ANALYSIS OF TRANSIENT WAVES IN LAYERED MEDIA WITH DIPPING STRUCTURE

Y.H. Pao (I)
F. Ziegler (II)
P.L. Chen (III)
Presenting Author: F. Ziegler

SUMMARY

The ground motion during the passage of seismic waves is significantly affected by the surface layers. Modified generalized ray theory is applied to analyze the transient SH-waves in layers with sloped surfaces. Incident waves can be generated by a line-source within the layers or in the half-space (bedrock). Numerical results are shown for the oblique incidence of a single layer over-laying a half-space and compared with those for parallel layers.

INTRODUCTION

For an elastic medium with parallel layers, the transient motion due to normally incident or obliquely incident waves from bedrock can be analyzed by applying the theory of generalized ray (TGR). In the theory the propagation of transient waves is sorted into a series of generalized ray integrals, each of which is the Fourier transformed or Laplace transformed wave function, representing the wave motion along a specific ray path between a line source and a point receiver. The arrival times of each ray undergoing multiple reflections and transmissions are then determined from the stationary value of the phase function with common slowness parallel to the layering. Inverse transform is completed by applying the Cagniard method (Ref. 1). See (Refs. 2, 3, 4, 5) for recent applications to multi-layered solids and (Ref. 6) for a comparison with other theories.

Recently the authors have modified the TGR to analyse the transient SH-waves in a wedge-shaped layer over-laying a half-space bedrock, (Ref. 7). By expressing the SH-source ray integral in two systems of cartesian coordinates, one for each of the two nonparallel surfaces, one can construct successively the ray integrals for waves multiply reflected within the layer. Phase functions and reflection coefficients are expressed in terms of the local wave slownesses, one group of them are slownesses along the horizontal surface, the other are along the sloped interface, each being dependent on the location of reflection. Through an invariance study of the ray-phases, (Ref. 8), all local wave slownesses are transformed to a common wave slowness, e.g. of source ray along either surface which then represents the integration variable. Likewise to the classical case inverse Laplace transforms of these ray integrals are then obtained by the Cagniard method.

(I) Professor of Mechanics, Cornell University, Ithaca, New York, USA.

(II) Professor of Mechanics, Technical University of Vienna, Austria.

(III) Visiting Scientist, National Taiwan University, Taiwan, China.

Extension of this newly developed method of modified TGR to a three layered medium, each of the two overlaying layers having a different dipping angle was recently performed (Ref. 9). Because of multiple refractions from layer to layer and reverberation within each layer, the number of generalized ray-integrals, however, increases sharply as the number of layers increases. If the diffractions from the apexes of the wedges are neglected, the solutions so obtained by superposition are exact within a finite observation time at a fixed receiver point.

After a short outline of the generalized modified TGR, numerical results are shown for rays of oblique incidence within a single layer over-laying a half-space. Results are compared to those for parallel layers. Special attention deserves the analysis of the effect on the obliquely incident transient waves due to a converging slope (waves toward the apex of the wedge), and a diverging slope. In the former case signals are detected from back-scattered rays, where the ray first goes up-dip, then returns to form a wave received at a point below the source. In the opposite a ray going down-dip after a few reflections may become critical thus forming head waves or becoming parallel to the interface opposite to last reflection. This type of phenomenon observed in local earthquake records thus becomes a natural explanation of site effects of sloping layers.

MODIFIED THEORY OF GENERALIZED RAY

We assume a three-layered elastic half-space with three cartesian coordinate systems of common origin at the free surface (1) and x-axis within free surface, x'axis parallel to interface (2) which is inclined against the free surface by angle α , x"-axis parallel to interface (3) which is further inclined by angle β , and therefore by $(\alpha + \beta)$ against the free surface. Material of the surface-layer has shearing modulus μ and the reciprocal speed of S-waves is $a = 1/c$. Material of layer-2 and bedrock-3 has shearing modulus μ_j and reciprocal of wave speed $a_j = 1/c_j$, $(j = 2,3)$. A line source S is located in the surface layer at $(x = 0,\ z = z_o)$, depth of layers at source location are denoted h , h_2 respectively. Hence, each plane surface has the equation: $z = z_1 = 0$ (free surface), $z' = z'_2 = h\cos\alpha$ (interface (2)), $z'' = z''_3 = (h + h_2)\cos(\alpha + \beta)$ (interface (3)). At interfaces stress and displacement are continuous, at free surface the shearing stress vanishes.

The Source Ray

The Laplace transformed displacement $\bar{v}(\underset{\sim}{x}, s)$ of the SH-wave radiated by the line source at $(0, z_o)$ with a time function $f(t)$ may be represented in terms of $x - z$ or $x' - z'$ coordinates, cf. (Ref. 6, Ch. 6)

$$\bar{v}_o(s) = \bar{F}(s) \int_{-\infty}^{\infty} S(\xi)\exp\{ sg_o(\xi)\} d\xi = \bar{F}(s) \int_{-\infty}^{\infty} S'(\xi')\exp\{ sg'_o(\xi')\} d\xi' \quad (1)$$

where $\bar{F}(s) = a^2\bar{f}(s)/4\pi$, and $S(\xi) = 1/\eta$, $S'(\xi') = 1/\eta'$ denote source function, $g_o(\xi) = i\xi x - \eta|z - z_o|$, $g'_o(\xi') = i\xi'(x' - z_o\sin\alpha) - \eta'|z - z_o\cos\alpha|$ denote the ray-phase function where $g_o = g'_o$ through the condition of invariance of the phase against rotation of coordinates. Employing the geometrical relations $x = x'\cos\alpha - z'\sin\alpha$, $z = x'\sin\alpha + z'\cos\alpha$ to the invariant phase the relations

$$\xi = \xi'\cos\alpha - i\eta'\sin\alpha\ , \quad \eta = -i\xi'\sin\alpha + \eta'\cos\alpha \tag{2}$$

between the pair of slowness $\xi, \eta = (a^2 + \xi^2)^{1/2}$ in x,z-directions and $\xi', \eta' = (a^2 + \xi'^2)^{1/2}$ in x',z'-directions are established, allowing for the transformation between the two representations in Eq. (1).

Train of Waves Multiply Reflected Within the Surface Layer

For a buried source in the upper layer, the upward source ray is reflected at the free surface and the displacement of the reflected ray, $\bar{v}_{-o}$, can be derived by inserting a reflection coefficient $R^{(1)}$ in the integrand of Eq. (1), and converting the phase function to $g_{-o}(\xi) = -\eta z_o + i\xi x - \eta z$, slowness ξ is common during reflection. Similary, the downward source ray is partially reflected at the interface (2), and partially transmitted to layer-2. The reflected ray, $\bar{v}_1$, is obtainable by inserting a reflection coefficient of plane wave $R_{(2)}(\xi_1')$ at the interface (2) in the primed integrand of Eq. (1) and a changing of the phase function to $g_1'(\xi') = -\eta z_o - (\eta' + \eta_1')z_2' + i\xi_1'x' + \eta_1'z'$, where $\xi_1' = \xi'$ is common slowness in x'-direction during reflection. Each ray is further reflected and the kth ray can be represented in unprimed receiver coordinates by, (Ref. 7),

$$\bar{v}_{\mp k}(s) = \bar{F}(s)\int_{-\infty}^{\infty} S'(\xi')\,\Pi_{\mp k}(\xi')\exp\left[sg_{\mp k}(\xi')\right]d\xi' \tag{3}$$

$$g_{\mp k}(\xi') = \mp\eta z_o - z_2'\sum_{\ell=0}^{k'}\eta_\ell + i\xi_k x + (-1)^{k+1}\eta_k z\ , \quad k' = \begin{cases} k\ (\text{odd}) \\ k-1,\ k\ (\text{even}) \end{cases} \tag{4}$$

$$\xi_{k+1}' = \xi_k' = \xi'\cos k\alpha + i\eta'\sin k\alpha,\ \eta_{k+1}' = \eta_k' = (a^2 + \xi_k'^2)^{1/2} = i\xi'\sin k\alpha + \eta'\cos k\alpha,$$

$$\xi_k = \xi_{k-1} = \xi'\cos(k-1)\alpha + i\eta'\sin(k-1)\alpha,\ \eta_k = \eta_{k-1} = i\xi'\sin(k-1)\alpha + \eta'\cos(k-1)\alpha,$$

$$k = 0,\ 2,\ 4\ \ldots\ (\text{even}) \tag{5}$$

A continuous product of reflection coefficients is formed during k reflections

$$\Pi_{\mp k} = \frac{R^{(1)}}{1}\ R_{(2)}(\xi')\ R^{(1)}R_{(2)}(\xi_2')\ \ldots \begin{cases} R_{(2)}(\xi_{k-2}')R^{(1)}\ k\ (\text{even}) \\ R^{(1)}R_{(2)}(\xi_{k-1}')\ k\ (\text{odd}) \end{cases} \tag{6}$$

where for plane SH-waves $R^{(1)} = +1$ and in terms of local wave slowness

$$R_{(2)}(\xi_k') = (\mu\eta_k' - \mu_2\zeta_k')/(\mu\eta_k' + \mu_2\zeta_k')\ , \tag{7}$$

the local slowness $\zeta_k' = (a_2^2 + \xi_k'^2)^{1/2}$ is an irreducible radical.

Secondary Train of Waves in the Surface Layer

Apparent source rays in layer-2, displacement denoted by $\bar{w}_{\mp ko}$, are the results of the ray $\bar{v}_{\mp k}$ of Eq. (3), k(zero,even), when partially transmitted through interface (2). Hence, we change Eq. (3) by inserting the transmission coefficient of plane SH-waves in terms of local slowness,

$$T_{(2)}(\xi_k') = 2\mu\eta_k'/(\mu\eta_k' + \mu_2\zeta_k'), \quad k = 0, 2, 4 \ldots \text{(even)} \tag{8}$$

and by changing the phase function to

$$h'_{\mp ko}(\xi') = \mp\eta z_o - z_2'\Big(\sum_{\ell=0}^{k}\eta_\ell' - \zeta_{ko}'\Big) + i\xi_k'x' - \zeta_k'z' . \tag{9}$$

which then contains the two irreducible radicals η' and $\zeta_{ko}' = \zeta_k'$.

Analogous to Eq. (3) we find the ray after a number j of reverberations within layer-2 of a multi-layered medium, $w_{\mp kj}$. Those odd numbered rays, $w_{\mp kj}$, are subject to transmission through interface (2) back into the surface layer. Therefore, we may call the displacement $\bar{v}_{\mp kjo}$ of a transmitted ray an apparent source ray in layer-1, number k, j fixed, and, considering q reflections within the surface layer, a secondary train of waves becomes likewise to $\bar{v}_{-k}$ of Eq. (3),

$$\bar{v}_{\mp kjq}(s) = \bar{F}(s)\int_{-\infty}^{\infty} S'(\xi')\Pi_{\mp k}T_{(2)}(\xi_k')\Pi_{kj}T^{(2)}(\xi_{kj}')\Pi_{kjq}\exp\{sg_{\mp kjq}(\xi')\}d\xi' , \tag{10}$$

k = 0, 2, 4 ... (even), j (odd). Last ray segment is downward when q is odd, q (even) renders last ray segment pointing upward. The transmission coefficient in terms of local slowness becomes, cf. Eq. (8),

$$T^{(2)}(\xi_{kj}') = 2\mu_2\zeta_{kj}'/(\mu_2\zeta_{kj}' + \mu\eta_{kjo}') , \tag{11}$$

$\eta_{kjo}' = (a^2 + \xi_{kj}'^2)^{1/2}$ is a new irreducible radical. The reflection coefficients in terms of local slowness in the product

$$\Pi_{kj} = R_{(3)}(\xi_{k1}'')R^{(2)}(\xi_{k2}') \ldots R_{(3)}(\xi_{kj}'') , \quad k\text{(even)} , \quad j\text{(odd)} \tag{12}$$

are given by $R^{(2)} = -R_{(2)}$ and at the interface (3) by

$$R_{(3)}(\xi_{kj}'') = (\mu_2\zeta_{kj}'' - \mu_3\chi_{kj}'')/(\mu_2\zeta_{kj}'' + \mu_3\chi_{kj}'') \tag{13}$$

where $\chi_{kj}'' = (a_3^2 + \xi_{kj}''^2)^{1/2}$ is an irreducible radical and

$$\xi_{kj}'' = \xi_{k(j-1)}'' = \xi_k'\cos j\beta + i\zeta_{ko}'\sin j\beta , \quad \zeta_{kj}'' = \zeta_{k(j-1)}'' = i\xi_k'\sin j\beta + \zeta_{ko}'\cos j\beta \tag{14}$$

The product of q reflection coefficients Π_{kjq} expressed in local slownesses ξ_{kjq}' starts with $R^{(1)}$ analogously to Eq. (6) and $R_{(2)}(\xi_{kjq}')$ is given by Eq. (7) when the local slownesses η_{kjq}' and $\zeta_{kjq}' = (a_2^2 + \xi_{kjq}'^2)^{1/2}$, which is another irreducible radical, are substituted. Similarly, the phase function is constructed likewise to Eq. (4) starting with Eq. (9) and adding the phase contributions of the ray reverberated in layer-2 and that of the transmitted ray:

$$g_{\mp kjq}(\xi') = \mp\eta z_o - z_2'\left(\sum_{\ell=0}^{k}\eta_\ell' - \sum_{\ell=0}^{j}\zeta_{k\ell}' + \sum_{\ell=0}^{q^*}\eta_{kj\ell}'\right) - z_3''\sum_{\ell=0}^{j}\zeta_{k\ell}''$$

$$+ i\xi_{kjq}x + (-1)^q z\eta_{kjq} \tag{15}$$

During the q reverberations in the surface layer the local pairs of wave slowness ξ_{kjq}, η_{kjq}; ξ_{kjq}', η_{kjq}' are subject to the transformations (5) when substituting the apparent source ray slowness ξ_{kj}', η_{kjo}' for ξ', η'.

Inverse Laplace Transform and Arrival Times

The infinite integrals (1), (3), (10) are given by the real part of a one-sided integration

$$\bar{v}_{\mp k\ldots}(s) = 2\bar{F}(s)\,\mathrm{Re}\int_0^\infty S'(\xi')\pi_{\mp k}\ldots\exp\left[sq_{\mp k\ldots}(\xi')\right]d\xi' \tag{16}$$

In the method of Cagniard (Ref. 1), the phase function is changed to t by the mapping

$$t = -q_{\mp k\ldots}(\xi') . \tag{17}$$

Assume that t is real and the inverse mapping $\xi' = \hat{\xi}'(t) = g^{-1}_{\mp k\ldots}(t)$ to complex ξ'-plane exists, the inverse Laplace transform can be put in the form of a convolution, $H(t)$ denotes Heaviside step function

$$v_{\mp k\ldots}(t) = (a^2/2\pi)H(t - t_A)\int_0^t \dot{f}(t-\tau)\left[\mathrm{Re}\int_\Gamma S'(\xi')\Pi_{\mp k}\ldots d\xi'\right]d\tau . \tag{18}$$

The integration on primed source ray slowness ξ' is over a contour Γ on the complex ξ'-plane, a portion of Γ, AM is along the imaginary axis and M denotes the saddle point, located at $\xi' = \xi_M' = \pm ib_M$, where b_M is the real (numerical) solution of $dt/d\xi' = 0$. The arrival time of a direct ray, when no branch point falls below ξ_M', is then given by

$$t_M = -g_{\mp k\ldots}(\xi_M') . \tag{19}$$

From the point M, the contour Γ extends in the complex plane to $\xi'(t)$.

Branch points of the integral, Eq. (16) are met, whenever local slownesses in z, z' or z'' direction vanish. Their locations on imaginary ξ'-axis are found by successive application of proper slowness tranformations. In case such $\xi' = \xi_E' = ib_E$ falls below ξ_M' a refracted ray occurs and the arrival time of the associated head wave is given from Eq. (17)

$$t_E = -g_{\bar{+}k\ldots}(\xi_E') \; . \tag{20}$$

Integration in ξ'-plane then starts at the lowest branch point which renders the last segment of the refracted ray under critical angle condition at the interface to a fast bottom.

NUMERICAL RESULTS

We evaluate the ray phases of Eq. (4) in terms of the source-ray slowness ξ' and $\eta' = (a^2 + \xi'^2)^{1/2}$ by estimating the local slownesses and find

$$g_{\bar{+}k}(\xi') = i\xi' X'_{\bar{+}k} - \eta' Z'_{\bar{+}k} \; , \tag{21}$$

showing the ray projections in imaging space onto the primed coordinates of Fig. 1,

$$X'_{\bar{+}k} = \pm z_o \sin\alpha - x_o \cos\alpha + (x + x_o)\cos(k^* - 1)\alpha - (-1)^k z \sin(k^* - 1)\alpha , \tag{22}$$

$$Z'_{\bar{+}k} = \pm z_o \cos\alpha + x_o \sin\alpha + (x + x_o)\sin(k^* - 1)\alpha + (-1)^k z \cos(k^* - 1)\alpha \; , \tag{23}$$

when $k^* = k$ (even), and $k^* = k+1$ if ray number k is odd, $x_o = h\cos\alpha/\sin\alpha$. The arrivals of direct rays of this first train of waves are calculated in analytical form, Eq. (19) becomes

$$t_M = aR_k \; , \quad R_k = (X_k'^2 + Z_k'^2)^{1/2} \; . \tag{24}$$

They are marked in Fig. 2 by numbers on the time axis, carrying the ray number as subscripts.

Depending on receiver distance to the source, head waves of various order are received. Arrivals of rays of number k refracted at the fast bottom are calculated from Eq. (20) whenever the branch point corresponding to $\xi_k' = ia_2$ falls below the stationary point. Early arrivals are indicated in Fig. 2 by time instants marked with asterisks and the subscript gives the ray number with last segment under critical angle condition at interface (2). Refracted rays with noncritical last segment are noted by two asterisks.

The total number n of rays of first wave train contributing to the summed signals shown in Fig. 2 depends on source-receiver distance, the geometry is kept constant (Fig. 1), and is given in the figure, the captions include information on geometrical- and material-properties of the layers. The time signature of an impulse source is chosen as the Dirac-δ-function.

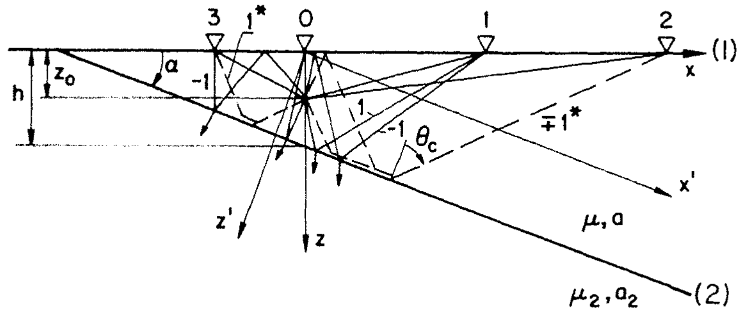

Fig. 1 Geometry of dipping surface layer, $\alpha = 20^{\circ}$, $h = 1$. Source location: $x=0$, $z_o=h/2$. Receiver locations: Epicenter 0, $x_1=2h$, $x_2=4h$, $x_3=-h$. Material parameters: $a=c^{-1}=1$, $a_2=c_2^{-1}=1/\sqrt{2}$, $\rho_2/\rho=1$. Direct rays ____. Refracted rays ----.

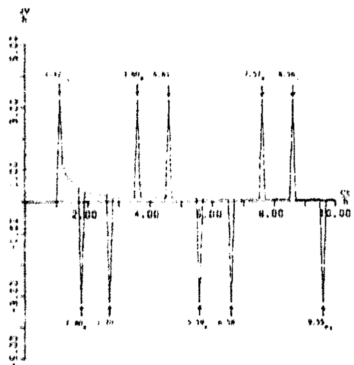

Fig. 2a Receiver location $x_3=-h$, n=..
Parallel layer: $\alpha = 0^{\circ}$, h=1

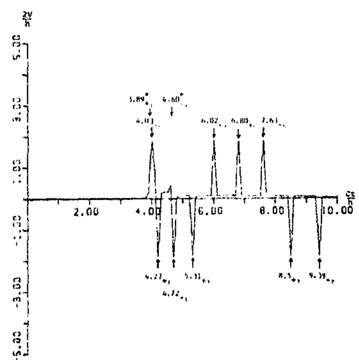

Fig. 2b Receiver location $x_2=4h$, n=..
Parallel layer: $\alpha = 0^{\circ}$, h=1

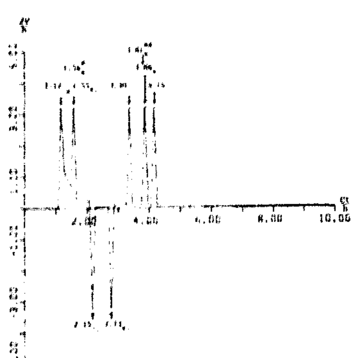

Fig. 2c Receiver location $x_3=-h$, n=..
Dipping layer: $\alpha = 20^{\circ}$, h=1

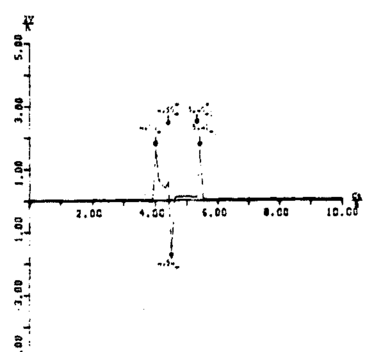

Fig. 2d Receiver location $x_2=4h$, n=..
Dipping layer: $\alpha = 20^{\circ}$, h=1

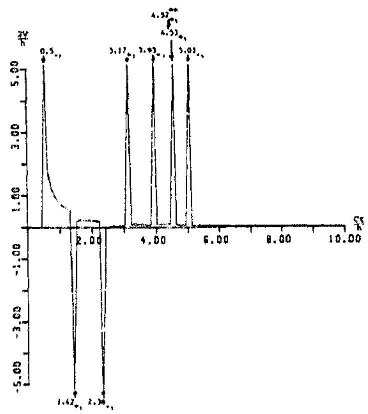

Fig. 2e Receiver location O, n=..
Dipping layer: $\alpha=20^{\circ}$, h=1

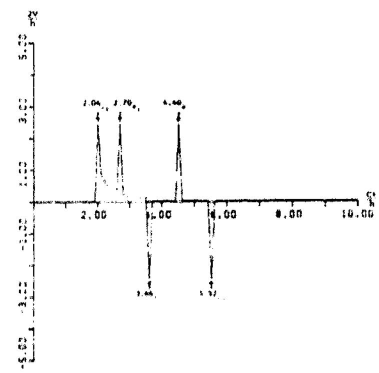

Fig. 2f Receiver location x_1=2h, n=..
Dipping layer: $\alpha=20^{\circ}$, h=1

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