DETERMINATION OF EARTHQUAKE DESIGN PARAMETERS FOR DIFFERENT LOCAL SOIL AND TOPOGRAPHIC CONDITIONS

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SUMMARY

This paper presents an improved theoretical model for the determination of design spectra for sites of known soil and topographic conditions. The model, stochastic in nature, is based on wave propagation theory and a method of superposition for complex modes. The numerical algorithm is based on a plane strain finite element formulation and accepts trains of surface or body waves arriving at given incidence angles. The applicability of the model to different geometries and soil properties distributions is discussed. Inelastic effects, important for strong seismic motions, are approximately considered using a least squares equivalent hysteretic damping for the soil. Uncertainties on the definition of the seismic excitation are handled by using, as input to the model, design spectra on firm soil previously determined from seismic risk studies.

INTRODUCTION

The definition of seismic design parameters for specific sites should include information on the effects that local soil and topographic conditions have in seismic motions. Its importance has been widely recognized; however, little attention has been devoted to practical ways to consider those effects on design response spectra.

Earthquake response of soil deposits has been the subject of research for more than two decades. Numerous applications and solution methods have evolved from these efforts. Unfortunately their goals have been restricted to the determination of amplification functions for Fourier amplitude spectra, which are not of direct interest in earthquake design of structures. The main reason for this drawback is that these functions are directly obtained from a dynamic response to steady state harmonic excitation, while the determination of amplification functions for response spectra involves dealing with the probability distributions of maximum response at firm soil and at sites with different soil characteristics.

In this paper, a different approach to the problem is proposed. It involves modelling aspects previously not considered and a realistic approximation to include the uncertainties in the nature of seismic excitations on firm soil. The numerical model used to solve the wave propagation problem is one of finite elements with conditions at artificial boundaries which approximate the free passage of waves. The strategy of solution is such that spurious reflec

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tions can be minimized using a recursive solution of the model.

The excitation to the model is given in terms of a power spectral density determined from the design spectrum using random vibration theory. Due to the discrete nature of the numerical model a complex mode superposition approach was chosen.

SOIL AMPLIFICATION MODEL

The determination of the response characteristics of soil deposits subjected to seismic excitation involves the solution of a complex wave propagation problem which is normally attained with semi-analytical and/or discretization methods (Refs. 1 and 2).

Of all the possible formulations those using integral methods and finite elements have been extensively used. Integral formulations have the advantage of being directly applicable to the kind of problems inasmuch as the solution is only required on the boundary. Nevertheless, their numerical implementation is complicated and their operation cost high. Additionally, it is not possible to formulate an algebraic eigenvalue problem required as a preliminary step to a complex mode superposition. Finite elements, on the other hand, are easy to implement and produce a good computational efficiency. They have, however, the difficulty of requiring the definition of conditions at artificial boundaries to simulate the infinite extent of domains associated to soil domains. The method has as an advantage that the formulation of the algebraic eigenvalue problem follows directly from the discretized equilibrium equations. In this paper a finite element formulation is used.

The discretized dynamic equilibrium equations for a linear viscoelastic medium resulting from a finite element displacement model can be written as

$$M \ddot{\underset{\sim}{U}} + C \dot{\underset{\sim}{U}} + K \underset{\sim}{U} = \underset{\sim}{P}\ (t) \tag{1}$$

where M, C and K are the respective mass, damping and stiffness matrices, $\underset{\sim}{U}$ is the vector of total nodal displacements, $\underset{\sim}{P}$ is the time dependent load vector, and dots on $\underset{\sim}{U}$ imply time derivatives.

Damping characteristics of soil media are generally of the hysteretic type. Their modelling leads, in time domain solutions, to a non linear set of equations impractical to solve. On the other hand, approximations, assuming a damping matrix of the Rayleigh type, do not reflect the frequency independence of hysteretic damping. This drawback can be overcome if, during the determination of the cofficients that define the damping matrix, a least squares criterion is used in which the independece is enforced in an approximate way.

Load vector $\underset{\sim}{P}$ is a function of the tractions along artificial boundaries, which, in turn, may be derived directly from the stresses on the continuous model. Thus, for a linear problem, stresses along the artificial boundary can be written as

$$\sigma_{ij} = \sigma_{ij}^{o} + \sigma_{ij}^{E} \tag{2}$$

where σ_{ij} is the stress tensor in the continuous model, the superscript 0 stands for the free field component, i.e. stresses associated to a simplified problem for which the solution is known. and superscript E for the complementary part, i.e. stresses in excess of the free field.

Stresses in excess of the free field may be approximately derived from an absorption condition similar to that used in the approximation of viscous boundaries (Ref 3). In this case, any information, normally intuitive on the range of incidence angles and/or the kind of waves in excess of the free field, is taken into account using a least squares approximation (Ref. 4). Thus, for a plane strain problem, stresses associated to waves propagating in the direction shown in Fig. 1 are

$$\begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{12} \end{Bmatrix}^E = -\rho \begin{bmatrix} C_p \cos^3\theta+(H+2C_s)\mathrm{sen}^2\theta\cos\theta & (C_p-2C_s)\mathrm{sen}\theta\cos^2\theta+H\,\mathrm{sen}^3\theta \\ (C_p-2C_s)\mathrm{sen}^2\theta\cos\theta+H\cos^3\theta & C_p\mathrm{sen}^3\theta+(H+2C_s)\mathrm{sen}\theta\cos^2\theta \\ (C_p-C_s-H)\mathrm{sen}\theta\cos^2\theta+C_s\mathrm{sen}^3\theta & (C_p-C_s-H)\mathrm{sen}^2\theta\cos\theta+C_s\cos^3\theta \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix}^E \tag{3}$$

or in compact form

$$\underset{\sim}{\sigma}^E = D\,\dot{\underset{\sim}{U}}^E \tag{4}$$

where $H = (C_p - 2\,s\,C_s)$ with $s = C_s/C_p$, $\dot{u}_1^E$ and $\dot{u}_2^E$ are the particle velocity components in the excess waves field, and θ is the angle that defines the propagation direction.

Elements of D in eq. 4 are dependent on soil properties and on θ, which is in general not known. This difficulty can be conveniently removed if a least squares criterion is used to approximate D with a new matrix D^* whose elements are θ independent. For this, we define the mean square error as (Ref. 5)

$$\varepsilon_{ij} = \int_{\theta_1}^{\theta_2} (\, d_{ij} - d_{ij}^* \,)^2\, W_i^2 \, d\theta \tag{5}$$

which has its minimum value when

$$d_{ij}^* = \int_{\theta_1}^{\theta_2} d_{ij}\; W_i^2 \; d\theta \,/ \int_{\theta_1}^{\theta_2} W_i^2 \; d\theta \tag{6}$$

where $W_i(\theta)$ is a weighting function reflecting the distribution of the waves arriving at a given point and θ_1 and θ_2 are the limits of the propagation range.

By assuming $W_i = \cos\theta$, the above formulation gives results equivalent to those obtained from the minimization of reflected energy at artificial boundaries (Ref. 3).

Once all elements of D are determined, tractions (components of the stress tensor referred to a local cartesian system) at a particular boundary are directly obtained using simple tensor transformations. For example, on a horizontal boundary we have, for the two dimensional case

$$\begin{Bmatrix} t_1 \\ t_2 \end{Bmatrix}^E = -\frac{8G}{15\pi} \begin{bmatrix} (3+2s)/C_s & 0 \\ 0 & (5+2s-2s^2)/sC_s \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix}^E \tag{7}$$

By the substitution of these traction boundary conditions in a conventional finite element formulation we get a load vector $\underset{\sim}{P}$, such that

$$\underset{\sim}{P} = \underset{\sim}{P}^{o} + C_e\ \dot{\underset{\sim}{U}}^{E} \tag{8}$$

where C_e is a viscous damping matrix, such that only the elements associated to degrees of freedom on artificial boundaries are different from zero.

Considering that the velocity field in excess of the free field can be expressed as

$$\dot{\underset{\sim}{U}}^{E} = \dot{\underset{\sim}{U}} - \dot{\underset{\sim}{U}}^{o} \tag{9}$$

the final dynamic equilibrium equations are

$$M\ \ddot{\underset{\sim}{U}} + \bar{C}\ \dot{\underset{\sim}{U}} + K\ \underset{\sim}{U} = \underset{\sim}{P}^{o} - C_e\ \dot{\underset{\sim}{U}}^{o} \tag{10}$$

where $\bar{C} = C - C_e$

The use of matrix C_e in eq. 10 represents only an approximation to the exact problem. An improved solution may be obtained, however, if the results from a first free field consideration are used as free field for a second approximation. This procedure can be recursively used until the desired precision is attained.

STOCHASTIC APPROXIMATION

Uncertainties present in the definition of expected earthquake motions at a given site are such that a deterministic analysis based on eq. 10 is not realistic. For this reason, in this investigation an stochastic approach for the seismic loading was taken.

Due to the discrete nature of finite element solutions, a modal superposition approach was considered convenient. For this purpose, the equilibrium equations given in eq. 10 can be rearranged in a system of order 2n as follows (Ref. 6)

$$\begin{bmatrix} 0 & M \\ M & \bar{C} \end{bmatrix} \begin{Bmatrix} \ddot{\underset{\sim}{U}} \\ \dot{\underset{\sim}{U}} \end{Bmatrix} + \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix} \begin{Bmatrix} \dot{\underset{\sim}{U}} \\ \underset{\sim}{U} \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{0} \\ \underset{\sim}{F}(t) \end{Bmatrix} \tag{11}$$

or, in compact form

$$A\ \dot{\underset{\sim}{Y}} + B\ \underset{\sim}{Y} = \underset{\sim}{Q} \tag{12}$$

If we consider as homogenous solution to eq. 12

$$\underset{\sim}{Y} = \exp(\alpha t)\ \underset{\sim}{\Phi} \tag{13}$$

the solution of the resulting eigenvalue problem gives 2n complex eigenvalues of the form

$$\alpha_j = -\beta_j + i\gamma_j \tag{14}$$

and the corresponding complex eigenvectors

$$\underset{\sim}{\Phi}_j = \begin{Bmatrix} \alpha_j \underset{\sim}{\phi}_j \\ \underset{\sim}{\phi}_j \end{Bmatrix} \tag{15}$$

The nonhomogeneous solution corresponding to eq. 12 can be expressed as a function of modal coordinates (t) by

$$\underset{\sim}{Y} = \sum_{j=1}^{2n} \xi_j(t)\ \underset{\sim}{\Phi}_j \tag{16}$$

Substituting eq. 16 in eq. 12 and premultiplying by Φ_k^T we obtain, after using the orthogonality property of the modes (Ref. 6)

$$m_k \dot{\xi}_k(t) - \alpha_k m_k\, \xi_k(t) = q_k(t) \tag{17}$$

If we consider $q_k(t)$ a harmonic stationary function given by $\exp(i\,\Omega t)$, the solution of eq. 17 for $\dot{\xi}_k$ is the frequency response function

$$H(\Omega) = -i\Omega \,/\, m_k(\alpha_k - i\,\Omega) \tag{18}$$

where Ω is the frequency and i the unit imaginary number.

Now, if the excitation is a stationary random process with zero mean, the cross spectral density function of $\dot{\xi}_i$, $\dot{\xi}_j$ can be expressed as (Ref. 7)

$$S_{\dot{\xi}_j \dot{\xi}_k}(\Omega) = H_j(\Omega)\ H_k^*(\Omega)\ S_{q_j q_k}(\Omega) \tag{19}$$

where $S_{q_j q_k}$ is the cross spectral density of q_j , q_k and * stands for complex conjugate.

On the other hand, since

$$q_j(t) = \underset{\sim}{\Phi}_j^T\ \underset{\sim}{Q} = \phi_j^T\ \underset{\sim}{F} \tag{20}$$

we obtain

$$S_{q_j q_k}(\Omega) = \sum_m \sum_n \phi_{mj}\ \phi_{nk}^*\ S_{F_m F_n}(\Omega) \tag{21}$$

where F_n stands for the nth component of the load vector.

Substituting eq. 21 in eq. 19 we obtain

$$S_{\dot{\xi}_j \dot{\xi}_k}(\Omega) = \sum_m \sum_n \phi_{mj}\ \phi_{nk}^*\ \ H_j(\Omega)\ H_k^*(\Omega)\ S_{F_m F_n}(\Omega) \tag{22}$$

Furthermore, from eqs 15 and 16, the nodal accelerations are defined as

$$\underset{\sim}{\ddot{U}} = \Sigma\ \alpha_j\ \phi_j\ \dot{\xi}_j(t) \tag{23}$$

Considering as an example a plane strain problem, the horizontal and vertical components of accelerations at a point a, u_{1a} and u_{2a} (Fig. 2) are

$$\ddot{u}_{1a} = \sum_{j=1}^{2n} \alpha_j\ \phi_{u_{1a},j}\,\dot{\xi}_j(t) \tag{24}$$

and

$$\ddot{u}_{2a} = \sum_{j=1}^{2n} \alpha_j \phi_{u_{2a},j}\,\dot{\xi}_j(t) \tag{25}$$

Thus, reasoning as before, the spectral density function for $\ddot{u}$ is

$$S_{\ddot{u}_{1a}\ddot{u}_{1a}} = \sum_j \sum_k \alpha_j \alpha_k \phi_{u_{1a,j}} \phi_{u_{1a,k}} S_{\dot{\xi}_j \dot{\xi}_k}(\Omega) \qquad (26)$$

and in a similar way for $\ddot{u}_{2a}$,

$$S_{\ddot{u}_{2a}\ddot{u}_{2a}} = \sum_j \sum_k \alpha_j \alpha_k \phi_{u_{2a,j}} \phi_{u_{2a,k}} S_{\dot{\xi}_j \dot{\xi}_k}(\Omega) \qquad (27)$$

Substituting eq. 22 in eqs. 26 and 27, the following relationships are obtained

$$S_{\ddot{u}_{1a}\ddot{u}_{1a}} = \sum_j \sum_k \sum_m \sum_n \alpha_j \alpha_k^* \phi_{u_{1a,j}} \phi^*_{u_{1a,k}} \phi_{m,j} \phi^*_{n,k} \quad H_j(\Omega)\, H_k^*(\Omega)\, S_{F_m F_n}(\Omega) \qquad (28)$$

and

$$S_{\ddot{u}_{2a}\ddot{u}_{2a}} = \sum_j \sum_k \sum_m \sum_n \alpha_j \alpha_k^* \phi_{u_{2a,j}} \phi^*_{u_{2a,k}} \phi_{m,j} \phi^*_{n,k} \quad H_j(\Omega)\, H_k^*(\Omega)\, S_{F_m F_n}(\Omega) \qquad (29)$$

In the determination of the cross spectral densities for the loading, i.e. $S_{F_mF_n}(\Omega)$, it is necessary to assume that the free field excitation is produced by a particular type of wave of known incidence angle. In what follows a procedure required to determine $S_{F_mF_n}(\Omega)$ is illustrated by considering, in a plane strain problem, that the free field displacement u_1^0 is produced by a train of P waves with an incidence angle β.

Using concepts of wave propagation theory, the velocity and stress fields for the above mentioned problem may be written as (Ref. 9)

$$\begin{aligned} \dot{u}^0_{\substack{1\\2}} &= f_{\substack{1\\2}}(x_1, x_2, \beta, C_p, C_s, \Omega)\; u_1^0(t) \\ \sigma^0_{11} &= f_3(x_1, x_2, \beta, C_p, C_s, \Omega)\; u_1^0(t) \\ \sigma^0_{22} &= f_4(x_1, x_2, \beta, C_p, C_s, \Omega)\; u_1^0(t) \\ \sigma^0_{12} &= f_5(x_1, x_2, \beta, C_p, C_s, \Omega)\; u_1^0(t) \end{aligned} \qquad (30)$$

where f_i are complex functions of their arguments not shown here due to space limitations.

In a finite element formulation, once the stresses at artificial boundaries are known the determination of the load vector $\underset{\sim}{P}^0$ is straightforward. On the other hand, since matrix $\underset{\sim}{C}_e$ can be conveniently approximated as diagonal, the mth component of load vector $\underset{\sim}{F}$, is

$$F_m(t) = P_m^0(t) - c_m \dot{u}_m^0(t) \qquad (31)$$

where subindex m indicates the component of the respective vectors and c_m is the corresponding diagonal element of matrix $\underset{\sim}{C}_e$.

Using concepts of the theory of stochastic processes, it can be shown that (Ref. 7), the cross spectral density function for F_m, F_n is

$$S_{F_mF_n}(\Omega) = S_{p^0_m p^0_n}(\Omega) - c_n S_{p^0_m \dot{u}^0_n}(\Omega) - c_m S_{\dot{u}^0_m p^0_n}(\Omega) + c_m c_n S_{\dot{u}^0_m \dot{u}^0_n}(\Omega) \qquad (32)$$

which expressed in terms of the spectral density function of the ground acceleration for the free field (condition at firm soil), is finally written as

$$S_{F_m F_n}(\Omega) = S_{p_m^0 p_n^0}(\Omega) - \frac{1}{\Omega} c_n S_{p_m^0 \ddot{u}_n^0}(\Omega) - \frac{1}{\Omega} c_m S_{\ddot{u}_m^0 p_n^0}(\Omega) + \frac{1}{\Omega^2} c_m c_n S_{\ddot{u}_m^0 \ddot{u}_n^0}(\Omega) \quad (33)$$

PROBABILISTIC AMPLIFICATION FOR RESPONSE SPECTRA

The foregoing paragraphs present a procedure for determining the amplification of power spectral densities due to local soil and topographic conditions. In practice, however, seismic design is based on a design spectrum for the site; therefore, it is necessary to study the relation between design spectrum and an equivalent power spectral density. For this, it will be considered that:

a) The acceleration record, corresponding to the design spectrum on firm ground, is a stationary stochastic process with spectral density $S_a(\Omega)$ and duration s.

b) $S_a(\Omega)$ is unknown, but it can be estimated from a simplified criterium that assumes the design spectrum ordinates as proportional to the square root of the variance of the response of a single degree of freedom system at time s, i.e., the end of the acceleration record (Ref. 8).

In accordance with the above mentioned criterium, ordinates of the pseudo acceleration spectrum are defined as

$$A^2(\Omega_i) = k\,\Omega_i^4 \text{ var } y(s) = k\,\Omega_i^4 \int S_a(\Omega) \left|H_{ay}(\Omega,\Omega_i,s)\right|^2 d\Omega \qquad (34)$$

where $H_{ay}(\Omega,\Omega_i,s)$ is the evolutionary transfer function proposed by Vanmarcke

$$\left|H_{ay}(\Omega,\Omega_i,s)\right| = \left[(\Omega_i^2 - \Omega^2)^2 + 4\zeta_s^2\Omega_i^2\Omega\right]^{-1} \qquad (35)$$

and ζ_s is an equivalent time dependent damping ratio (Ref. 10).

Thus, given $A(\Omega_i)$, the determination of $S_a(\Omega)$ follows from the solution of the integral equation given by eq. 36. To do this, an efficient algorithm based on a least squares collocation method and an analytical integration has been implemented. In a strict sense, the proportionality coefficient k needs to be known. However, by considering k only a function of spectral damping and redefining the power spectral density ordinates as

$$\bar{S}_a(\Omega) = k\, S_a(\Omega) \qquad (36)$$

the formulation presented in this paper becomes independent of k.

FINAL REMARKS

The approach proposed herein permits the direct transformation of design spectra on standard conditions (firm ground, flat topography) into design spectra corresponding to more general local conditions. The tranformation is based on the assumption of proportionality between the peak response values (for given probabilities of been exceeded) of simple systems to random ground motion an the square roots of the variances of the responses of those systems to finite segments of stationary gaussian ground motion assumed to be equivalent to the earthquake excitation. As presented here, the approa ch implies determining a steady state frequency domain transformation function for spectral densities of ground motion, using an equivalent damping which accounts for the finite duration of the excitation.

Alternative approaches can be formulated, capable of accounting in a more accurate way for the characteristics of the evolutionary response .of local formations and structural systems to the base excitation, as well as to the probability of exceedence of spectral ordinates. These approaches are more complicated, and sound recommendations about the adequacy of the various approaches must arise from their calibration with the results of systematic simulation studies.

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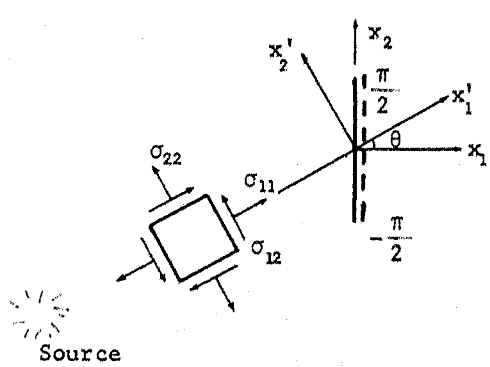

Fig. 1

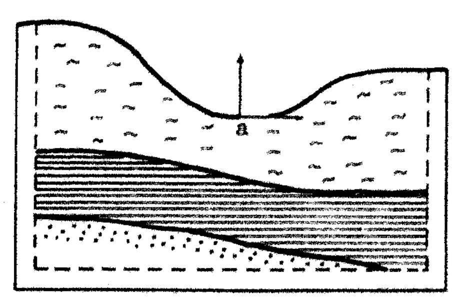

Fig. 2