THE FUZZY COMPREHENSIVE EVALUATION OF EARTHQUAKE INTENSITY AND ITS APPLICATION TO STRUCTURAL ASEISMIC DESIGN

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SUMMARY

Earthquake intensity to be evaluated, as a global grade classification, has strong fuzziness and should be considered as a fuzzy subset of the intensity reference set. This paper presents methods of single-stage and two-stage multifactorial evaluations for earthquake intensity. The process of human thoughts and experiences to rate earthquake *intensity is scientifically* and *concretely* expressed *in mathematical form on* the basis of these methods. Besides, the methods are very convenient to use. In addition, using the membership levels in the obtained fuzzy vector of intensity, a method to determine the values of parameters used in structural aseismic design is given.

INTRODUCTION

In 1965, American scholar L. A. Zadeh put forward the concept of fuzzy set for the first time (Ref. 1), then a new branch of science—fuzzy mathematics growing rapidly was not only extended into various fields of mathematics but also used immediately in many applied sciences achieving good results.

All objects of the same kind u_i $(i=1, 2, \cdots, m)$ to be considered in any problem being studied constitute a "reference set"

$$U=\{u_1,\ u_2,\ \cdots\cdots,\ u_m\} \tag{1}$$

According to Zadeh's suggestion, fuzzy subset $\underset{\sim}{A}$ of reference set U may be defined as

$$\underset{\sim}{A}=\frac{\mu_A(u_1)}{u_1}+\frac{\mu_A(u_2)}{u_2}+\cdots+\frac{\mu_A(u_m)}{u_m} \tag{2}$$

in which the quotient and sum are not meant by the symbols of division and addition, the equation is used only to show every element u_i and its membership level to fuzzy subset $\underset{\sim}{A}$. The latter is indicated as $\mu_A(u_i)$ satisfying

$$0\leqslant\mu_A(u)\leqslant1,\ u\in U. \tag{3}$$

When $\mu_A(u)=1$, u is entirely a member of $\underset{\sim}{A}$; $\mu_A(u)=0$, μ is *absolutely not*.

Fuzzy subset $\underset{\sim}{A}$ may be defined by its membership function and represented usually by fuzzy vector

$$\underset{\sim}{A}=[\mu_A(u_1),\ \mu_A(u_2),\ \cdots\cdots,\ \mu_A(u_m)] \tag{4}$$

Earthquake intensity is influenced by so many factors that it is very difficult to

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synthesize them into a judgement in a common way, for results contradictory with each other may be obtained according to different factors, by no way to coordinate them. It is the comprehensive judgement in fuzzy mathematics that is the most suitable tool to solve this problem.

Given intensity reference set

$$V=\{v_1,\ v_2,\ \cdots\cdots,\ v_n\} \tag{5}$$

we may consider an earthquake intensity to be evaluated as a fuzzy subset of V;

$$\underset{\sim}{B}=\frac{b_1}{v_1}+\frac{b_2}{v_2}+\cdots\cdots+\frac{b_n}{v_n} \quad (0\leqslant b_j\leqslant 1) \tag{6}$$

where v_j $(j=1,2,\cdots\cdots,n)$ are intensity degrees, and $n=12$ while whole scale is considered. For the purpose of engineering we may consider only

$$V=\{v_1,v_2,v_3,v_4,v_5\}=\{\text{VI},\ \text{VII},\ \text{VIII},\ \text{IX},\ \text{X}\} \tag{7}$$

$$\underset{\sim}{B}=\frac{b_1}{\text{VI}}+\frac{b_2}{\text{VII}}+\frac{b_3}{\text{VIII}}+\frac{b_4}{\text{IX}}+\frac{b_5}{\text{X}} \tag{8}$$

or fuzzy intensity vector

$$\underset{\sim}{B}=[b_1,b_2,b_3,b_4,b_5] \tag{9}$$

where Roman numerals indicate intensities, and b_j is the membership level of v_j to the fuzzy intensity $\underset{\sim}{B}$ to be evaluated.

Let U in Eq. 1 indicate the factor reference set, which consists of the all intensity evaluating factors considered. Then, $\underset{\sim}{A}$ in Eq. 2 will be the fuzzy factor subset

$$\underset{\sim}{A}=\frac{a_1}{u_1}+\frac{a_2}{u_2}+\cdots\cdots+\frac{a_m}{u_m} \quad (0\leqslant a_i\leqslant 1) \tag{10}$$

or fuzzy factor vector

$$\underset{\sim}{A}=[a_1,a_2,\cdots\cdots,a_m] \tag{11}$$

where a_i is the membership level of u_i to $\underset{\sim}{A}$. In fact, the value of a_i indicates the importance of the role played by factor u_i in the intensity evaluation. In other words, it indicates the ability of factor u_i to evaluate the intensity independently.

Our objective is starting from the data of factors u_i $(i=1,2,\cdots\cdots,m)$ to evaluate the fuzzy intensity vector $\underset{\sim}{B}$ through the fuzzy relation between U and V.

SINGLE-FACTOR EVALUATION

Multi-factor evaluation is based on single-factor ones. As examples, we demonstrate the method of single-factor evaluation with the following intensity evaluating factors individually;

$u_1=I$—mean earthquake damage index of common buildings,

$u_2=a_H$—peak value of horizontal ground acceleration (cm/s^2);

$u_3=a_V$—peak value of vertical ground acceleration (cm/s^2);

$u_4=v_H$—peak value of horizontal ground velocity (cm/s).

To differentiate from the result of comprehensive evaluation $\underset{\sim}{B}$, we denote the fuzzy intensity subset obtained from single factor u_i as

$$\underset{\sim}{r_i}=\frac{r_{i1}}{\text{VI}}+\frac{r_{i2}}{\text{VII}}+\frac{r_{i3}}{\text{VIII}}+\frac{r_{i4}}{\text{IX}}+\frac{r_{i5}}{\text{X}} \tag{12}$$

or

$$\underset{\sim}{r}_i = [r_{i1}, r_{i2}, r_{i3}, r_{i4}, r_{i5}] \tag{13}$$

where r_{ij} is the membership level of intensity v_j to the evaluating fuzzy intensity $\underset{\sim}{r}_i$ according to the value of u_i, that is

$$r_{ij}(u_i) = \mu_{r_i}(v_j) \tag{14}$$

Concrete data at present has to be count up to gain the membership function mentioned above. Table 1 shows the specifications of three physical variables in the Seismic Intensity Scale of China(1980)(Ref. 2). They were worked out through analizing and arranging the informations coming from macroscopic investigations and instrumental observations at home and abroad. So they summarized the international practical experiences, and have legal meaning in China.

Idealizing the data in Table 1 in light of their natural meaning and in view of fuzzy mathematics, we obtain following results;

1. Since the data of observed physical variables I, a_H, a_V and v_H are considerably scattered within the interval of one degree of intensity and influenced by a lot of elements, their membership functions belong to normal type, that is

$$\mu(x) = e^{-\left(\frac{x-m}{c}\right)^2} \tag{15}$$

where e is the base of natural logarithm, m and c are constants.

2. According to the normal membership function shown in Eq. 15, when $x=m$, $\mu=1$, membership level reaches maximum, so "m" is the mean of physical variable listed in Table 1.

3. Between two adjacent degrees of intensity, the boundary value of each physical variable has the same membership level 0.5. Then constant "c" in Eq. 15 may be obtained (see Table 2) from the condition

$$e^{-\left(\frac{x_H-x_L}{2c}\right)^2} \approx 0.5 \tag{16}$$

where x_H and x_L are the higher and lower boundary values of the corresponding physical variable (Table 1) for a given intensity degree.

4. Statistical result based on more than two hundred strong earthquake observations in Ref. 3 states

$$(a_V)_{mean} \approx 0.5(a_H)_{mean} \tag{17}$$

So, half of the index for a_H in Table 1 may be used for a_V.

Table 1 Specifications in Seismic Intensity Scale of China(1980)

Intensity	Damage index I		Peak hori. acc. a_H		Peak hori. vel. v_H	
	mean	interval	mean	interval	mean	interval
V			31	22—44	3	2—4
VI	0.05	0—0.1	63	45—89	6	5—9
VII	0.2	0.11—0.3	125	90—177	13	10—18
VIII	0.4	0.31—0.5	250	178—353	25	19—35
IX	0.6	0.51—0.7	500	354—707	50	36—71
X	0.8	0.71—0.9	1000	708—1414	100	72—141
XI	0.95	0.91—1.0				

Table 2 Parameters in the Membership Functions

Intensity	I (u_1)		a_H (u_2)		a_V (u_3)		v_H (u_4)	
	m_{1j}	c_{1j}	m_{2j}	c_{2j}	m_{3j}	c_{3j}	m_{4j}	c_{4j}
Ⅵ (v_1)	0.05	0.07	63	31	31	16	6	3
Ⅶ (v_2)	0.2	0.14	125	63	63	31	13	6
Ⅷ (v_3)	0.4	0.14	250	125	125	63	25	13
Ⅸ (v_4)	0.6	0.14	500	250	250	125	50	25
Ⅹ (v_5)	0.8	0.14	1000	500	500	250	100	50

Thus, according to the value of u_i the membership level of intensity degree v_j to the fuzzy intensity subset to be evaluated is

$$r_{ij}(u_i)=e^{-\left(\frac{u_i-m_{ij}}{c_{ij}}\right)^2} \quad \begin{pmatrix} i=1,\ 2,\ 3,\ 4 \\ j=1,2,3,4,5 \end{pmatrix} \tag{18}$$

The values of m_{ij} and c_{ij} can be taken from Table 2.

It is impossible to form the membership functions for some evaluating factors, such as the human reactions, phenomena of the ground surface, etc. The fuzzy intensity vector according to such factors can be given directly by experience or by some statistical data. It is not difficult for a single factor.

Example 1 Suppose we have $u_1=I=0.45$, $u_2=a_H=160cm/s^2$, $u_3=a_V=90cm/s^2$, and $u_4=v_H=25cm/s$ from a region in certain earthquake, try to find the corresponding fuzzy tensity vectors for the four factors individually.

substituting the given values of u_i and the corresponding parameters m_{ij} and c_{ij} from Table 2 into Eq. 18, we may obtain the elements of following fuzzy intensity vectors:

$$\underset{\sim}{r}_1=[0.00,\ 0.04,\ 0.88,\ 0.32,\ 0.00]$$

$$\underset{\sim}{r}_2=[0.00,\ 0.73,\ 0.60,\ 0.16,\ 0.06]$$

$$\underset{\sim}{r}_3=[0.00,\ 0.47,\ 0.73,\ 0.19,\ 0.07]$$

$$\underset{\sim}{r}_4=[0.00,\ 0.02,\ 1.00,\ 0.37,\ 0.11]$$

These are just the results of the single-factor evaluations of the earthquake intensity.

MULTI-FACTOR EVALUATION

The method of multi-factor evaluation contains modification and synthesis of the results of single-factor ones $\underset{\sim}{r}_i(i=1,2,\cdots,m)$ to obtain the comprehensive fuzzy vector $\underset{\sim}{B}$. We have analysed and compared(Ref. 4)the available mathematical models (Ref. 5) of multifactorial evaluations and concluded that the "weighted mean model" is the most suitable one to assess earthquake intensity.

According to the weighted mean model,

$$\underset{\sim}{B}=\underset{\sim}{A}\underset{\sim}{R} \tag{19}$$

that is

$$[b_1,b_2,\cdots,b_n]=[a_1,a_2,\cdots,a_m]\begin{pmatrix} r_{11} & r_{12}\cdots & \cdots r_{1n} \\ r_{21} & r_{22}\cdots & \cdots r_{2n} \\ \cdots & \cdots & \cdots \\ r_{m1} & r_{m2}\cdots & \cdots r_{mn} \end{pmatrix} \tag{20}$$

In this equation the elements a_i of the weight vector

$$\underset{\sim}{A} = [a_1, a_2, \cdots\cdots a_m]$$

are the weighting coefficients indicating the importance of the role played by factor u_i in the intensity evaluation. They satisfy

$$\sum_{i=1}^{m} a_i = 1 \tag{21}$$

The fuzzy relation matrix

$$\underset{\sim}{R} = \begin{pmatrix} \underset{\sim}{r}_1 \\ \underset{\sim}{r}_2 \\ \vdots \\ \vdots \\ \underset{\sim}{r}_m \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12}\cdots\cdots r_{1n} \\ r_{21} & r_{22}\cdots\cdots r_{2n} \\ \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots \\ r_{m1} & r_{m2}\cdots\cdots r_{mn} \end{pmatrix} \tag{22}$$

is simply the assemblage of the results of single-factor evaluations $\underset{\sim}{r}_i$ $(i = 1, 2, \cdots, m)$.

The right side of Eq.(20) is the common multiplication of matrices, that is

$$b_j = \sum_{i=1}^{m} a_i r_{ij} \qquad (j = 1, 2, \cdots, n) \tag{23}$$

Here we do not discuss other mathematical models of the multifactorial evaluation, for the limited space.

Example 2 Find the fuzzy intensity vector $\underset{\sim}{B}$ by means of the comprehensive evaluation according to the four factors given in Example 1.

Using the results of Example 1, we obtain the fuzzy relation matrix

$$\underset{\sim}{R} = \begin{pmatrix} \underset{\sim}{r}_1 \\ \underset{\sim}{r}_2 \\ \underset{\sim}{r}_3 \\ \underset{\sim}{r}_4 \end{pmatrix} = \begin{pmatrix} 0.00 & 0.04 & 0.88 & 0.32 & 0.00 \\ 0.00 & 0.73 & 0.60 & 0.16 & 0.06 \\ 0.00 & 0.47 & 0.73 & 0.19 & 0.07 \\ 0.00 & 0.02 & 1.00 & 0.37 & 0.11 \end{pmatrix}$$

Taking weight vector $\underset{\sim}{A} = [0.4, 0.3, 0.15, 0.15]$ and substituting $\underset{\sim}{R}$ and $\underset{\sim}{A}$ into Eq.(19), we obtain the fuzzy intensity vector

$$\underset{\sim}{B} = [0.000,\ 0.309,\ 0.792,\ 0.260,\ 0.045]$$

From the distribution of the membership levels in the obtained vector, the earthquake intensity may be judged as degree Ⅷ.

TWO-STAGE MULTI-FACTOR EVALUATION

In fact, we may consider much more factors in the evaluation of earthquake intensity. In this case, we may use the method of multi-stage comprehensive evaluation (Ref. 5). The steps of two-stage evaluation are as follows:

Step 1 In accordance with the properties of the factors divide the set of factors U into s nonintersecting subsets:

$$U = \{U_1, U_2, \cdots, U_s\} \tag{24}$$

Step 2 By means of single-stage evaluation on the basis of each factor subset find individually the corresponding fuzzy intensity vectors $\underset{\sim}{B}_i$ $(i = 1, 2, \cdots, s)$:

$$\underset{\sim}{A}_i \underset{\sim}{R}_i = \underset{\sim}{B}_i = [b_{i1}, b_{i2}, \cdots, b_{in}] \tag{25}$$

The possibility of using other mathematical models of comprehensive evaluation for some factor subsets cannot be ruled out here.

Step 3 Carry out the second stage of evaluation

$$\underset{\sim}{A}\underset{\sim}{R}=\underset{\sim}{B}=[b_1,b_2,\cdots,b_n] \tag{26}$$

where fuzzy relation matrix is

$$\underset{\sim}{R}=\begin{pmatrix}\underset{\sim}{B}_1\\ \underset{\sim}{B}_2\\ \vdots\\ \underset{\sim}{B}_s\end{pmatrix}=\begin{pmatrix}b_{11} & b_{12} & \cdots\cdots & b_{1n}\\ b_{21} & b_{22} & \cdots\cdots & b_{2n}\\ \cdots & \cdots & \cdots & \cdots\\ b_{s1} & b_{s2} & \cdots\cdots & b_{sn}\end{pmatrix} \tag{27}$$

That means the s factor subsets are considered as s single factors in this step.

The fuzzy intensity vector $\underset{\sim}{B}$ obtained by Eq.(26) is just the required final result.

We make a preliminary suggestion that the intensity evaluating factors may be divided into 4 categories,each of which is considered as a factor subset. They are as follows:

Subset U_1 of damage indexes for different kinds of structures. Find the membership functions of the earthquake damage indexes for several typical structures(such as common buildings, brick chimneys, tower type structures, etc.) like the example I given above. Each of these damage indexes is considered as a single factor, with their membership function carry out their single-factor evaluation and find the corresponding fuzzy vector $\underset{\sim}{r}_i$. Then, find the fuzzy intensity vector $\underset{\sim}{B}_1$ for this subset U_1 by Eq. 25.

Subset U_2 of peak values of ground motion. This subset of factors consists of a_H, a_V, v_H and some parameters of the response spectrum to the earthquake. Find the vector $\underset{\sim}{B}_2$ for this subset by Eq. 25 with the method given in the above section.

Subset U_3 Sum up the effects of the factors of the magnitude, depth of origin, epicentral distance and duration of the earthquake, and give the fuzzy intensity vector $\underset{\sim}{B}_3$ of this subset directly by experience.

Subset U_4 Comprehensively consider the factors of the human reaction,ground fissures, faulting process,terrain condition, etc. and give vector $\underset{\sim}{B}_4$ directly by experience.

After that, we may carry out the second stage of evaluation by Eqs 26 and 27 with a given weight vector $\underset{\sim}{A}$ to obtain the final fuzzy intensity vector $\underset{\sim}{B}$.

Example 3 Find the fuzzy intensity $\underset{\sim}{B}$ with the method of two-stage multi-factor evaluation based on the data given in Example 1 and the following supplement:

$$\underset{\sim}{B}_3=[0.00,\ 0.36,\ 0.48,\ 0.32,\ 0.00]$$

$$\underset{\sim}{B}_4=[0.00,\ 0.48,\ 0.50,\ 0.12,\ 0.00]$$

Since only the mean damage index of common buildings is given ($I=0.45$) in this example, here the vector $\underset{\sim}{B}_1$ is just the result of single-factor evaluation $\underset{\sim}{r}_1$ in Example 1, that is

$$\underset{\sim}{B}_1=[0.00,\ 0.04,\ 0.88,\ 0.32,\ 0.00]$$

According to the given subset of peak values of ground motion(a_H, a_V and v_H)and using the weighted mean method of multi-factor evaluation with weight vector $\underset{\sim}{A}=[0.5,\ 0.3,\ 0.2]$, we obtain

$$\underset{\sim}{B}_2=[0.5,\ 0.3,\ 0.2]\begin{pmatrix}0.00 & 0.73 & 0.60 & 0.16 & 0.06\\ 0.00 & 0.47 & 0.73 & 0.19 & 0.07\\ 0.00 & 0.02 & 1.00 & 0.37 & 0.11\end{pmatrix}=[0.00,\ 0.51,\ 0.52,\ 0.21,\ 0.07]$$

in which the elements $\underset{\sim}{r}_i$ of the fuzzy relation matrix are taken directly from Example 1.

Fuzzy vectors $\underset{\sim}{B}_3$ and $\underset{\sim}{B}_4$ are given in this problem directly by experience in accordance with the related phenomena of the earthquake.

Then, the second stage of evaluation (Eqs 26 and 27) with weight vector $\underset{\sim}{A}=(0.4,\ 0.3,\ 0.15,\ 0.15)$ gives

$$\underset{\sim}{B}=[0.4,\ 0.3,\ 0.15,\ 0.15]\begin{pmatrix} 0.00 & 0.04 & 0.88 & 0.32 & 0.00 \\ 0.00 & 0.51 & 0.52 & 0.21 & 0.07 \\ 0.00 & 0.36 & 0.48 & 0.32 & 0.00 \\ 0.00 & 0.48 & 0.50 & 0.12 & 0.00 \end{pmatrix}=[0.00,\ 0.30,\ 0.66,\ 0.26,\ 0.02]$$

This fuzzy intensity vector is the final results of two-stage evaluation, from which we may judge that the earthquake intensity is degree Ⅷ.

DETERMINATION OF THE DESIGN PARAMETERS FOR ASEISMIC STRUCTURES

Above procedure results in a fuzzy subset of the earthquake intensity reference set. From either theoretical or practical point of view such a result is more reasonable than the results rated by existing methods, in which the fuzziness of intensity and the property of gradual transition from one degree to another were not considered. Therefore, for the same earthquake the intensity might be easily rated with difference of one degree by different people or in accordance with different factors. There was similar case in deciding the design intensity for aseismic structures. But as a consequence, the design parameters and earthquake loads would be doubled. This serious shortcoming has been known for a long time, but no reasonable replacement was available.

Fuzzy multi-factor evaluation of earthquake intensity makes it possible to overcome this difficulty. Now that there exist membership levels to several intensity degrees for an earthquake, it will be more reasonable to determine the design parameters for structures by using membership levels comprehensively. To stress the part played by the predominant intensity, we suggest to use following equation to determine design parameter α

$$\alpha=\sum_{k=\mathrm{VI}}^{\mathrm{X}} b_k^2\alpha_k \Big/ \sum_{k=\mathrm{VI}}^{\mathrm{X}} b_k^2 \qquad (28)$$

where b_k is the membership level to intensity degree k, and α_k the value assigned to design parameter α when intensity degree is k. Of course, the exponent 2 of membership levels in the equation may be discussed as well.

Take the maximum value of earthquake influence coefficient α_{max} as example, according to our design Code for aseismic structures (Ref. 6), Table 3 may be obtained through extrapolation.

Table 3 **Values of α_{max}**

Intensity	Ⅵ	Ⅶ	Ⅷ	Ⅸ	Ⅹ
α_{max}	0.11	0.23	0.45	0.90	1.80

Thus for Example 3 in above section

$$\alpha_{max}=\frac{(0.30)^2\times 0.23+(0.66)^2\times 0.45+(0.26)^2\times 0.90+(0.02)^2\times 1.80}{(0.3)^2+(0.66)^2+(0.26)^2+(0.02)^2}=0.47$$

This result means that in view of parameter α_{max}, the intensity is slightly stronger than Ⅷ degree.

CONCLUSION

The advantages of fuzzy multifactorial evaluation of earthquake intensity can be seen through the procedure offered by this paper as follows:

1. To replace the present method so called "accurate" intensity by the fuzzy vector of intensity, this approach conforms the fuzzy property of the intensity borders and resolves some contradictions which can not be done by the existing methods.

2. Almost all evaluating factors can be considered by means of two-stage comprehensive evaluation of intensity presented in this paper. In addition, the method is very convenient to use.

3. Parameters adopted in design of aseismic structures may be determined reasonably on the basis of membership levels of all intensity degrees. This is extremly important for engineering design practice.

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