is consistent with a uniform level of ground acceleration. This uniform level is defined as the effective cyclic acceleration. It is shown that the value of the proposed effective cyclic ground acceleration is equal to $\sqrt{2}$ RMS accelerations, where the RMS acceleration is defined over the proposed strong motion duration. Based on the maximum spectral response of single degree systems, a necessary condition (in terms of an inequality) for the defined strong motion duration and the effective cyclic acceleration is derived. The proposed strong motion durations and their corresponding effective cyclic accelerations are calculated for 60 horizontal components of 30 earthquake recordings. It is shown that in all cases the calculated values satisfy the necessary condition. The results for the strong motion duration are also compared with their corresponding values as defined by several other investigators. It is found that the proposed definition for the strong motion duration yields durations which are in general agreement with those defined by Trifunac-Brady but longer than those defined by McCann-Shah. It is also found that the mean normalized effective cyclic acceleration $\dfrac{a_e}{a}$, for the 60 records considered, is 0.32.

Strong Motion Duration

It can be shown that the quantity $I(t_r) = \int_0^{t_r} \ddot{u}_g^2(\tau)d\tau$, where $\ddot{u}_g(t)$ is the ground acceleration and t_r is the duration of its record, is related to the total energy input to all single degree of freedom structures during an earthquake (Refs. 7 and 10). A plot of $I(t)$ for the N-S component of the 1940 El Centro earthquake is given in Figure 1. It is noted that the slope of this curve at any time is $\ddot{u}_g^2(t)$; therefore, a constant slope for this curve implies uniformly strong shaking, i.e., uniform maximum acceleration as is shown in Figure 2. To define a unique slope, it is noted that after

some time, the quantity $\int_0^t \ddot{u}_g^2(\tau)d\tau$ and also area A shown in Figure 1 become almost constants. Area A is defined by

$$A = \frac{1}{a^2} \int_0^{t_r} [I(t_r) - \int_0^t \ddot{u}_g^2(\tau)d\tau]d\tau \quad . \tag{1}$$

Then the duration of strong shaking phase t_s may be defined by

$$t_s = \frac{A}{(\frac{1}{2})I(t_r)/a^2} \quad . \tag{2}$$

It is noted that during this time the quantity $I(t_r)a^2 = \int_0^{t_r} \ddot{u}_g^2(t)d\tau/a^2$, which according to Arias (Ref. 7) is representative of the total energy associated with the ground motion, $\ddot{u}_g(t)$, is reached with the uniform slope $\int_0^{t_r} \ddot{u}_g^2(\tau)d\tau/a^2 t_s$, where a is the peak ground acceleration.

Effective Cyclic Acceleration

The normalized mean square acceleration a_{rs}/a over the defined strong motion duration t_s is

$$\frac{a_{rs}}{a} = (\frac{\int_0^{t_s} \ddot{u}_g^2(\tau)d\tau}{a^2 t_s})^{\frac{1}{2}} \quad . \tag{3}$$

If the ground acceleration is represented by a harmonic series, i.e.,

$$\ddot{u}_g(t) = \sum_i a_i \sin\Omega_i t \quad , \tag{4}$$

then

$$(\frac{a_{rs}}{a})^2 = \frac{1}{a^2 t_s} \int_0^{t_s} \ddot{u}_g^2(\tau)d\tau = \frac{1}{a^2 t_s} \int_0^{t_s} (\sum_i a_i \sin\Omega_i\tau)^2 d\tau \quad . \tag{5}$$

But for sufficiently long t_s,

$$\frac{1}{a^2 t_s} \int_0^{t_s} (\sum_i a_i \sin\Omega_i\tau)^2 d\tau = \frac{1}{2} \sum_i (\frac{a_i}{a})^2 \quad . \tag{6}$$

Substitution of expression (6) into expression (5) yields

$$(\frac{a_{rs}}{a})^2 = \frac{1}{2} \sum_i (\frac{a_i}{a})^2 \quad . \tag{7}$$

If the normalized effective cyclic acceleration $\frac{a_e}{a}$ is defined by

$$(\frac{a_e}{a})^2 = \sum_i (\frac{a_i}{a})^2 \quad , \tag{8}$$

then expression (7) in conjunction with expression (3) yield

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$$\frac{a_e}{a} = \sqrt{2}\ \left(\frac{\int_0^{t_s} \ddot{u}_g^2(\tau)d\tau}{a^2 t_s}\right)^{\frac{1}{2}} \quad . \tag{9}$$

To be on the conservative side in estimating the effective cyclic acceleration, one may replace the upper limit of the integral in the above expression by the duration of the record, t_r. Therefore, this conservative estimate is given by

$$\frac{a_e}{a} = \sqrt{2}\ \left(\frac{\int_0^{t_r} \ddot{u}_g^2(\tau)d\tau}{a^2 t_s}\right)^{\frac{1}{2}} = \sqrt{2}\ \left(\frac{I(t_r)}{a^2 t_s}\right)^{\frac{1}{2}} \quad . \tag{10}$$

This conservative estimate of $\frac{a_e}{a}$ is the one which is used from now on.

Of the three parameters, strong motion duration, level of ground motion shaking, and the frequency content, the first two have been defined. Assuming that the ground motion is fed to a single degree of freedom structure which has a natural frequency equal to the dominant ground motion frequency, i.e., the frequency imparting the highest amount of energy (the frequency corresponding to the maximum spectral velocity, SV_{max}, on the spectral plot), then it may be surmised that the structural response will be in tune with the ground motion. It can be shown that the maximum velocity response of a single degree of freedom structure which is in tune with an applied harmonic excitation of duration t_s is given by (Ref. 11)

$$SV_{max} = \frac{at_s}{2} \quad , \tag{11}$$

where a is the amplitude of the input excitation. Replacing a in this expression by the effective cyclic amplitude of the ground motion, a_e, yields

$$SV_{max} = \frac{a_e t_s}{2} \quad . \tag{12}$$

Since, in general, only a portion of the total ground motion energy is released in the dominant frequency in tune with the structure, it may be conjectured that

$$SV_{max} < \frac{a_e t_s}{2} \quad \text{or}$$

$$\frac{SV_{max}}{a_e t_s/2} < 1 \quad . \tag{13}$$

Of course, the satisfaction of the above inequality does not provide a strong quantitative estimate of the quality of the proposed definitions for a_e and t_s per se, except that since this necessary condition is independent of the proposed definitions, its satisfaction ensures that the definitions are not in contradiction with it.

Results and Comparisons

The earthquake strong motion records used in this study are presented in Table 1. The strong motion duration and the effective cyclic acceleration as defined by expressions (2) and (10) together with the quantity $a_e t_s/2$ and SV_{max} for 60 horizontal components of 30 earthquake recordings are presented in Table 2. The pertinent parameters are taken from Volume III of the CIT-EERL reports (Ref. 12). These records are the same records which McCann and Shah (Ref. 5) have used in presenting their estimates and those of Trifunac-Brady, Bolt and Vanmarcke-Lai of the strong motion durations. As may be noted from the last two columns of this table, in all cases the necessary condition $SV_{max}/(a_e t_s/2) < 1$ is satisfied. It should be noted that the satisfaction of this inequality by itself does not imply that the proposed definitions for the strong ground motion duration and the effective cyclic acceleration are necessarily satisfactory. However, its satisfaction ensures that the proposed definitions are consistent. For comparison purposes, the strong motion durations for the 60 records as given by Trifunac-Brady and McCann-Shah are also presented in Table 2.

The proposed durations for the 60 records named in Table 2 are plotted versus their corresponding values, as defined by Trifunac-Brady and McCann-Shah, in Figure 3. It may be observed that the proposed method for estimating the strong ground motion duration yields durations which are in general agreement with those estimated by Trifunac-Brady but longer than those estimated by McCann-Shah. The normalized effective cyclic acceleration (normalized with respect to the peak ground acceleration), for the 60 records considered, varies between 0.18 and 0.44 with a mean of 0.32 and a standard deviation of 0.06. To study the effect of peak ground acceleration on the effective cyclic acceleration, the data presented in Table 2 is rearranged in Table 3. From this table, it is observed that the mean normalized effective cyclic acceleration for lower and higher intensity motions are about the same.

Conclusions

Based on energy considerations, the duration of strong ground motion and its effective level are defined. It is found that the proposed definition for duration yields estimates which are, in general, of the same order as those given by Trifunac and Brady. However, the proposed definition for the duration of the strong motion phase ties in the duration with a uniform level of ground motion. This uniform level is defined as the effective cyclic acceleration. For the 60 records considered, it is found that the mean normalized effective cyclic acceleration, a_e/a, is 0.32 with a standard deviation of 0.06.

Acknowledgement

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TABLE 3

EFFECT OF PEAK GROUND ACCELERATION a ON

EFFECTIVE CYCLIC ACCELERATION a_e

Peak Ground Acceleration	Number of Records	Mean a_e/a	Standard Deviation a_e/a
All records	60	.32	.06
a > .15 g	23	.31	.06
a < .15 g	37	.33	.05

TABLE 2

STRONG MOTION DURATION AND EFFECTIVE CYCLIC ACCELERATION

EERL No.	a/g	T_s (sec) Proposed	T_s (sec) Trifunac-Brady	T_s (sec) McCann-Shah	a_e/a	$a_e t_s/2$ in/sec	SV_{max} in/sec
A001	.348	18.68	24.42	24.84	.33	414.09	95.4
A001	.214	23.86	24.52	25.12	.40	394.25	74.2
A002	.104	13.76	16.74	6.30	.28	77.35	18.0
A002	.112	13.71	16.96	6.90	.30	88.92	21.8
A003	.047	49.37	30.00	32.92	.28	125.41	20.6
A003	.053	45.86	29.62	24.42	.34	159.52	44.4
A004	.156	25.82	30.50	11.02	.34	264.36	44.8
A004	.179	23.20	28.82	12.40	.33	264.53	55.8
A005	.090	32.14	29.60	14.90	.35	195.43	46.7
A005	.131	31.41	33.68	23.32	.27	214.45	70.5
A006	.055	45.28	32.12	29.16	.31	149.02	26.0
A006	.044	50.30	35.82	37.30	.37	158.07	28.8
A007	.059	44.36	30.68	31.36	.30	151.56	29.30
A007	.042	48.25	34.00	35.36	.40	156.47	29.40
A008	.168	13.56	13.48	3.62	.34	149.51	40.70
A008	.258	12.97	9.56	3.22	.33	213.16	62.60
A009	.159	22.07	17.94	10.68	.35	237.08	113.00
A009	.201	21.74	19.52	17.60	.24	202.44	68.20
A010	.102	6.61	12.42	0.62	.38	49.46	17.40
A010	.108	6.48	9.74	0.64	.30	40.53	10.40
A014	.043	12.18	27.92	1.46	.31	31.34	8.90
A014	.046	12.74	28.50	2.92	.25	28.28	7.90
A015	.083	4.78	3.02	0.92	.33	25.27	10.00
A015	.105	5.09	2.86	1.12	.34	35.08	10.40
A016	.085	8.49	27.46	1.50	.34	47.36	18.10
A016	.056	11.74	27.70	1.56	.34	43.15	12.00
A017	.040	6.86	11.58	0.52	.34	18.01	6.30
A017	.024	8.83	14.02	3.30	.38	15.54	6.30
B021	.133	12.47	18.50	11.82	.37	118.45	37.30
B021	.154	14.13	21.10	6.96	.27	113.41	33.20
B023	.033	15.09	19.70	12.30	.28	26.91	8.30
B023	.027	15.19	20.28	6.54	.35	27.71	7.50
B024	.160	20.96	21.06	12.90	.37	239.52	42.80
B024	.183	23.08	19.76	15.36	.33	269.05	49.00
B025	.146	5.56	2.20	2.48	.28	43.87	14.00
B025	.145	5.39	1.98	1.78	.37	55.82	16.50
B026	.144	10.37	10.18	9.82	.24	69.18	17.50
B026	.089	11.46	12.04	4.02	.33	64.97	14.60
B030	.054	18.20	16.80	5.12	.35	66.40	18.10
B030	.075	18.96	18.44	10.80	.25	68.62	24.50
B031	.065	19.17	16.46	6.60	.25	60.13	12.70
B031	.068	19.10	15.92	10.04	.25	62.68	13.30
B034	.355	16.83	7.46	2.66	.20	230.66	48.10
B034	.434	16.23	6.68	3.84	.18	244.74	60.00
B035	.237	13.04	13.70	7.80	.24	143.17	29.30
B035	.275	11.67	10.82	3.84	.24	148.68	31.30
B036	.053	23.90	29.52	29.94	.33	80.69	20.20
B036	.064	25.98	27.94	28.62	.27	36.66	21.70
B037	.269	9.21	5.58	1.64	.24	114.78	27.40
B037	.347	9.02	4.44	1.40	.23	138.96	57.80
B038	.014	12.95	18.96	9.92	.38	13.30	3.30
B038	.012	13.22	20.44	6.88	.38	11.64	2.20
C048	.255	22.02	16.56	16.90	.34	368.52	80.90
C048	.134	24.92	20.72	17.80	.44	283.62	100.00
D056	.315	10.97	15.86	3.22	.28	186.77	62.30
D056	.271	14.50	16.90	13.92	.35	265.48	73.00
D057	.106	16.37	17.48	11.26	.41	137.33	31.20
D057	.151	14.17	14.82	7.48	.40	165.21	52.40
D058	.171	13.45	13.52	5.52	.37	164.27	28.30
D058	.211	12.81	13.14	5.52	.38	198.26	54.10

TABLE 1

EARTHQUAKE STRONG-MOTION RECORDS USED IN THIS STUDY

| Earthquake	Date	EERL No.
El Centro	May 18, 1940	A001
Northwest California	October 7, 1951	A002
Kern County	July 21, 1952	A003, A004, A005, A006, A007
Eureka, CA	December 21, 1954	A008, A009
San Jose, CA	September 4, 1955	A010
San Francisco	March 22, 1957	A014, A015, A016, A017
Long Beach, CA	March 10, 1933	B021
Southern California	October 2, 1933	B023
Lower California	December 30, 1934	B024
Helena, MT	October 31, 1935	B025
Northwest California	September 11, 1938	B026
Northern California	September 22, 1952	B030
Wheeler Ridge, CA	January 12, 1954	B031
Parkfield, CA	June 27, 1966	B034, B035, B036, B037, B038
San Fernando, CA	February 9, 1971	C048, D056, D057, D058
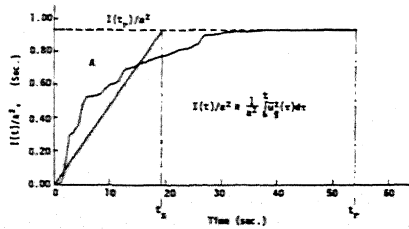
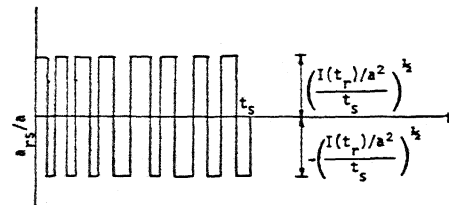Fig. 1. Variations of $I(t)/a^2$ with time for N-S Component of El Centro 1940 Earthquake

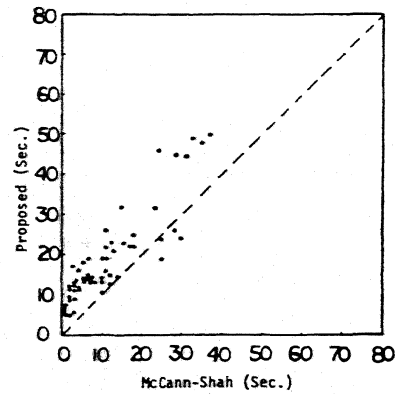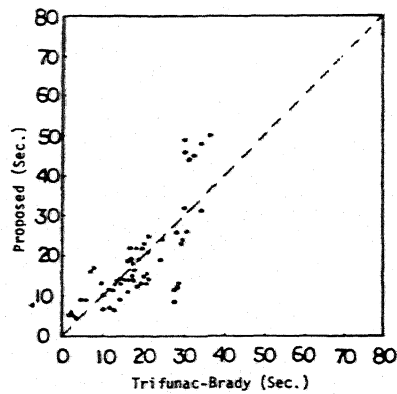Fig. 2. Uniformly Strong Shaking Phase

Fig. 3. Comparisons of durations based on the proposed method and the other methods for records named in Table 2.