PROBABILISTIC MODELING OF SPATIAL VARIATION OF STRONG EARTHQUAKE GROUND DISPLACEMENTS

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SUMMARY

This paper presents a probabilistic description of spatial variation of strong earthquake-generated ground displacement. The intent is to establish a practical procedure to deal with the effects of earthquake on long extended structures. Applying the procedure into the analyses of the dense instrument array data and of the damage statistics collected on the buried water supply pipelines during past earthquakes, the maximum amplitude spectra of ground strain and relative displacement between two points on ground surface are estimated. These spectra can be calculated when the predominant ground period and the peak ground acceleration can be assigned.

INTRODUCTION

The seismic ground motion is known to display a complex pattern of variation in space as well as variation with time as shown schematically in Fig. 1. Earthquake engineers have focused their major attention on its temporal variation nature, for instance, the characteristics of ground motion acceleration time history, which are the fundamental factors in dynamic analyses and seismic design of building type of structures. The state-of-the-art report [1] has recently revealed that the spatial variation of seismic ground displacements is the main source of seismic forces acting on underground structures such as subways, pipelines and tanks below ground, etc., which by nature extend for large distance and are widely spaced. The spatial variation has also been understood to be an important factor for the torsional seismic response of tall buildings and of nuclear containment structures having wide spread foundations. Furthermore, the spatially varying seismic ground motion may have a profound influence on the response behaviors of long extended bridge structures. This paper presents a probabilistic method for describing the spatial variability of strong earthquake ground displacement, intended to establish a practical procedure to deal with the effects of earthquake on extended structures.

BACKGROUND

In this section, the fundamentals for the space-time random process are summarized in brief. They are needed to understand the structure of spatial correlation function frequently used in this paper.

Assuming the strong earthquake ground motion displacement as depicted in Fig. 1 is resulted from a homogeneous Gaussian space-time process with

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zero mean, $U(x,t)$, which depends on space coordinate x and on time t, the space-time correlation function $Q(\xi,\tau)$ is defined as

$$Q(\xi,\tau) = \mathrm{E}[\ U(x,t)U(x+\xi,t+\tau)\ ] \tag{1}$$

where $\mathrm{E}[U]$ represents the expectation of random variable U. The quantities ξ and τ are the separation distance and the time lag, respectively. An obvious generalization is to add the two coordinates and to take into consideration the other two orthogonal displacements. By transforming the time lag τ into the frequency ω, the spatial cross spectral density function $P(\xi,\omega)$ can be obtained. This transform is the Wiener-Khintchine relation and can be written as [2],

$$\begin{aligned} P(\xi,\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} Q(\xi,\tau)e^{-i\omega\tau}d\tau \\ Q(\xi,\tau) &= \int_{-\infty}^{\infty} P(\xi,\omega)e^{i\omega\tau}\ d\omega \end{aligned} \tag{2}$$

If the separation distance ξ is zero, the cross spectral density function coincides with the two sided point spectral density function, that is, $P(0,\omega) = S(\omega)$. By normalizing the spatial cross spectral density function with respect to the value at $\xi = 0$, the frequency-dependent spatial correlation function can be defined as

$$r_{\omega}(\xi) = P(\xi,\omega)\ /\ S(\omega) \tag{3}$$

and the spatial correlation function $R(\xi)$ associated with the frozen wave profile along the space coordinate can be expressed as follows, by taking $\tau = 0$ in Eq.(2) and using Eq.(3),

$$R(\xi) = Q(\xi,0) = \int_{-\infty}^{\infty} S(\omega)r_{\omega}(\xi)\ d\omega \tag{4}$$

From Eq.(4), it may be interpreted that the spatial correlation function $R(\xi)$ is a weighted integration of the frequency-dependent spatial correlation function $r_{\omega}(\xi)$ with the point spectral density function $S(\omega)$ as a weighting function. This interpretation yields the explanation that $r_{\omega}(\xi)$ defined by Eq.(3) quantifies the degree of spatial correlation associated with each frequency component of the space-time process $U(x,t)$. When the space-time process is assumed to be ergodic, the spatial correlation function may be estimated from the temporal average as

$$R(\xi) = \lim_{T\to\infty}\frac{1}{T}\int_{0}^{T} u(x,t)u(x+\xi,t)\ dt \tag{5}$$

in which $u(x,t)$ represents a sample function of $U(x,t)$.

EXTREMES OF GROUND DISPLACEMENT, RELATIVE DISPLACEMENT AND GROUND STRAIN

The other statistics can be obtained by making use of the fundamental statistics described in the previous section. From the practical engineering and the preliminary analyses points of view, however, the maximum values for the spatially varying ground displacement, the relative displacement between two points on ground surface and the ground strain, at a given instant $t = t_0$, may be of major interest.

Maximum Ground Displacement

The maximum amplitude of ground motion displacement u_{max} at a given time $t = t_0$ is related to the rms (root mean square), σ_U, of the random process $U(x,t_0)$ in a probabilistic fashion. Specifically, the value of u_{max}/σ_U which has a probability of 1/e (0.37) of no exceeding during an interval distance s_0 is approximately expressed as

$$u_{max} / \sigma_U = \begin{cases} \sqrt{2\ln(2s_0 / L)} & s_0 \geq 1.36L \\ \sqrt{2} & \text{otherwise} \end{cases} \tag{6}$$

where L is the predominant wave length of ground displacement $U(x,t_0)$ which is given by

$$L = 2\pi\sigma_U / \sigma_{\dot{U}} \tag{7}$$

in which σ_U and $\sigma_{\dot{U}}$ can be obtained by making use of the following well known relationships [4] if the spatial correlation function defined by Eqs.(4) or (5) are specified.

$$\sigma_U^2 = R(0) \quad , \quad \sigma_{\dot{U}}^2 = - d^2R / d\xi^2|_{\xi=0} \tag{8}$$

Equation (6) is derived on the basis of the common assumption that the crossings of a specified level occur as a Poisson arrival process [3] and Eq.(7) corresponds to a zero-upcrossing interval of the random instant ground displacement $U(x,t_0)$.

Maximum Relative Displacement and Ground Strain

The relative displacement $D(\xi,x)$ between two points on ground surface at a given instant $t = t_0$ may be defined as

$$D(\xi,x,t_0) = U(x + \xi,t_0) - U(x,t_0) \tag{9}$$

and the apparent ground strain may be written as

$$B(\xi,x,t_0) = D(\xi,x,t_0) / \xi \tag{10}$$

where D and B are also the homogeneous Gaussian random processes with zero mean because $U(x,t_0)$ is the same random process as assumed before. If the separation ξ goes to zero, $B(\xi,x)$ approaches the local ground strain defined by $dU(x)/dx$. For simplicity, the time t_0 will be omitted hereafter. Then, the spatial correlation function, R_D, for the random relative displacement $D(\xi,x)$ is obtained from that of the random ground displacement by the definition $E[D(\xi,x)D(\xi,x+\eta)]$, that is,

$$R_D(\xi,\eta) = 2R(\eta) - R(\eta + \xi) - R(\eta - \xi) \tag{11}$$

The variances of D and its derivative with respect to the separation can be obtained as follows, similar to Eq.(8).

$$\sigma_D^2 = R_D(\xi,0) \quad , \quad \sigma_{\dot{D}}^2 = - d^2R_D / d\eta^2|_{\eta=0} \tag{12}$$

Thus, the maximum amplitude of relative displacement $d_{max}(\xi)$ during the interval s_0 can be calculated from the following equations, similar to Eqs.(6) and (7).

$$d_{max}(\xi) \;/\; \sigma_D = \begin{cases} \sqrt{2\ln(2s_0 \;/\; L_D)} & s_0 > 1.36L_D \\ \sqrt{2} & \text{otherwise} \end{cases} \tag{13}$$

$$L_D = 2\pi\sigma_D \;/\; \sigma_{\dot{D}}$$

where L_D is the predominant wave length of relative displacement. From Eq.(10), the maximum ground strain $\varepsilon_{max}(\xi)$ can be written as,

$$\varepsilon_{max}(\xi) = d_{max}(\xi) \;/\; \xi \tag{14}$$

By making use of Eqs.(11) to (14), the required maximum amplitude of relative ground displacement and ground strain at a given instant can be obtained as a function of the spatial correlation function $R(\xi)$ given by Eqs.(4) or (5). Therefore, it can be found that the main question at now is to determine the spatial correlation function. This is done by analyzing the array instrument data and the damage statistics of buried water supply pipelines during past earthquakes which are described in later sections.

MEAN BREAK RATE OF BURIED PIPES

It is well known [1] that the seismic axial stress in buried small pipes such as water supply pipes is much higher than the bending stress and that the effect of pipe mass is negligible indicating a quasi-static forces resulting from the spatial variation of seismic ground displacement is of major importance. Therefore, if the pipe is assumed to be a straight slender beam on flexible foundation, the spatial spectral density function of the pipe displacement in axial direction can be written as [5]

$$S_p(k) = S_g(k)\; n^4 \;/\; (\,k^2 + n^2\,)^2 \;\;,\;\; n^2 = K_h \;/\; (EA) \tag{15}$$

where K_h = the stiffness of soil per unit pipe length longitudinal to the pipe axis, E = the modulus of elasticity of pipe, A = the cross sectional area of pipe material. In Eq.(15), the quantity $S_g(k)$ is the spatial spectral density function with the wave number k, for the random ground displacement at instant $t = t_0$ which is connected with the spatial correlation function $R(\xi)$ by the Wiener-Khintchine relation as follows.

$$S_g(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(\xi)e^{-ik\xi}\, d\xi \tag{16}$$

From the study of Rice [4], the mean number of crossings of the tensile strain in buried pipe over a limiting fracture strain, ε_a, in unit pipe length, v_f , is given by

$$\begin{aligned} v_f &= L_\varepsilon^{-1} \exp[\, -\, \varepsilon_a^2 \;/\; (2\sigma_\varepsilon^2)\,] \\ L_\varepsilon &= 2\pi\sigma_\varepsilon \;/\; \sigma_{\dot{\varepsilon}} = 2\pi\Big[\int_{-\infty}^{\infty} k^2 S_g(k)dk \;/\; \int_{-\infty}^{\infty} k^4 S_g(k)dk\Big]^{1/2} \end{aligned} \tag{17}$$

where L_ε represents the predominant wave length of randomly varying pipe strain. From Eqs.(15) to (17), the mean pipe break rate during unit pipe length can be calculated when the spatial correlation function $R(\xi)$ and the pipe-soil mechanical-geometrical properties (n, ε_a) may be given. In inverse, from Eqs.(15) to (17), the spatial correlation function may be estimated if the mean pipe break rate and the other parameters (n, ε_a) can be assigned.

APPLICATIONS TO ARRAY DATA ANALYSIS

To illustrate the application procedures of the formulation results presented in previous sections, the seismic array data from the SMART-1 [7] are analysed here. The normalized spatial correlation function was first determined by using Eq.(5) and the NS ground displacements produced from the acceleration time histories at stations C00, I03 to 12, M03 to 09 and 004 to 07 during the earthquake of 29 January 1981. In this data set, the average maximum ground acceleration and displacement are a_{max} = 102 cm/s^2 and u_{max} = 1.8 cm, the interval is s_0 = 2.5 km. The time window T = 10 sec. was used so that the maximum displacement in each station was included in this time interval. The result is plotted in Fig. 2 as a function of separation distance between two stations which is along the wave propagation axis. In the SMART-1 data, the wave propagation direction is estimated to be the range of azimuth between 154° and 176° from Ref. [7]. Thus, the separation along wave propagation axis varies depending on its direction, so that the variation range of separation distance is shown in Fig. 2 by the length of each horizontal line.

From Fig. 2, the observed spatial correlation tends to decrease and to exhibit negative value as the separation increases. The solid curve approximation in Fig. 2 for the spatial correlation function is,

$$R(\xi) = \sigma_U^2 \exp[\,-\,(b\xi)^2\,][\,1 - 2(b\xi)^2\,] \tag{18}$$

where $b = 8.8388\text{x}10^{-4}\ \text{m}^{-1}$. The normalized value R/σ_U^2 in Eq.(18) is shown in Fig. 2. By making use of Eqs.(7), (8) and (18), the wave length can be obtained as L = 2.9 km. Then, the rms ground displacement can be determined as σ_U = 1.3 cm from Eq.(6) with s_0 = 2.5 km, L = 2.9 km and u_{max} = 1.8 cm. Next, the maximum relative ground displacement was calculated by Eqs.(11) to (13) and Eq.(18). This theoretical result is shown in Fig. 3 by a solid curve. And the directly computed maximum relative ground displacement are also shown in Fig. 3 as a function of the separation distance similar to that in Fig. 2. It is noted here that all scales in Fig. 3 are logarithmic. In this logarithmic representation, horizontal lines give the value of d_{max}, 45° lines going up from left to right give ε_{max}, because of the relationship between the maximum relative displacement and the maximum ground strain given by Eq.(14). The average value of maximum ground strain is found from Fig. 3 to be about 5x10^{-5} having the wide variation range of about 5x10^{-6} to 5x10^{-4}. It is suggested from this analysis that the data from the more dense instrument array with the interval within 100 m are needed to be cumulated.

APPLICATIONS TO PIPE DAMAGE DATA ANALYSIS

From the statistical analysis on damage data for buried water supply pipelines during past earthquakes, an empirical relation between the mean pipe break rate during one km and the peak ground acceleration is established by Kubo and Katayama [1]. This empirical relation is used to estimate the spatial behaviors of ground displacement. In this analysis, the spatial correlation function is assumed to be the same form as given by Eq.(18). However, the parameter values are necessary to be adjusted as describing below.

Combining Eq.(6) (s_0 = 1000 m) with the study of Kanai [8] that shows the relation among u_{max}, a_{max} and the predominant ground period T_g, yields the relation between the rms ground displacement and the peak ground acceleration as follows.

$$\sigma_U = \begin{cases} 2.53[\ 2\ln b + 13.318]^{-1/2}\ T_g^2\ a_{max} \times 10^{-2}\ , & L < 735 \\ 1.75\ T_g^2\ a_{max}\ \times 10^{-2} & \text{otherwise} \end{cases} \tag{19}$$

where σ_U is in cm, a_{max} in cm/s^2 and b in m^{-1}. Substituting Eq.(19) into Eqs.(15) to (17) gives the required relation between v_f and a_{max} when the parameters T_g, b and n, ε_a in Eqs.(15) to (17) are given. After the numerical study for the numerous combinations of value for these parameters, the final relation is determined as

$$\log b = -\ [\ 1.533\ \log T_g + 2.159\ ] \tag{20}$$

The detail description can be seen in Refs. [5,6]. Substituting Eqs.(19) and (20) into Eq.(18) gives the spatial correlation function as a function of the predominant ground period T_g and the peak ground acceleration a_{max}. Therefore, the maximum relative ground displacement and strain spectra can be established by the similar way of drawing Fig. 3, provided T_g and a_{max}. The resulting spectra are shown in Fig. 4 for the four ground conditions (T_g = 0.67, 0.50, 0.33 and 0.25 sec.) and the peak ground acceleration a_{max} = 100 cm/s^2. From Eqs.(11) to (14) and Eqs.(18), (19), the spectra for a_{max} = α x 100 cm/s^2 can be found to be obtained by multiplying the spectral values in Fig. 4 into α.

CONCLUSION

A probabilistic description is presented for the seismic ground displacement varying in space as well as time. Applying this description into the analyses of the dense instrument array data and of the damage statistics of underground water supply pipelines during past earthquakes, the maximum amplitude spectra of ground strain and relative ground displacement between two points on ground surface are estimated. These spectra can be established if the predominant ground period and the peak ground acceleration are assigned. However the more precise characterization of these spectra may be needed. Doing this is possible by cumulating the data from the dense instrument array having the shortest interval within 100 m, and analysing them by the procedure presented in this paper.

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REFERENCES

(1) Kubo,K., Katayama,T., and Ohashi,M., "Lifeline Earthquake Engineering in Japan", Proc. of ASCE, Vol.105, No. TC1, pp.221-238, 1979.
(2) Capon,J., "High-Resolution Frequency-Wavenumber Spectrum Analysis", Proc. of IEEE, Vol.57, No. 8, pp.1408-1418, 1969.
(3) Vanmarcke,E.H., and Lai,S.S.P., "Strong-Motion Duration and RMS Amplitude of Earthquake Records", Proc. of BSSA, Vol.7, No. 4, pp.1293-1307, 1980.
(4) Vanmarcke,E.H., "Properties of Spectral Moments with Applications to Random Vibration", Proc. of ASCE, Vol.98, No. EM2, pp.425-446, 1972.
(5) Harada,T., "Spatial Correlation and Spectrum of Strong Earthquake Ground Displacements", Proc. of the 6th Japan Earthquake Engineering Symposium, Tokyo, pp.353-360, 1982.
(6) Harada,T., Oda,T., and Shimada,T., "Determination of Apparent Wave propagation Velocity From Damage Statistics of Buried Pipelines", Conference Volume PVP-77, the 4th National Congress on PVPT, ASME, Portland, 1983.
(7) Bolt,B.A., Loh,C.H., Penzien,J., Tsai,Y.B., and Yeh,Y.T., "Preliminary Report on the SMART 1 Strong Motion Array in Taiwan", EERC-82/13, University of California, Berkeley, 1982.
(8) Kanai,K., Seismological Engineering, Kyoritsu Press, Tokyo, pp.97, 1973, (in Japanese).

Fig. 1 Schematic Diagram Showing the Nature of Seismic Ground Motion

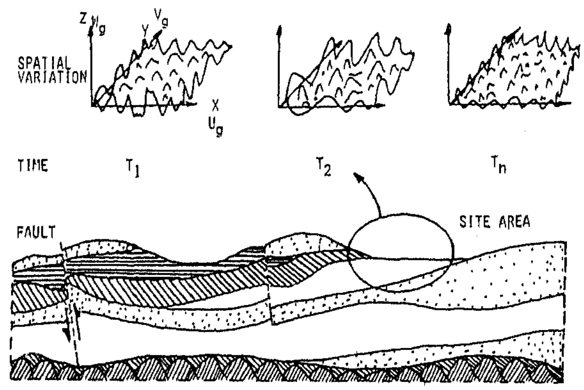
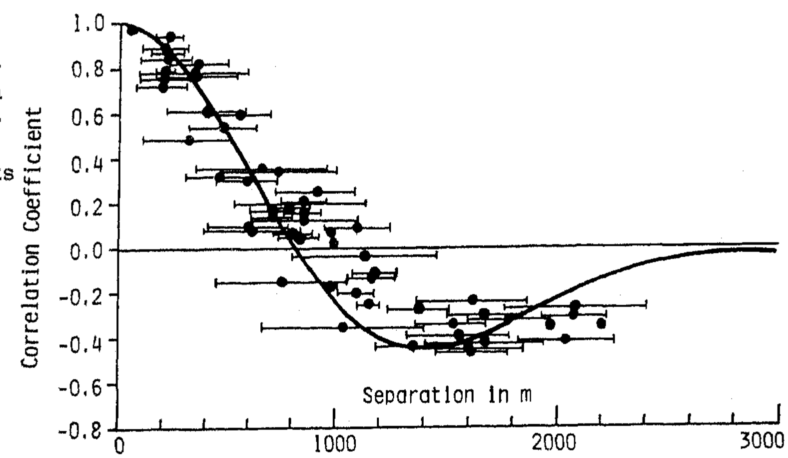


Fig. 2 Spatial Correlation Function from the SMART-1 Data (NS, Event 5) and Its Approximation



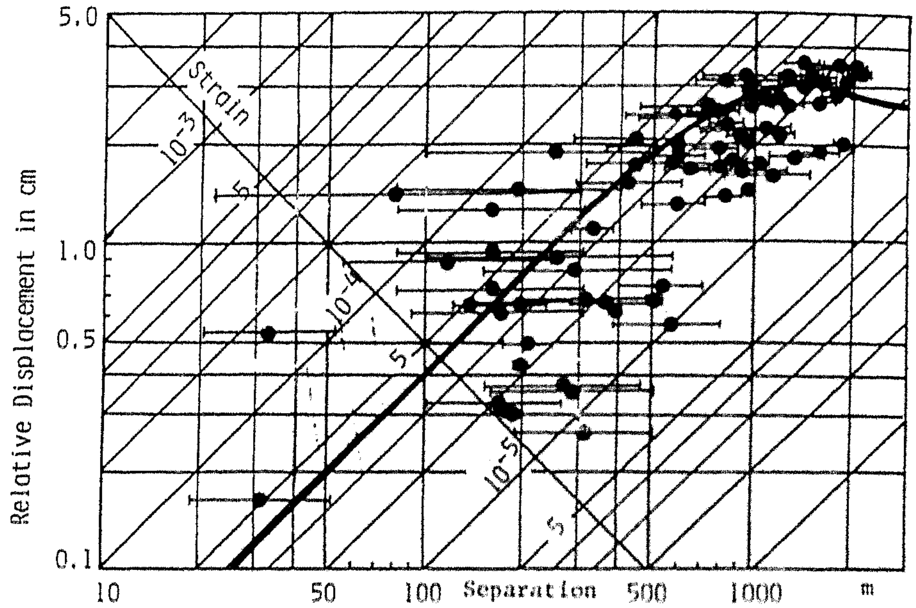


Fig. 3 Spectra of Relative Displacement and Ground Strain from the SMART-1 Data (NS, Event 5).

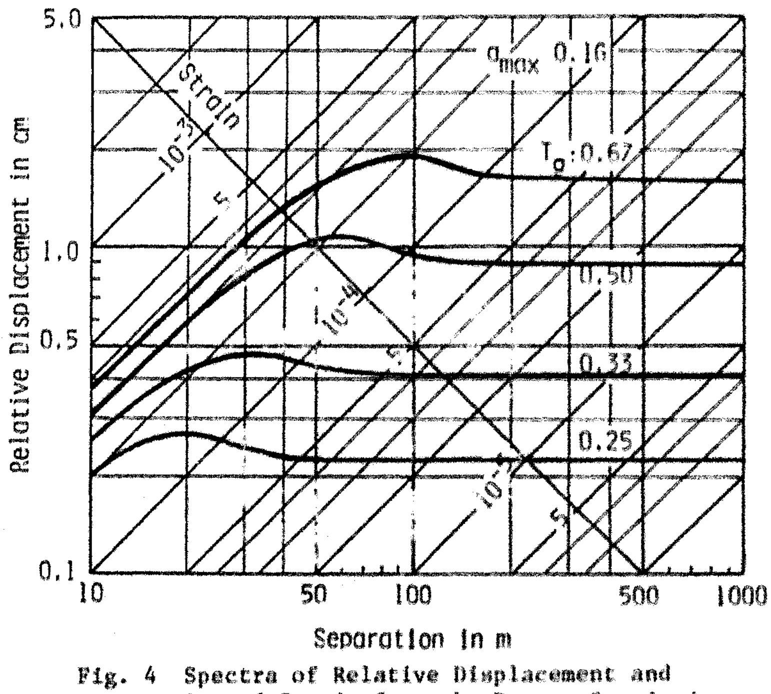


Fig. 4 Spectra of Relative Displacement and Ground Strain from the Damage Statistics.