BASE MOTION EXCITATION IN ONE-DIMENSIONAL SOIL DYNAMICS

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SUMMARY

Uncertainties exist in the use of recorded surface accelerograms as the forcing function at sub-surface locations for soil response studies. These are addressed in connection with one-dimensional shear wave propagation in this paper. The merits of the transmitting boundary vs the nontransmitting boundary are discussed. Inherent in the use of the transmitting boundary is the decision as to the magnitude of the incident wave and its relationship to the surface accelerogram. The limitations associated with the use of fifty per cent of the recorded motion as the incident wave are presented.

INTRODUCTION

One set of traditional boundary conditions used in the computation of one-dimensional dynamic soil behavior during seismic events is a prescribed total velocity at the soil base and a prescribed shear stress at the soil surface; then computations produce the response in the intermediate soil layers. If the total velocity prescribed at the soil base was obtained from a measured seismograph at that level in the substrata, there would be no uncertainty about the correctness of the imposed boundary conditions. However, measurements are seldom made at the desired soil base level; therefore rock surface seismographs, either recorded in the vicinity of the soil mass or transposed from other locations, are often imposed as the actual total velocity at the soil base. The name of nontransmitting boundary (NTR) is given to this type of boundary, i.e., the case in which a prescribed total velocity is applied at the soil base. Another set of boundary conditions for one-dimensional studies involves the use of a prescribed incident velocity at the soil base and a prescribed shear stress at the soil surface, Joyner and Chen (Ref. 1). The incident velocity is either assumed to be one-half of the recorded surface velocity, or it is obtained by inverting the surface velocity and then decomposing the result. The name of transmitting boundary (TR) is given to this type of boundary, i.e., the situation in which a prescribed incident velocity is applied.

Although the topic of nontransmitting, partial transmitting, and perfect transmitting boundaries has received some attention in the literature, it relates mostly to the representation of a semi-infinite space in the study of surface foundations. The works of Kuhlemeyer (Ref. 2), Lysmer and Wass (Ref. 3), Roesset and Whitman (Ref. 4), and Kausel and Tassoulas (Ref. 5) are of particular note. Joyner and Chen (Ref. 1) present the TR boundary in the context of seismic shear wave excitation in one-dimensional finite difference analysis. Tsai, Lam, and Martin (Ref. 6), embrace the superior qualities of the TR boundary without expanding on implementation details.

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This presentation begins with a discussion of total, incident, and reflected waves in layered media, followed by a demonstration of the differences generated by the two types of boundaries (NTR and TR). It is demonstrated that the transmitting boundary is generally more applicable than the nontransmitting boundary if the total velocity at soil base is not available. Then a method is discussed to invert the rock surface velocity to get the base incident velocity. Finally, the usage of one-half of the rock total surface velocity as the base incident velocity is evaluated.

TOTAL, INCIDENT AND REFLECTED WAVES

The fundamental condition relating the component waves to the total wave is:

$$V_T = V_R + V_I \tag{1}$$

in which, V_T = total velocity, V_R = reflected velocity and V_I = incident velocity. Applying the method of characteristics, as described in Refs. 7 and 8, for waves transmitting across two distinct media both in a semi-infinite domain, the relations amongst total, incident and reflected velocities, if energy dissipation is neglected, are:

$$V_T = \frac{2}{1 + \frac{\rho_{i-1} \, v_{s_{i-1}}}{\rho_i \, v_{s_i}}} V_I \tag{2}$$

$$V_T = \frac{2}{1 - \frac{\rho_{i-1} \, v_{s_{i-1}}}{\rho_i \, v_{s_i}}} V_R \tag{3}$$

in which, ρ = mass density, v_s = shear wave velocity, and the indices i and i-1 refer to the bottom and top layer numbers, respectively.

Figure 1 is a sketch illustrating the one-dimensional shearing waves propagating in the different media in the physical z -x plane. Two parallel conditions are depicted: a rock-rock interface and a rock-soil interface. Since the lower layer is homogeneous and subject to the same assumed seismic source, points a, b, c and d receive the same incident velocity; thus,

$$V_{I_a} = V_{I_b} = V_{I_c} = V_{I_d} \tag{4}$$

Use of Eqs.(2), (3), and (4) shows that the total velocity, V_{T_b}, at the interface of rock and rock is different from the total velocity, V_{T_d}, at the interface of rock and soil. A logical and important conclusion from this observation is that it is more reasonable, in general, to assume a common incident velocity at level b-d in the substrata, Fig. 1, than to assume a common total velocity.

DIFFERENCES GENERATED BY THE TWO BOUNDARY CONDITIONS

It is desired to illustrate the differences created by the two base boundaries (NTR and TR) as precisely as possible; therefore, it is assumed that the soil and rock are linear and elastic. This is reasonable for rock behavior during siesmic events, but it is quite unrealistic for the soil whose response is of primary concern. The assumption of linear, elastic, and homogeneous conditions in the soil is not necessary; it is done only as a convenience so that the differences in results can be properly attributed to the differences in boundary conditions.

A subsurface condition is illustrated in Fig. 2 in which the soil response is desired. Material properties and strata thicknesses are shown and the rock surface velocity is assumed known. Separate studies utilizing NTR and TR boundaries are performed. Assumed rock surface velocities in the study are either a sine wave motion with unit amplitude, or the 1940 N-S El Centro earthquake.

Nontransmitting Boundary. By use of the shear wave compatibility equations along the characteristic lines shown in Fig. 3, one can obtain (Ref. 8):

$$V_p = 2\ V_q - V_u \qquad (5)$$

When the entire domain between the top free surface and the base nontransmitting boundary, Fig. 3a, is visualized as a one-dimensional sub-strata system it can be seen that this boundary entirely isolates the system from the sub-base conditions. Energy introduced at the base is trapped in the system. In a non-dissipative system, if the excitation continues at the base, the amplitude of the surface motion may be expected to reach a high magnitude. Figure 4 illustrates such a case in which a sine wave is imposed. The response shows a beat representing the combination of the forcing frequency and the soil natural frequency. In the special case of the natural frequency of the system matching the frequency of the forcing motion at the base, an unbounded resonance occurs in this nondissipative system.

Transmitting Boundary. Application of the method of characteristics with the TR boundary, Fig. 3(b), leads to

$$V_q = \frac{2}{1 + \dfrac{\rho_1\, v_{s_1}}{\rho_2\, v_{s_2}}}\, V_I + \frac{1}{1 + \dfrac{\rho_2\, v_{s_2}}{\rho_1\, v_{s_1}}}\, V_u \qquad (6)$$

The velocity V_p at the surface may be obtained by use of Eq. (5). The transmitting boundary at the base permits an interchange of energy between the substrata system of interest and the underlying medium. Only a portion of the energy input at the base contributes to the ultimate surface motion as some of the input that is reflected back to the base is transmitted out of the system at that level. Even with the nondissipative medium the amplitude of the free surface motion can be expected to remain small compared with the motion calculated with the NTR boundary, Fig. 5. In the special case of matched frequencies, the amplitude of the free surface motion remains bounded.

Another feature that emphasizes the difference between two boundaries may be noted when the forced motion at the base stops. With the TR boundary the soil motion ultimately returns to a static condition, whereas the NTR boundary does not allow energy to escape and therefore the computed motion continues indefinitely. The former effect is illustrated in Fig. 6. With the NTR boundary, the forcing motion, V_q, is zero when the excitation ceases. In Eq. (5), at any j th step:

$$V_{p,j} = - V_{u,j} \tag{7}$$

This shows that $V_{p,j}$ will never change magnitude, but will change sign at each step.

These examples demonstrate differences generated by the two boundaries, and generally confirm the rationality of the transmitting boundary. However, a requirement for use of the TR model is knowledge of the actual incident wave at the base of the soil to be studied. The next section presents equations to compute the incident velocity at level B or C, Fig.2.

DETERMINATION OR INCIDENT VELOCITY

In this section it is assumed that the surface velocity is known at a seismological station at a rock outcry. The total velocity and shear stress at the bottom of a layer can be related to that at the top of a layer by use of the method of characteristics:

$$V_{i+1,j} = \frac{1}{2} (V_{i,j+1} + V_{i,j}) + \frac{1}{2\rho_i v_{s_i}} (\tau_{i,j+1} - \tau_{i,j}) \tag{8}$$

$$\tau_{i+1,j} = \frac{1}{2} (\tau_{i,j+1} + \tau_{i,j}) + \frac{\rho_i v_{s_i}}{2} (V_{i,j+1} - V_{i,j}) \tag{9}$$

in which, index i represents the layer number measured from top to bottom, and index j represents increments in the time direction. Computation begins at the rock free surface where V is given and $\tau = 0$. At the first step V and τ are computed at the interface between layer 1 and 2, with $i = 1$ and $j=1,2,..,2^{n-1}$. By repeating the same procedure, V and τ may be determined at the interface of all layers in the rock. Once the total velocity and shear stress at an interface are computed, the method of characteristics may be applied to find the incident velocity at the interface:

$$V_{I_{i+1,j}} = \frac{1}{2} V_{i+1,j} + \frac{1}{2\rho_i v_{s_i}} \tau_{i+1,j} \tag{10}$$

It may be noted in Eq. (10) that if the rock consists of one homogeneous material the incident velocity of one-half the surface velocity may be obtained by inserting $\tau = 0$. Figure 7 does not invert the total rock surface velocity to get the incident velocity at the level C, Fig. 2, but simply use one-half the total rock surface velocity as the incident velocity at level B, Fig. 2. The soil surface response in Fig. 7 is similar to that in Fig. 5. Due to the simplification in computation and the similarity in results, recommendations may be possible to favor this model, rather than use of the more precise incident wave model. It is thus the final objective of this study to find the ratio of rock surface velocity that should be used to represent an actual base incident velocity.

FREQUENCY DOMAIN METHOD

The equation that describes the vertical propagation of shearing waves without dissipation is:

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} \tag{11}$$

The solution to Eq. (11) for a sinusoidal motion of frequency ω is available in a number of the standard mathematics texts (also Ref. 4) and is:

$$u\,(z,t) = (Ee^{ikz} + Fe^{-ikz})\; e^{i\omega t} \tag{12}$$

in which $k = (\rho\omega^2/G)^{1/2}$. The time derivative of Eq. (12) gives the velocity, $V(z,t)$, and the shear stress is given by, $\tau(z,t) = G\ \partial u/\partial z$. The incident and reflected waves traveling in the negative (upwards) and positive (downwards) z-direction may be identified in Eq. (12).

The amplitude of the total velocity and shear stress at the top of a layer V_{U_m} and τ_{U_m}, can be related to that at the bottom of a layer, V_{D_m} and τ_{D_m}:

$$\begin{Bmatrix} V_{D_m} \\ \tau_{D_m} \end{Bmatrix} = \begin{bmatrix} \cos k_m h_m & iZ_m \sin k_m h_m \\ i\,\dfrac{\sin k_m h_m}{Z_m} & \cos k_m h_m \end{bmatrix} \begin{Bmatrix} V_{U_m} \\ \tau_{U_m} \end{Bmatrix} \tag{13}$$

in which, $Z_m = \omega/k_m h_m$, and h_m = layer thickness. It is convenient if Eq. (13) is expressed as

$$\{D\}_m = [F]_m \{U\}_m \tag{14}$$

The total velocity and shear stress at the bottom of the (m-1) th layer is the total velocity and shear stress at the top of the m th layer. When the computation proceeds from layer 1 to layer n, repeated application yields

$$\{D\}_n = [F]_1 [F]_2 \cdots [F]_n \{U\}_1 \tag{15}$$

Since the objective is to find the velocity ratio, it is convenient to use a sine wave of unit amplitude as the total surface velocity. Then the vector U , at the free surface is

$$\{U\}_1 = \begin{Bmatrix} -i \\ 0 \end{Bmatrix} \tag{16}$$

If [J] represents the product of the matrices shown in Eq. (15), the incident wave E_n may be obtained:

$$E_n = \frac{1}{2}\left(\frac{J_{11}}{\omega} + \frac{J_{21}}{k_n G_n}\right) e^{-ik_n G_n} \tag{17}$$

Once E_n is defined, the amplitude of the incident velocity at the n th layer is defined as the modulus of ωE_n. The velocity ratio, r, between base incident velocity and total surface velocity is

$$r = |\omega E_n| \tag{18}$$

In Eq. (18), ω and E_n are independent of time; thus, the ratio, r, is also independent of time. This method provides the layered system response in the frequency domain instead of the time domain. Among other things it allows one to investigate the variation of r with frequency and to find the critical frequencies at which the ratio is maximum or minimum.

Based on the method, several numerical examples were tested with ω in the range 1-125 rad/sec, G in the range $9(10)^8$ $-7(10)^9$ psf and specific gravity in the range 2.5 -3.5. Two types of rocks were investigated: one in which the properties become weaker towards the surface, and the other in which the properties become stronger towards the surface. Results show that $0.15 < r < 0.5$ for the former. This means that the use of one-half the measured surface velocity would be the same as using a forcing function always greater than or equal to the actual incident velocity. In the unlikely situation of the properties becoming weaker toward the surface $0.5 < r < 1.65$. In this case the use of one-half the measured surface velocity would be the same as using a forcing function always less than or equal to the actual incident velocity. The ratio, r, is obtained from a sinusoidal velocity with frequency, ω, but it can be extended to a seismic motion by using Fourier analysis.

CONCLUSION

Measured surface records of previous earthquakes are often used as the excitation source for prediction of soil behavior with numerical methods. With these seismological records, the appropriate boundary condition for one-dimensional seismic analysis of soil was examined in this study.

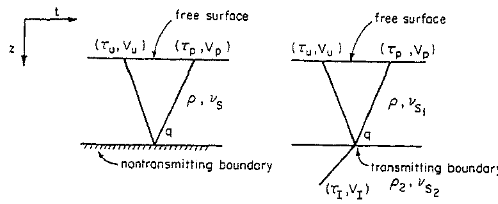
1. The use of the actual rock surface velocity as the prescribed motion at a nontransmitting boundary is not recommended. Alternatively, if the actual physical motion (total velocity) at the soil-rock interface were known then the use of the nontransmitting boundary would be justified.

2. The use of the original incident velocity as the forcing function at a transmitting boundary is most precise. The location of the transmitting boundary is at an interface in the substrata below which the rock might be assumed to be homogeneous. The incident velocity is obtained by inverting the rock surface velocity.

3. If field data shows that the rock property is of the type that becomes weaker toward the surface, the usage of one-half surface velocity to represent the actual incident velocity is proper in the engineering sense, since the approximation is conservative.

4. If field data shows that the rock property is opposite, the usage of 165% of the rock surface velocity would provide a conservative computation. It is noted that rock of this type is very unusual and probably seldom exists.

Fig.3. Method of characteristics in z-t plane, free surface with base boundaries (a) NTR and (b) TR.



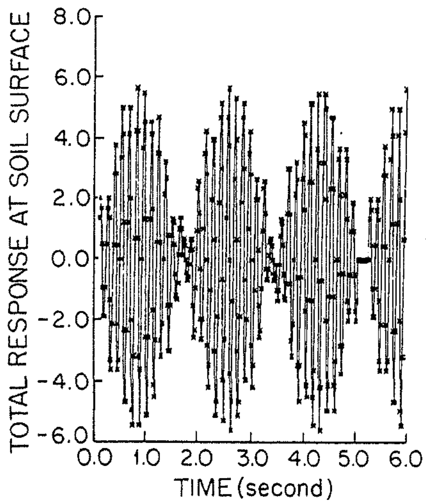


Fig. 4. NTR boundary at base B. Total velocity at the rock surface, Fig. 2, used as prescribed motion at level B, ω=43.5.

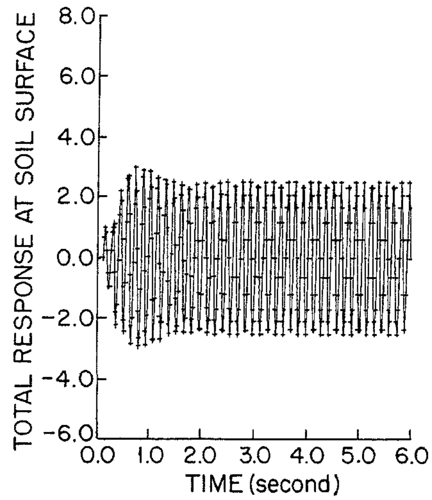


Fig. 5. TR boundary. Original incident velocity used at level C as the pres cribed motion, Fig. 2. (total rock surface V=1 sin 43.5 t).

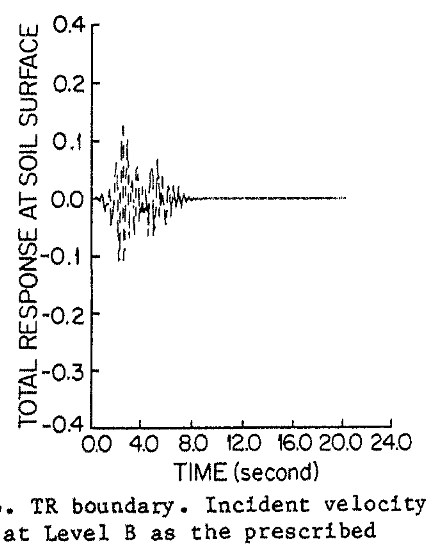


Fig.6. TR boundary. Incident velocity used at Level B as the prescribed motion, Fig.2.(total rock surface velocity is N-S EL Centro Earthquake.)

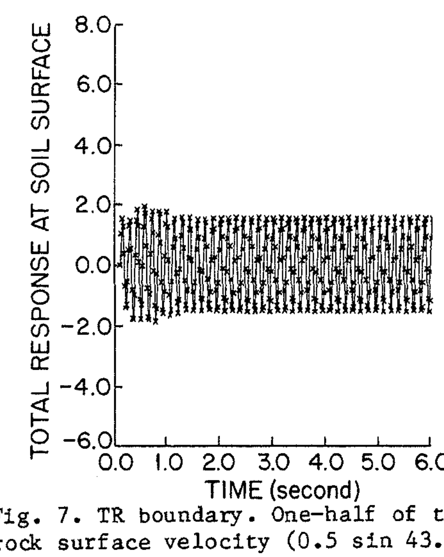


Fig. 7. TR boundary. One-half of total rock surface velocity (0.5 sin 43.5t) used as incident velocity at level B, Fig.2.

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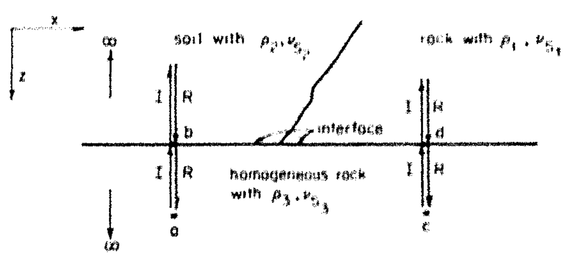

Fig.1. Incident wave (I) propagates upward and reflected wave (R) downward.

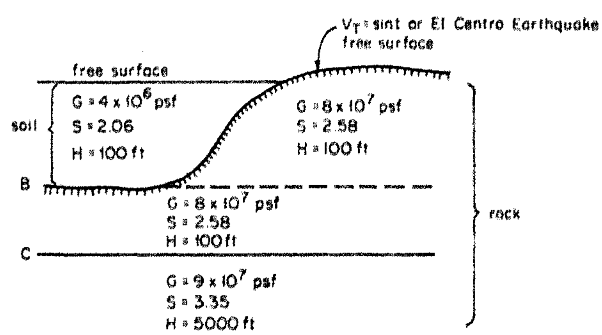

Fig.2. Soil and rock subsurface conditions.