QUANTITATIVE EVALUATION OF EARTHQUAKE INTENSITY BASED ON THE FUZZY STANDARDS OF SEISMIC SCALE

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SUMMARY

In this paper, based on the concept of the degree of approaching for normal fuzzy sets and by means of fuzzy multifactorial evaluation, the conventional evaluation of earthquake intensity is researched and treated quantitatively. The abundant macroscopic observational data have been collected and analysed. Setting the damage conditions of buildings, the fissure conditions of earth surface and the man-sensibility conditions as three main kinds of macroscopic standards, the corresponding quantitative comparison tables for different degrees in the seismic scale had been compiled respectively. Then, the general method for quantitative evaluating the earthquake intensity by use of the fuzzy standards is described.

INTRODUCTION

In the evaluation of earthquake intensity some qualitative indices, such as the building damage, the human sensibleness, the rupture of earth surface and others are used mainly. These fuzzy indices or standards are full of difficulties for quantitative evaluation, and, therefore, the earthquake intensity is a typical fuzzy scale in the engineering seismology. Fortunately, the concepts and methods of fuzzy mathematics developed rapidly in recent years remain to be tried in suchlike quantitative evaluations (Ref.1.5).

QUANTITATIVE COMPARISON TABLES

We have selected about 700 building damage data, 37 earth fissure data and 80 man-sensibility data corresponded to the different intensities in the different cases. These macroscopic data have been collected from the seismological literatures and reports on intensities of Haiyuan(M=8.5), Jixian(M=8.5), Chayu(M=8.5), Atushi(M=8.25), Huaxian(M=8), Sanhe(M=8), Pinlo (M=8), Sonming(M=8), Gulang(M=8), Huwen(M=8), Luhuo(M=7.9), Tangshan(M=7.8), Longling(M=7.6), Haicheng(M=7.3), Xintai(M=7.2), Sonpan(M=7.2), Sandan(M=7.25), Yangkiang(M=6.4), Liyang(M=6.0), Ushi(M=6.1) and other earthquakes

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occurred in China. According to statistics obtained on the basis of these abundant macroscopic observational data, we can suppose that the macroscopic fuzzy standards corresponded to a determinate degree in earthquake intensity scale are characterized approximately by nomal distribution function as:

$$N_i = e^{-\left(\frac{x_i - a}{b}\right)^2}, \tag{1}$$

where x_i is the value of i-th kind sample, a is the average value of x_i, i=1,2,...n, b is the standard deviation, N_i is the total mumber of i-th kind samples with value x_i. Then, taking the damage conditions of buildings of different types, the fissure conditions of earth and the man-sensibility conditions as three main kinds of macroscopic standards used for determination of earthquake intensity, we have compiled the corresponding quantitative comparison tables for different ranges of degrees in the seismic scale respectively, as shown in Tables 1-4.

Building Damage Standard

Table 1 is the quantitative comparison table for building damage standard in the range of degrees VI--XI, where 0+ is a infinitely small value, and the buildings are classified into I, II, III types according to the Chinese Seismic Scale. This table can be applied to evaluating the high intensities of strong earthquakes, when the building damage condition may be used as a main kind of macroscopic standards. Besides, it must be noted that some data in Table 1 were revised from the Table 1 in Ref.2.

In some special cases, for example, in the case of evaluating the intensity of a historical earthquake by using the historied materials, the building damage conditions can't be classified in detail, and the somewhat rough comparison tables may be applied. One of them is shown in Table 2.

Earth Fissure Standard

As is well-known, for evaluating the high intensities, such as XI and XII degrees, the building damage standard can't be applied as a main standard, because almost all buildings are quite destroyed. Obviously, in this case the disaster, especially the fissure conditions of earth surface must be taken as the main kind of macroscopic standards.

The general characteristics of disaster including the fissure conditions after a series of earthquakes in China had been researched in Ref.4. The result obtained shows that the ground fissure condition is one of sensitive macroscopic indices for very high intensities. Therefore, we can take the earth fissure condition as a main macroscopic standard for evaluating the XI and XII degrees in seismic scale. Based on the analysis of collected data for destructive earthquakes with magnitude larger than 8, we have taken the wide of earth fissure as an important characteristic for its quantitative evaluation, and classified the earth fissures into 2 types: rock fissures and general ground fissures. Then, we have compiled the quantitative comparison table of XI and XII degrees for earth fissure standard, as shown in Table 3. This Table can be used for evaluating the extremely high intensity in the epicentral region of a disastrous earthquake.

Table 1. Quantitative Comparison Table from VI to XI Degrees for Building Damage Standard

Type of buildings	Damage condition	XI		X		IX		VIII		VII		VI	
		a	b	a	b	a	b	a	b	a	b	a	b
I	No damage	0	0	0	0	0	0	0	0	0	0	0.020	0+
	Slight damage	0	0	0.003	0.008	0.014	0.041	0.08	0+	0.240	0+	0.100	0+
	Damage	0	0	0.012	0.028	0.030	0.045	0.197	0.160	0.573	0.109	0.398	0.155
	Destroy	0	0	0.038	0.052	0.130	0.036	0.268	0.274	0.319	0.084	0.135	0.060
	Severe destroy	1	0+	0.959	0.052	0.904	0.094	0.494	0.312	0.097	0.077	0.019	0.019
II	No damage	0	0	0	0	0	0	0	0	0.045	0.141	0.143	0.223
	Slight damage	0	0	0.009	0.042	0.067	0.115	0.218	0.110	0.410	0.066	0.470	0.148
	Damage	0	0	0.038	0.065	0.150	0.076	0.363	0.232	0.524	0.258	0.286	0.189
	Destroy	0	0	0.105	0.114	0.228	0.215	0.525	0.287	0.186	0.166	0.055	0.050
	Severe destroy	1	0+	0.772	0.269	0.510	0.300	0.194	0.159	0.067	0.071	0	0
III	No damage	0.023	0.068	0.046	0.079	0.134	0.150	0.282	0.155	0.367	0.267	0.420	0.310
	Slight damage	0.003	0.011	0.079	0.078	0.245	0.194	0.299	0.171	0.260	0.147	0.177	0.122
	Damage	0.023	0.033	0.211	0.145	0.299	0.156	0.261	0.226	0.188	0.108	0.088	0.085
	Destroy	0.169	0.119	0.362	0.193	0.247	0.243	0.137	0.107	0.041	0.058	0	0
	Severe destroy	0.784	0.124	0.262	0.171	0.040	0.057	0.010	0.059	0.006	0.010	0	0

Table 2. A Rough Comparison Table from VI to XI Degrees for Building Damage Standard

Type of buildings	Damage conditions	XI		X		IX		VIII		VII		VI	
		a	b	a	b	a	b	a	b	a	b	a	b
I	Destroy and	1	0+	0.991	0.052	0.944	0.065	0.762	0.293	0.400	0.081	0.154	0.040
II	Severe destroy	1	0+	0.877	0.192	0.738	0.258	0.560	0.223	0.260	0.119	0.055	0.025
III		0.953	0.122	0.557	0.182	0.296	0.550	0.136	0.083	0.050	0.034	0	0

Table 3. Quantitative Comparison Table for Earth Fissure Standard Represented by Its Wide

Type of fissures	XII		XI	
	a (cm)	b (cm)	a (cm)	b (cm)
Rock fissure	163	100	50	27
Ground fissure	720	260	140	100

Man-sensibility Standard

In the cases of low intensities smaller than VI degree the building damage standard also can't be used as the main standard continuously, because the majority of buildings are not damaged. The man-sensibility can be applied mainly for evaluating the intensities in this range. The corresponding quantitative comparison table,obtained on the basis of collected data, is shown in Table 4, in which the man-sensibility condition is represented by percent of population sensible of the earthquake. This table may be applied especially for evaluating the low intensities in seismic scale.

Table 4. Quantitative Comparison Table from I to VI Degrees for Man-sensibility Standard

VI		V		IV		III		II		I	
a	b	a	b	a	b	a	b	a	b	a	b
100%	0+	85%	23%	50%	23%	15%	5%	5%	0+	0%	0

METHOD

Let us suppose that the macroscopic standards for different degrees in seismic scale form a series of normal fuzzy sets A_j, $j=1,2,\ldots m$, and the evaluated sample set also forms a normal fuzzy set A_0. Then, we can take the corresponding distribution functions (1) as their membership functions, namely:

$$A_j(x) = e^{-\left(\frac{x-a_j}{b_j}\right)^2} \qquad (2)$$

For two normal fuzzy sets A_j and A_0 with parameters (a_j, b_j) and (a_0, b_0) respectively, the degree of approaching can be defined as

$$(A_j, A_0) = \left[e^{-\left(\frac{a_j - a_0}{b_j + b_0}\right)} + 1 \right] /2 \qquad (3)$$

When the parameters a_j, a_0 and b_j, b_0 all are equal to zero, we may take $(A_j, A_0)=0$.

According to the principle of approaching, if we have equation

$$r_i = (A_0, A_i) = \max \ (A_0, A_j), \quad j=1,2,\ldots m, \qquad (4)$$

so it can be conclused that the fuzzy sample set A_0 is most approaching to the fuzzy model set A_i.

Based on the above-mentioned concepts of fuzzy sets, we can apply a simple method for quantitative evaluating the earthquake intensity by use of the fuzzy macroscopic standards. This method consist of the following main steps.

Step I: Collection and treatment of Observational Data

Firstly, we need to collect the enough macroscopic observational data as samples of the given standards after occurrence of an earthquake. Secondly, we must determine the membership functions, i.e. the parameters a_j, b_j in (2) for different cases. Among them the parameters a_0, b_0 for sample fuzzy set can be obtained from the collected macroscopic data, and the parameters a_j, b_j for different model fuzzy sets can be taken directly from the corresponding quantitative comparison tables (see Tables 1-4).

Step II: Operation of Degree Matrix of Approaching

All elements of the degree matrix of approaching are calculated on the basis of formula (3). Then, we can obtain this matrix R. For example, the degree matrix of approaching for building damage standard can be written as:

$$R = \left\| \begin{matrix} r_{0,1,1} & r_{0,1,2} & \cdots\cdots & r_{0,1,6} \\ \cdots & \cdots & \cdots & \cdots \\ r_{0,5,1} & r_{0,5,2} & \cdots\cdots & r_{0,5,6} \end{matrix} \right\| = (r_{0,i,j}), \qquad (5)$$

where the sign i denotes the degree of building damage, i=1,2,...,5 correspond to no damage, slight damage, damage, ...severe destroy respectively; and the sign j denotes the degree in seismic scale, j=1,2,...,6 correspond to XI, X,..., VI degrees respectively.

Moreover, the matrix R may be reformed by means of average and normalization. In the case of using the building damage standard, we may reform the matrix (5) into:

$$Q = \left\| \begin{matrix} q_{1,1} & q_{1,2} & \cdots\cdots & q_{1,6} \\ \cdots & \cdots & \cdots & \cdots \\ q_{3,1} & q_{3,2} & \cdots\cdots & q_{3,6} \end{matrix} \right\| = (q_{k,j}), \qquad (6)$$

where k=1,2,3 denote the type I,II,III of buildings respectively.

Step III: Fuzzy Multifactorial Evaluation

Taking account of certain weight distribution function W of different cases, we can make the fuzzy multifactorial evaluation the basis of matrix Q obtained above. If we use the building damage standard, the corresponding formula for fuzzy multifactorial evaluation is

$$P = W \circ Q = (P_1, P_2, \ldots\ldots, P_m) = P_j \quad , \qquad (7)$$

where $W=(W_1, W_2, W_3)$ is the weight vector for I,II,III types of buildings, and the sign "o" denotes the operation "combination" which defination is following: suppose $R=(r_{ij})$ and $S=(s_{ij})$ are the nxm and mxp fuzzy matrices respectively, so the matrix $T=(t_{ij})$ with elements

$$t_{ik} = \max_j \left(\min \left(r_{ij}, s_{jk} \right) \right) , \quad j=1,2, \ldots, m \qquad (8)$$

$$i = 1,2,\ldots,n; \; k = 1,2,\ldots,p$$

can be called a combinational matrix of R and S and denoted as:

$$T = R \circ S \quad .$$

The vector P may be normalized and reformed into:

$$H = (h_1, h_2, \ldots\ldots, h_m) = h_j \quad , \qquad (10)$$

where
$$h_j = p_j / (\sum_{j=1}^{m} p_j) \quad , \quad j = 1, 2, \ldots\ldots, m. \qquad (10.a)$$

Step IV: Evaluation of Earthquake Intensity

By using the principle of approaching, we can obtain the judged earthquake intensity, i.e. the degree in seismic scale for a given event. Obviously, this intensity must correspond to the maximal component of the vector P in (7) or vector H in (10):

$$P_k = \max_j \; P_j \qquad (11)$$

or

$$h_k = \max_j \; h_j \quad . \qquad (12)$$

EXAMPLES

Intensity of Songpan M=7.2 Earthquake in 1976

For evaluating the intensity of songpan M=7.2 earthquake at Beima and Wanbachu points together, we used the data about building damage conditions at these points. From these data we have obtained the result shown in Table 5. where a is the mean value in percent of all taken indices and b is its mean standard deviation. Then, we can get the degree matrix of approaching as

$$R = \begin{Vmatrix} 0.5 & 0.547 & 0.678 & 0.893 & 0.826 \\ 0.5 & 0.577 & 0.589 & 0.959 & 0.837 \end{Vmatrix}$$

and its normalized form as

$$Q = \begin{Vmatrix} 0.15 & 0.16 & 0.20 & 0.26 & 0.24 \\ 0.15 & 0.17 & 0.17 & 0.28 & 0.24 \end{Vmatrix}$$

Table 5. Parameters of Membership Function Obtained for Songpan Earthquake at Beima and Wanbachu Points

Type of buildings	Slight damage	Damage	Destroy	Severe destroy
II-type buildings	$a = 0.412$ $b = 0.059$	$a = 0.47$ $b = 0.228$	$a = 0.018$ $b = 0.025$	$a = 0$ $b = 0$
III-type buildings	$a = 0.529$ $b = 0.426$	$a = 0.08$ $b = 0.042$	$a = 0.0025$ $b = 0.0035$	$a = 0$ $b = 0$

Supposing that both II-type and III-type buildings have equal weight for evaluating the earthquake intensity, i.e. taking $W=(0.5,\ 0.5)$, we can obtain the normalized vector H for fuzzy multifactorial evaluating the intensity of studied earthquake:

$$H = (\ 0.14,\ 0.16,\ 0.18,\ 0.27,\ 0.24\ )\ .$$

The components of H correspond to X,IX,VIII,VII,VI degrees respectively. From these components we can see that the $H_4=0.27$ corresponding to VII degree is largest. Therefore, the intensity at Baima and Wangbachu Together must be evaluated as VII degree, but it also may be approaching to VI degree. This result agrees well with the result of macroscopic intensity investigation obtained by the Szechuan Seismological Bureau.

Intensity of Hozhei M=7.0 Earthquake in 1937

According to the historied material, after this earthquake about 30% of buildings in the Hozhei region and 20% of buildings in the Donming and Dintau regions had been destroyed. Let us suppose that the destroyed buildings in given regions in 1937 were II type buildings basically, and the conditions of building damage may be researched as destroy and severe destroy together. By using Table 2, we can obtain the degree matrices of approaching as

$$\begin{array}{cccccccc} & & XI & X & IX & VIII & VII & VI \\ R_1 & = (& 0.5 & 0.5 & 0.528 & 0.628 & 0.947 & 0.5\ ) \end{array}$$

for the Hozhei region, and

$$\begin{array}{cccccccc} & & XI & X & IX & VIII & VII & VI \\ R_2 & = (& 0.5 & 0.5 & 0.506 & 0.537 & 0.888 & 0.5\ ) \end{array}$$

for the Donming and Dintau regions together.

Obviously, from the components of matrices-vectors Q_1 and Q_2 we can come to the conclusion that the Hozhei earthquake intensity degrees in seismic scale are equal to VII+ in the Hozhei region and VII in the Donming and Dintau regions.

Intensity of Haiyuan M=8.5 Earthquake in 1920

According to the historied data about disaster of Haiyuan earthquake we may take approximately the average wide of earth fissure in the Haiyuan region as: a=250cm, b=0 for rock fissure and a=550cm, b=0 for ground fissue. By using Table 3, we can obtain the degree matrix of approaching

$$R = \begin{array}{cl} \left\| \begin{array}{cc} 0.735 & 0.5 \\ 0.826 & 0.5 \end{array} \right\| & \begin{array}{l} \text{Rock fissure} \\ \text{Ground fissure} \end{array} \\ \begin{array}{cc} \text{XII} & \text{XI} \end{array} & \end{array}$$

Supposing that the rock fissure and ground fissure have equal weight for evaluating the earthquake intensity, i.e. taking $W=(0.5, 0.5)$, we can obtain the vector for fuzzy multifactorial evaluation

$$P = W \circ Q = (0.5, 0.4) \quad .$$

According to the principle of approaching, the intensity in the Haiyuan region can be evaluated as XII degree.

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