SCATTERING OF ELASTIC WAVES BY THREE-DIMENSIONAL TOPOGRAPHIES

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SUMMARY

A boundary method is applied for solving the scattering and diffraction of elastic waves by three-dimensional irregularities on the surface of a half-space. The method makes use of the completeness of a family of wave functions in order to construct the scattered fields. Boundary conditions are satisfied in a least-squares sense. Numerical results are presented for two types of irregularities under vertical incidence of P or SV waves.

INTRODUCTION

There is no doubt that topographical irregularities may induce significant changes in the nature of ground motion at nearby places during earthquakes. In the last few years the problem has attracted the interest of engineers as they are concerned with the design of important facilities in which is mandatory the proper assessment of ground motion. Lateral heterogeneities may generate large amplifications and spatial variations of seismic ground motion and this is also relevant in planning and microzonation studies.

Most of the work in this area has been devoted to study the ground motion at two-dimensional irregularities for various incident wave fields (*e.g.* Refs. 1-5). This work has emphasized the physical understanding of local effects so that quantitative predictions can be made in many cases.

Three-dimensional studies have received less attention because of the increased difficulties which arise in solving this class of problems. Some works dealing with three-dimensional problems have appeared. For small-slope cavities or inclusions at the surface of an elastic half-space a perturbation approach have been presented (Ref. 6) and reasonable estimates were obtained of the scattered Rayleigh waves, as compared with observations (Ref. 7). A finite difference analysis of axisymmetric irregularities has been presented (Ref. 8) for vertically incident shear waves. It was assumed a rigid base at certain depth and lateral transmiting boundaries. Spectral ratios were obtained and comparison with some observations gives reasonable agreement. The exact solution has been obtained for incidence of P waves at a semi-spherical cavity under the acoustic approximation (Ref. 9). For the same geometry and assuming an elastic medium, a solution has been presented for incident P and S waves (Ref. 10). It seems, however, that this approach is limited to small frequencies.

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In this paper the scattering and diffraction of elastic waves by a three-dimensional irregularity on the surface of a half-space are considered. A boundary method recently developed for two-dimensional problems (*e.g.* Refs. 11-16) and extended to three-dimensional cases (Ref. 17) is presented here. The scattered fields are constructed with linear combinations of functions which form a complete family (Ref. 18) of solutions of the reduced Navier equation. Coefficients of the linear forms are obtained from a collocation, least-squares matching of boundary conditions. The mentioned solutions are given in terms of spherical Hankel and Bessel functions, associated with Legendre and trigonometric functions (Refs. 19 and 20). Since each one of these solutions does not satisfy in itself the free-boundary conditions, the numerical treatment is extended to part of the half-space surface.

In the present approach, axial symmetry of the scatterer is assumed in order to allow azimuthal decomposition; the problem is split into "two-dimensional" ones. For normal incidence of P or SV waves only one azimuthal number is required. In what follows, the problem is formulated and the method described in brief. Some numerical examples are given for vertically incident P or SV waves on different surface irregularities.

FORMULATION OF THE PROBLEM

Consider the elastic half-space and a three-dimensional surface irregularity which in Fig. 1 are denoted by E and R, respectively. Let $\partial_1 E$ and $\partial_1 R$ be the free boundaries of the regions, and $\partial_2 E = \partial_2 R$ be the common boundary between them. Under incidence of elastic waves the total field is obtained by superposition of diffracted waves on the *free-field* solution, *i.e.* on the solution in absence of irregularity.

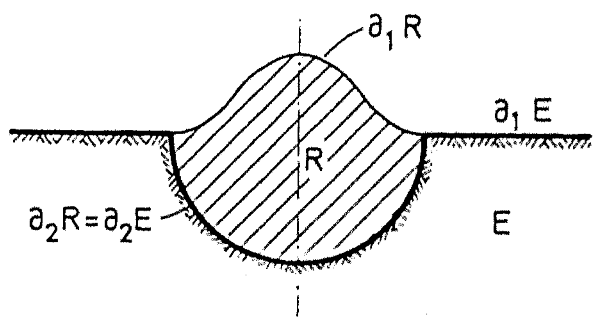

Fig 1. Definition of regions E and R and its boundaries

For harmonic dependence of time, the displacement vector, $\bar{u}$, must satisfy the reduced Navier equation:

$$\mu\nabla^2\bar{u} + (\lambda + \mu)\nabla\nabla\cdot\bar{u} + \rho\omega^2\bar{u} = 0, \tag{1}$$

where λ, μ = Lamé constants, ρ = mass density and ω = circular frequency. The elastic constants and the mass density should be particularized for each medium.

Boundary conditions are those of zero tractions at $\partial_1 R$ and $\partial_1 E$ and continuity of displacements and tractions across $\partial_2 R = \partial_2 E$. In addition, the diffracted fields must satisfy the Sommerfeld-Kupradze elastic radiation condition at infinity (Refs. 21 and 22).

METHOD OF SOLUTION

Let us write the total fields as

$$\bar{u}^E = \bar{u}^{(0)} + \sum_{j=0}^{2} \sum_{n=0}^{N} \sum_{m=0}^{n} A_{jnm} \bar{w}^E_{jnm} \tag{2}$$

for the region E, and

$$\bar{u}^R = \sum_{j=0}^{2} \sum_{n=0}^{M} \sum_{m=0}^{n} B_{jnm} \bar{w}^R_{jnm} \tag{3}$$

for the region R. In Eq. 2, $\bar{u}^{(0)}$ = displacement vector of the free-field solution, $\bar{w}^E_{jnm}$ = displacement vector of a scattered field. In Eq. 3, $\bar{w}^R_{jnm}$ = displacement vector of a refracted field. A_{jnm} and B_{jnm} are unknown coefficients and N, M are the orders of approximations. The first index in the given linear forms corresponds to the type of the field . There are three types of solutions; toroidal of S waves and spheroidal of P and S waves (Ref. 20). The solutions depend on other two indexes; n and m, the radial and azimuthal numbers, respectively. The scattered or refracted solutions can, in general, be written in the form

$$f_n(r)\ \bar{F}^m_n(\theta, \phi) \tag{4}$$

in which $f_n(r)$, the radial function, is given in terms of spherical Hankel or Bessel functions for the regions E or R, respectively (Ref. 17). $\bar{F}^m_n(\theta, \phi)$ is a vector function which is given in its different forms in terms of the function

$$Y^m_n(\theta, \phi) = P^m_n(\cos\theta)e^{im\phi} \tag{5}$$

and its derivatives. Here $P^m_n(\cdot)$ = Legendre function and $m = 0, \pm 1, \pm 2, \ldots, \pm n$. It can be seen, from Eqs. 2 to 5, that the scattered or refracted fields contain sinus and cosinus of $m\phi$, where ϕ = azimuthal angle (Fig. 2).

By imposing boundary conditions at a finite number of points on the boundaries, a system of linear equations for the unknown coefficients is obtained, in which the independent part is given in terms of the free-field solution. It is convenient to form an overdetermined system and solve it in the least-squares sense. This "collocation and least-squares" approach has been useful in two-dimensional problems (Refs. 11-16, 23, 24).

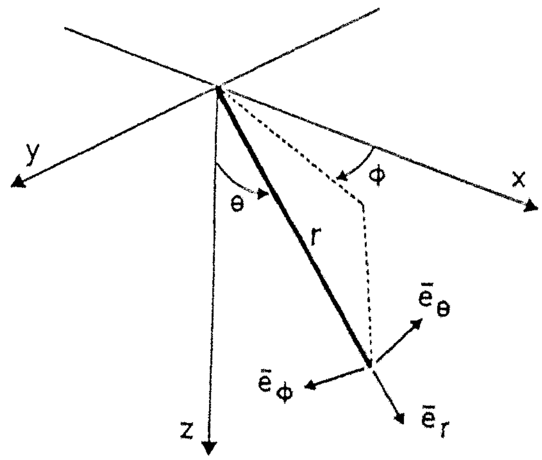

Fig 2. Cartesian and spherical coordinate systems. Unitary vectors in the spherical system

THE AZIMUTHAL DECOMPOSITION

If the shape of the irregularity is independent of ϕ, that is to say, axisymmetric with respect to the z-axis the orthogonality of azimuthal functions allows a complete decomposition of the problem in terms of the azimuthal number. It can be shown that any component of the free-field can be expanded in a Fourier series of azimuthal functions (Ref. 17), and also that the *evenness* and *oddness* properties of the free-field solution hold for the diffracted and refracted fields. Then, if the scatterer is axisymmetric, boundary conditions also have these properties, *i.e.* being even or odd. In this form it suffices to solve a "two-dimensional" problem for each azimuthal number.

For vertically incident plane waves only one azimuthal number is required. In this case, for P waves, only $m = 0$ is needed; for SV waves, it suffices to take $m = 1$. If the incidence is nearly vertical only a few azimuthal terms are needed to obtain good results. For almost grazing incidences or for Rayleigh waves with large horizontal wave number, this approach would require many azimuthal numbers. However, even in this critical case, four or five azimuthal terms can give a good approximation if the horizontal wave-lengths of the incident field are of the order of the maximum horizontal dimension of the irregularity.

EXAMPLES

In this section some results are presented for normal incidence of P or SV waves upon two types of surface irregularities; a ridge and an alluvial deposit. Results are given in terms of normalized frequencies for each type of incoming waves. These are

$$\eta_q = \frac{qa}{\pi} = \frac{2a}{\Lambda_p} \quad \text{and} \quad \eta_k = \frac{ka}{\pi} = \frac{2a}{\Lambda_s} \tag{6}$$

for P and SV waves, respectively. Here, q, k = wave numbers, Λ_p, Λ_s = wavelengths and 2a = maximum diameter of the irregularity. In all cases, the examples correspond to η_q or η_k equal to one.

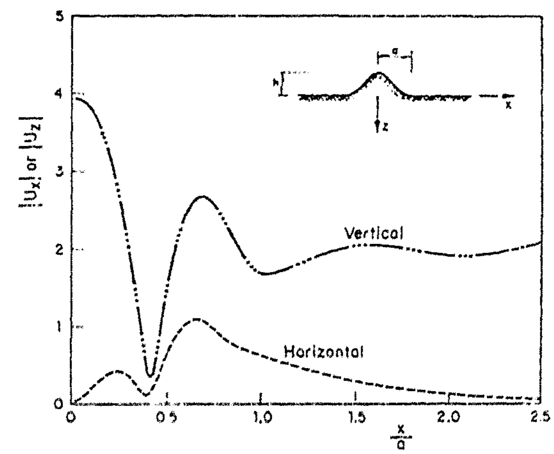

Fig 3. Surface amplitudes of vertical and horizontal displacements. Ridge with h/a = 1.0 Normalized frequency η_q = 1.0. Poisson coefficient ν_E = 0.25.

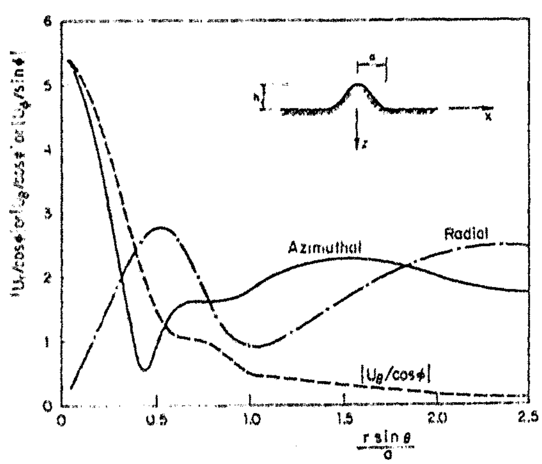

Fig 4. Surface amplitudes of spherical displacement components. Ridge with h/a = 1.0. Normalized frequency η_k = 1.0. Poisson coefficient ν_E = 0.25

The order of expansions, the number and location of collocation points are obtained using a "trial and error" procedure which is based upon the error analysis of boundary conditions and the stability of the surface displacement field. The results were obtained with an order for the expansions of ten and 30 collocation points placed uniformly at $\partial_1 R$, $\partial_2 R$ and at $\partial_1 E$ in a length of 2a. The calculated residual tractions do not exceed the six per cent of the maximum stress in the free-field solution.

Fig. 3 shows the normalized amplitudes of displacements for incidence of P waves upon a ridge which shape is given by $z=h[1-3\xi^2+2\xi^3]$, where $0\le\xi\le1$, h=heigth of the ridge, and $\xi=(x^2+y^2)^{1/2}/a$. Results are given for h/a = 1 and a Poisson coefficient ν_E = 0.25. For incidence of SV waves upon the same irregularity the results are shown in Fig. 4. Here the displacements are normalized with the sinus or cosinus of the azimuthal angle ϕ showing the three-dimensional nature of the surface field. Amplifications of about 200 and 250 per cent can be observed for the incidences of P and SV waves, respectively.

For an alluvial deposit with the depth given by the same shape of the studied ridge, Figs. 5 and 6 show the normalized amplitudes of horizontal and vertical displacements on the surface for

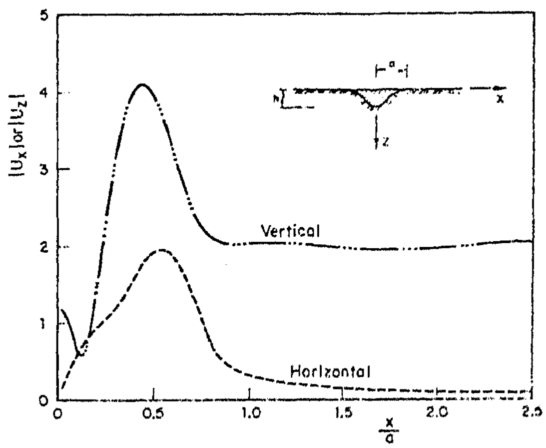

Fig 5. Surface amplitudes of vertical and horizontal displacements. Alluvial deposit with $h/a = 0.5$. Normalized frequency $\eta_q = 1.0$. Poisson coefficients $\nu_R = 0.30$ and $\nu_E = 0.25$. Material properties $\mu_R/\mu_E = 0.25$ and $\rho_R/\rho_E = 0.75$

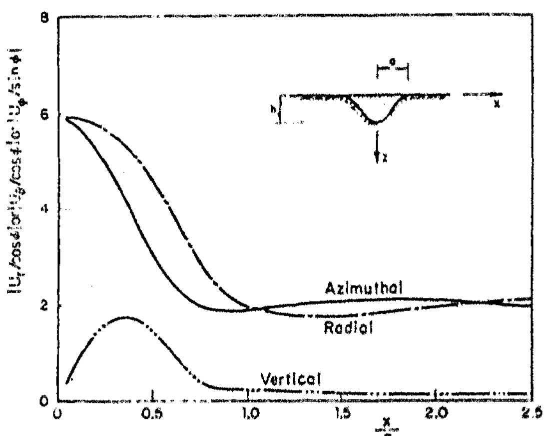

Fig 6. Surface amplitudes of spherical displacement components. Alluvial deposit with $h/a = 0.5$. Normalized frequency $\eta_k = 1.0$. Poisson coefficients $\nu_R = 0.30$ and $\nu_E = 0.25$. Material properties $\mu_R/\mu_E = 0.25$ and $\rho_R/\rho_E = 0.75$

incidence of P and SV waves, respectively. Parameters are given by $h/a = 0.5$, $\mu_R/\mu_E = 0.25$, $\rho_R/\rho_E = 0.75$, $\nu_R = 0.3$ and $\nu_E = 0.25$. In this case, amplifications of 200 and 300 per cent are observed. Note that important mode conversion takes place given significant horizontal and vertical displacements for incident P and SV waves, respectively.

CONCLUSIONS

A boundary method has been applied for solving the scattering and diffraction of elastic waves by axisymmetric surface irregularities. The method makes use of a complete family of wave functions, which are solutions of the reduced Navier equation, to construct the scattered fields. An azimuthal decomposition allows to solve the problem as a sequence of "two-dimensional" ones.

Some results were given for vertical incidence of P or SV waves upon two types of irregularities. Large amplifications of motion were found at the top of the ridge and, for the studied normalized frequency the incidence of SV waves appears as critical. The examples of the alluvial deposit show the important effect of the mode conversion which takes place there. Although more results are needed, the obtained ones suggest that the influence of local irregularities on ground motion cannot be disregarded.

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