PARAMETRIC TIME SERIES MODELS FOR EARTHQUAKE STRONG GROUND MOTIONS AND THEIR RELATIONSHIP TO SITE PARAMETERS

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SUMMARY

Characterization of strong earthquake ground motions through parametric time series models provides an efficient description of observed motion through a small set of parameter values. In particular, the application of ARMA models to strong motion accelerograms, after processing by a variance stabilizing transformation, yields statistical estimates of parameters which can be compared among different earthquakes recorded at the same site or for a single earthquake recorded at different sites. By fitting similar models to a large number of recorded accelerograms, relationships between model parameters and site conditions can be estimated and tested for statistical significance.

INTRODUCTION

The effects of strong earthquake ground motions on engineering structures depend largely on the possible loadings and motions to which that structure might be subjected. To better understand and estimate these motions, it seems reasonable to consider a number of factors such as local site conditions or the type of seismic activity that might be expected to affect the recorded motions. However, the uncertain nature of such factors render the deterministic modelling for seismic loading on a given structure at a particular site extremely difficult. The alternative is to construct a stochastic model which accurately represents the earthquake-induced forces and motions.

Extensive studies [1,2,3] have been performed to create an appropriate model describing the characteristics of any particular earthquake record such as that shown in Fig. 1(a). A relatively new method of analysis employs the general class of autoregressive moving average (ARMA) models to analyze strong motion accelerations [4]. These models consist of a discrete stationary transfer function applied to a sequence of white noise. The output is a zero-mean, stationary process with frequency characteristics which depend on the transfer function parameters.

Box and Jenkins [5] discuss in detail the features of these ARMA processes as models representing empirical time series.

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However, evaluation of ARMA and related parameters, on their own, show little physical significance. In this study, the authors have developed a methodology that relates the estimated model parameters to variables such as earthquake intensity and duration, distance to the fault and local geology.

Because this paper represents the initial investigation aimed at evaluating model and site parametric relationships, studies were simplified by limiting the number of parameters to be linked. Furthermore, comparison of record characteristics for numerous earthquakes is greatly facilitated if the model parameters are assumed to be constant over time.

In this section, a methodology developed by Cakmak and Polhemus is presented, which stabilizes the variance of the observed series so that time-invariant ARMA(2,1) models may be applied. The estimated parameters from the model and variance stabilizing transformation are then compared and their relationships to site and earthquake-specific characteristics are evaluated.

DETERMINATION OF THE ARMA MODEL

From the observed accelerations $\{z_t\}$ a new series $\{z_t^2\}$ is created by squaring each of the original acceleration values. A general power transform, suggested by Box & Cox [6], using a transform parameter λ, is then applied to the squared series. In this study, it was determined that a 10th-order polynomial provided a consistent representation of the expected values of the transformed squared accelerations for both horizontal and vertical components of the Imperial Valley Earthquake of 1979.

Application of the reverse Box-Cox transform provides an estimate of the standard deviation function $\hat{\sigma}_z(t)$. Finally, a variance stabilized series is constructed by dividing each $\{z_t\}$ by a local estimate of its standard deviation according to

$$z_t^* = z/\hat{\sigma}_z(t) .$$

Having stabilized the variance of the original series, an ARMA(2,1) model, which yields model parameters ϕ_1, ϕ_2 and θ, can then be constructed to characterize the dynamic behavior of the stabilized series. However, some difficulty was encountered in applying the ARMA model to the full duration of the stabilized data, and it was necessary to model only a representative segment from the full stabilized series.

Figure 1(b) shows typical standard deviation functions corresponding to the acceleration series plotted above them. The duration T of the modelling segment was defined by the interval formed (B to B') between points of zero-slope or inflection immediately before and after the "bell" portion of the function. A parameter α is defined to be the absolute maximum value of the function (A to A') and represents the amplitude of strong ground accelerations. Additionally, as labelled on the figure, the time interval between the first inflection points before and after the function peak (C to C') is taken as the duration of strong ground motion and is termed τ.

The methodology outlined above is applied to all three components of twelve arrays for the Imperial Valley earthquake of 1979 [7]. Thus, for each record analyzed, seven model parameters are obtained: λ, ϕ_1, ϕ_2, θ, T, α, and τ.

EVALUATION OF FUNCTIONAL RELATIONS FOR THE PARAMETERS

For this study, it is postulated that α, τ, and T depend on the distance of the recording station to the fault and that λ is assumed to depend on the type of faulting. Furthermore, θ seems to be related to the specific earthquake and ϕ_1 to depend on the site.

To determine the validity of the relationship set forth above, the parameter values obtained for each component are assumed to be independent. In other words, no correlation between parameters of one component and another component was initially assumed. Each model parameter is then plotted as a function of the perpendicular distance of its corresponding recording station to the fault line. A mean square line is fitted through each set of values and a statistical test on the slope of the line is performed, as shown in Figure 2. It was postulated that if the test showed that the slope was insignificant (T-statistic < 2.0), then the parameter may be assumed to be a constant and therefore independent of distance. However, a significant slope (T-statistic ≥ 2.0) indicates that the parameter is distance-dependent and is given as a simple linear function of distance from the fault.

Definite similarities in the parameter-distance relations began to appear amongst all three components. The striking agreement between the two horizontal components led to the construction of a comprehensive parameter-distance relationship that was independent of which horizontal component was being considered. For some parameters, λ for instance, the correlations between parameter values and distance were so close for all components that component, and therefore, direction - invariant relationships were assumed.

The derived relations for each parameter are listed in Table 1. A logical way to test these functional relations is to analyze a different earthquake recorded at the same Imperial Valley site and investigate whether or not its parameters exhibit the same assumed functional dependencies.

The identical methodology used to analyze the 1979 Imperial Valley earthquake was successfully applied to the three components of the 1940 El Centro series [8]. Since the direction of horizontal recordings were not the same for the two earthquakes, the modelling parameters for the 1940 records were resolved along coincident components of the 1979 records. The parameter estimates for the 1940 series were then plotted against the Imperial Valley results. The comparisons, although certainly not conclusive, do lend support to the original suggestions that θ is earthquake-specific, ϕ_1 is site-specific, and the other relations given in Table 1.

Before further addressing the physical significance of the model parameters, a procedure testing the validity of our methodology is discussed in the following section.

SIMULATION OF ORIGINAL RECORDS

ARMA modelling has a distinct advantage in that the model parameters can be used to generate simulations of the original acceleration series in a recursive manner. These simulations are produced by first generating a stabilized acceleration series from the fitted ARMA model and then multiplying the stationary series by an estimate of the standard deviation function $\hat{\sigma}_z(t)$.

However, since direct application of the relations of Table 1 permit an estimate of the standard deviation function only of duration T seconds, one can only create a simulation representing those T seconds. Since generating a simulation whose duration is equal to that of the original record is much more useful, some extension of the standard deviation function is required.

Using the parameters α and τ, one can construct a standard deviation function $\hat{\sigma}'(t)$ of the form

$$\hat{\sigma}'(t) = c_1 \alpha t^3 e^{-\frac{c_2}{\tau} t} \quad \text{where} \quad c_1 = \frac{8 e^3}{3\sqrt{3}}, \quad c_2 = 2\sqrt{3}$$

for time t over the time interval T. The standard deviation is then set at constant values beyond the endpoints until a function spanning the entire duration of the original record is constructed (Fig. 1(c)). In this way, one can generate simulations for any duration of shaking or distance from the fault, to be compared with any of the original Imperial Valley Series.

For records to be used as input for structural experiments and design purposes, a useful test to determine how closely the simulated series matches the original data is through response spectra. If the simulated series yield spectra similar to those of the original record, then the use of these simulations in such studies might be justified.

Before calculating response spectra, the simulated acceleration series are passed through a filter similar to that used in the processing of the original record before it was recorded on the publicly available tapes [7,9]. Figure 3 shows the response spectra of the original data and its simulated acceleration series at 5 per cent damping. In general, the spectra for the simulated and original records seem to match quite closely. However, the similarity of the response spectra is an encouraging result that lends credance to their possible use in comparative studies across many earthquakes.

CONCLUSIONS

The search for parametric relations, in both the qualitative and deterministic sense, has formed the basis of this investigation. By no means should the functional dependencies formulated in Table 1 or the postulation that ϕ_1 is earthquake-specific and θ is site-specific, be complacently accepted as the comforting conclusion of the study.

The authors have presented a methodology that appears to be a successful step in the quest to accurately define the relationships between model and physical parameters. The results obtained in this investigation seem promising from the descriptive standpoint and could have significant utility in the design of engineering structures.

To better interpret the parametric relationships, it will be necessary to consider and analyze many more earthquake records. The application of even higher order or time-dependent ARMA representations might assist in further understanding the apparent links that exist between model and physical parameters.

REFERENCES

[1] P.C. Jennings, G.W. Housner and N.C. Tsai, "Simulated Earthquake Ground Motion," California Inst. of Technology Tech. Report, Pasadena, CA (1968).

[2] M.K. Chang, J.W. Kwiatkowski, R.F. Nau, R.M. Oliver and K.S. Pister, "ARMA Models for Earthquake Ground Motions," Univ. of Calif. Tech, Report ORC 79-1, Operations Research Center, Berkeley, CA (1979).

[3] F. Kozin, "Estimation and Modelling of Non-Stationary Time Series," Proc. Symp. Computer Meth. in Eng., Univ. of Southern California (1979).

[4] N.W. Polhemus, A.S. Cakmak, "Simulation of Earthquake Ground Motions Using Autoregressive Moving Average (ARMA) Models," Earthquake Eng. and Structural Dynamics, Vol. 9, 343-354 (1981).

[5] G.E.P. Box and G.M. Jenkins, Time Series Analysis Forecasting and Control, Holden-Day, San Francisco, California (1970).

[6] G.E.P. Box and D.R. Cox, "An Analysis of Transformations," J.R. Statist. Soc. B 26, 211-252 (1964).

[7] A.G. Brady, V. Perez and P.N. Mork, "Seismic Engineering Data Report," The Imperial Valley Earthquake, Oct. 15, 1979, Open-File Report 80-703, series IIZ003-IIZ021, U.S. Geological Survey, Menlo Park, CA (April 1980).

[8] Caltech, Vol. II data; Corrected Acceleration, Velocity and Displacement, Earthquake Engineering Research Laboratory, Calif. Inst. of Technology, Pasadena, CA EER 73-05 (May 1973).

[9] D.E. Hudson, Reading and Interpreting Strong Motion Accelerograms, Earthquake Engineering Research Institute (1979).

Table 1: Relations between model parameters and distance to fault, the direction(s) for which the relation is valid, and possible other physical links.

H - horizontal
V - vertical

PARAMETER	RELATION to DISTANCE (d)	DIRECTION	Possibly a Function of
λ	= 0.086	H and V	Fault Type (eg strike-slip)
l	= 15.68	H and V	
ϕ_2	= -0.40	H and V	
α	= (-2.27)d + 81.35 = (-3.12)d + 67.09	H V	
τ	= (0.11)d + 3.41	H and V	
θ	= -0.93 = -0.59	H V	Specific Earthquake
ϕ_1	= 0.69 = 0.22	H V	Site Conditions

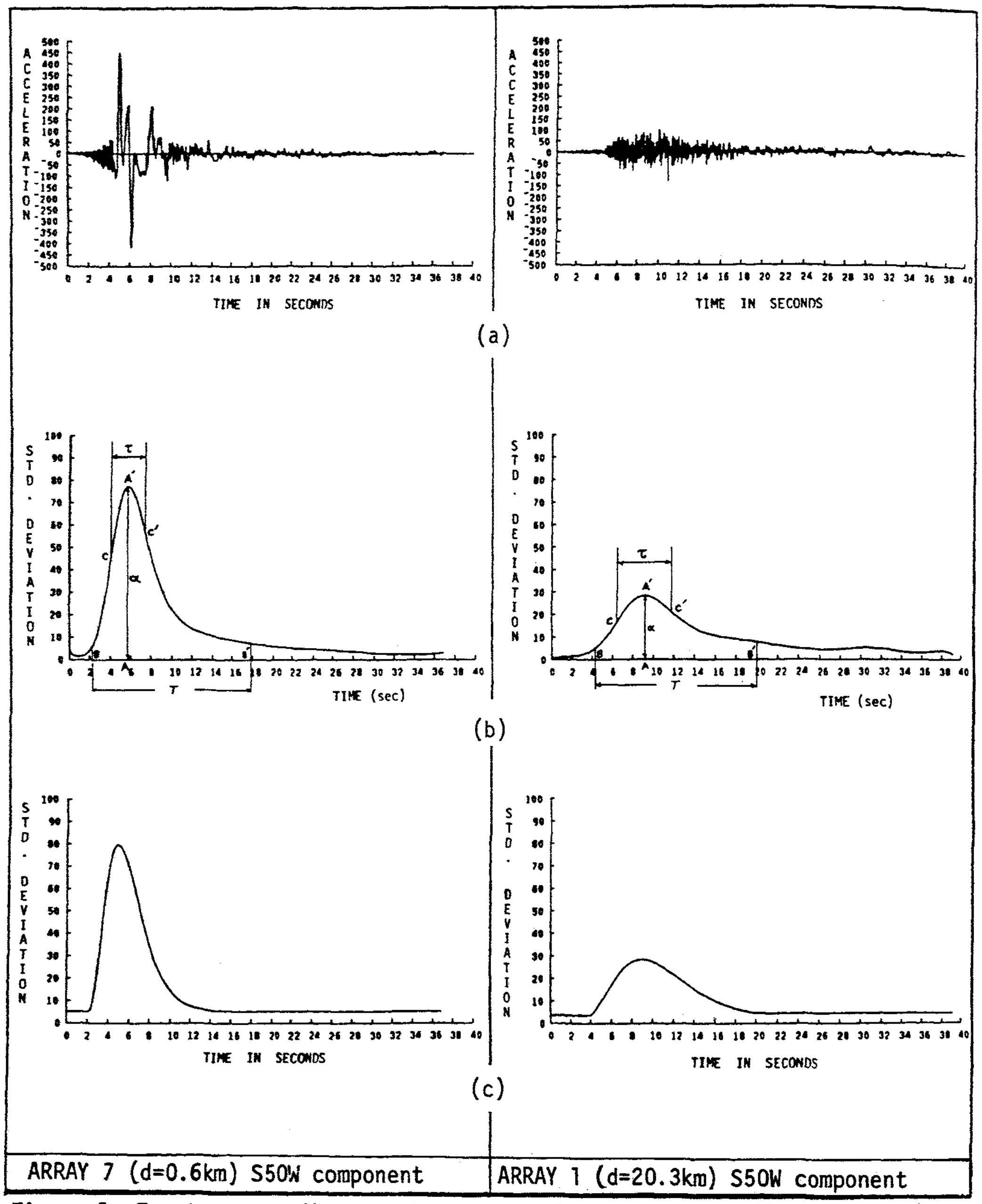

Figure 1: For two recording stations of the 1979 Imperial Valley earthquake
(a) shows recorded acceleration (cm/s/s) as a function of time,
(b) shows standard deviation as obtained by Cakmak-Polhemus method,
(c) shows standard deviation as generated from parametric relations.

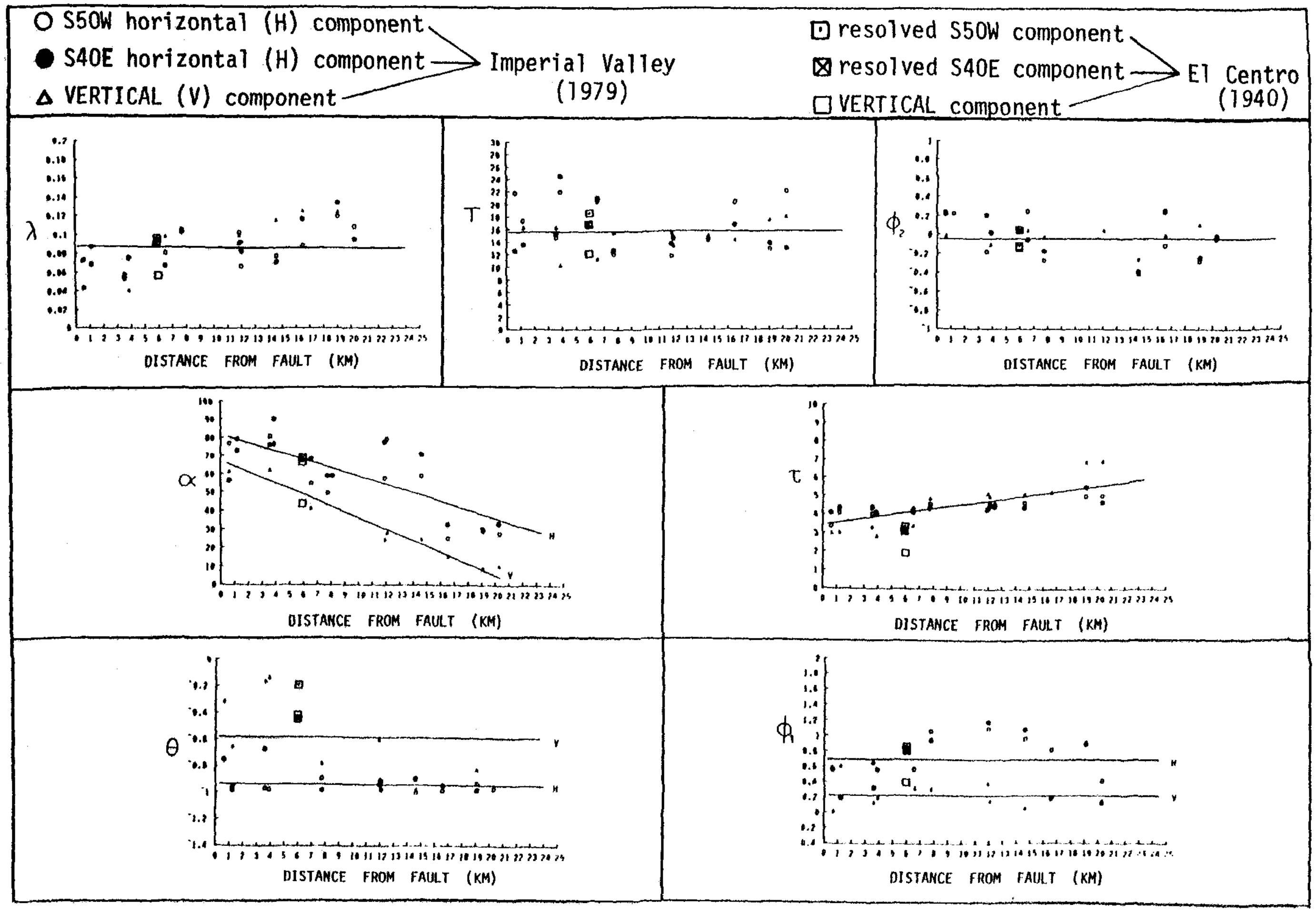

Figure 2: Plots relating model parameters to distance from fault.

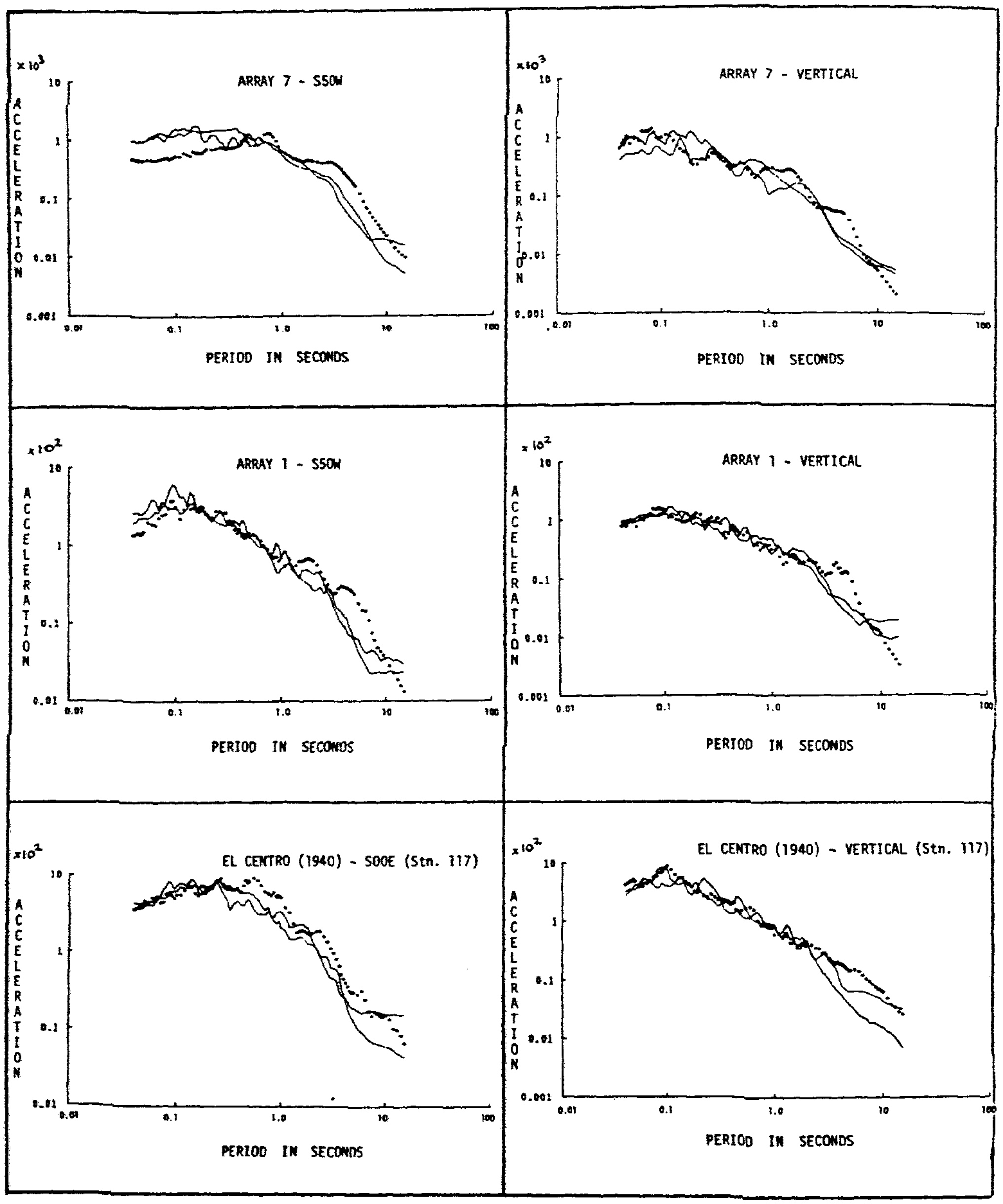

Figure 3: Response spectra for accelration (cm/s/s) at 5 per cent damping.

●●● observed series

—— simulated series generated using parametric relations