APPLICATION OF PHASE DIFFERENCES TO THE ANALYSIS OF NONSTATIONARITY OF EARTHQUAKE GROUND MOTION

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SUMMARY

Physical meaning of Fourier phase differences of earthquake motions is verified through deriving the relationship between the phase differences and the envelope function of narrow band components containing a certain frequency of the Fourier spectra. The relationship can be applied to analyze nonstationary frequency content of recorded earthquake motion accelerograms. The results are examined by comparing with evolutionary spectra.

INTRODUCTION

The nonstationary frequency content of earthquake ground motion is important in seismic design of engineering structures, since it has a significant effect on structural response, especially on inelastic structural response. Various techniques, such as evolutionary spectrum (Ref.1) and physical spectrum (Ref.2), have been proposed in order to analyze the nonstationary frequency content of earthquake motions.

In recent studies, Fourier phase angles of earthquake motions have received attention and they have been applied to analyze the nonstationarity in time and frequency regions of earthquake motions. Ohsaki and others have illustrated from numerical examples that the probability distribution of Fourier phase differences for earthquake ground motion is closely related to the envelope function of the motion (Ref.3). Izumi and others have also clarified the physical meaning of phase differences by numerical examples, and applied the concept to synthesize earthquake motions (Ref.4). However, they have not given theoretical verification for the physical meaning of phase differences.

In the present study, the narrow band components containing a specified frequency of the Fourier spectra for earthquake motion are approximated by a group of harmonic waves, and then a relationship between Fourier phase differences of wave components and the peak appearing time of the envelope of the wave is derived from comparison of the Fourier phase angles of them. Then, by making use of the relationship, the nonstationary frequency content of recorded earthquake motion accelerograms is evaluated and the results are examined by comparing with evolutionary spectra.

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RELATIONSHIP BETWEEN PHASE DIFFERENCES AND NONSTATIONARITY OF EARTHQUAKE MOTION

When earthquake motion, $x(t)$, is defined for N equally spaced discrete times, $t_m=m\Delta t$, discrete Fourier transforms of $x_m=x(t_m)$ can be represented as follows (Ref.5)

$$x_m = \sum_{k=-N/2+1}^{N/2} X(f_k)\cdot exp(i2\pi f_k t_m)\cdot\Delta f \tag{1}$$

in which Δf = frequency increment, $\Delta f=1/T$, T = total duration, and f_k = the k-th frequency, $f_k=k\Delta f$. Then, discrete spectra, $X(f_k)$, $k=0,1,\cdots,N/2$, are as follows

$$X(f_k) = X_k\cdot\exp(i\phi_k) \tag{2}$$

in which $X_k=|X(f_k)|$ and ϕ_k= Fourier phase angle for the k-th frequency. Phase difference, $\Delta\phi_k$, for the k-th frequency can be defined by

$$\Delta\phi_k = \phi_{k+1} - \phi_k\ , \qquad k = 0,\ 1,\ 2,\ \cdots\cdots,\ N/2-1 \tag{3}$$

Ohsaki and others have illustrated that the shape of the probability density function of the phase differences $\Delta\phi_k$ is very similar to that of the envelope of time history, where $\Delta\phi_k$ are assumed to be ranging between -2π and 0. In Fig.1, the probability distribution of phase differences of recorded accelerogram is illustrated in comparing with their time history. It is found from this figure that the distribution of the phase differences is fairly matching with the envelope of the time history.

In the following, the relationship between the probability distribution of phase differences and the envelope function of time history is clarified by deriving the relation between the phase differences and envelope function of the narrow band components containing a specified frequency of Fourier spectra for earthquake motion.

Consider the Fourier spectrum for a group of harmonic waves, which has a constant amplitude and linearly varying phase angle within the frequency range of $f_n\pm\Delta f'$ or $-f_n\pm\Delta f'$, where f_n= central frequency (Ref.6)

$$F_n(f) = \begin{cases} a_n\cdot exp[-i2\pi(f-f_n)t_n+i\Psi_n]\ , & f_n-\Delta f'\leq f\leq f_n+\Delta f' \\ a_n\cdot exp[-i2\pi(f+f_n)t_n-i\Psi_n]\ , & -f_n-\Delta f'\leq f\leq -f_n+\Delta f' \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

in which a_n = Fourier amplitude, Ψ_n = random phase angle ranging of $0-2\pi$, and t_n= time parameter as illustrated later. In Fig.2, Fourier spectrum represented by Eq.4 is illustrated only in the positive frequency range. Fourier inverse transform of Eq.4 yields the following equation.

$$y_n(t) = \int_{-\infty}^{\infty} F_n(f)\cdot exp(i2\pi ft)\, df = Q_n(t)\cdot cos(2\pi f_n t + \Psi_n) \qquad (5)$$

in which $Q_n(t)$ = the envelope function of $y_n(t)$ and is represented by

$$Q_n(t) = \frac{2a_n}{\pi}\cdot\frac{sin\, 2\pi\Delta f'(t-t_n)}{t - t_n} \qquad (6)$$

where t_n = time parameter, which is the same as in Eq.4. In Fig.3, time history $y_n(t)$ as well as its envelope function $Q_n(t)$ are illustrated, in which $Q_n(t)$ has the maximum value $4a_n\Delta f'$ at the time $t = t_n$.

Now, we extract $2l$ discrete Fourier spectra located in the both sides of k-th frequency f_k from N spectra $X(f_k)$ of the eathquake motion, as shown in Fig.4(a). They are approximated by the spectrum of a group of hamonic waves, as shown in Fig.4(b). By comparing the amplitudes X_k in Eq.2 with a_n in Eq.4, following equation can be obtained.

$$a_n = \left(\sum_{j=k-l}^{k+l-1} |X_k|^2 / 2l \right)^{1/2} / T \qquad (7)$$

Similarly, phase angles of Eq.2 and 6 correspond as follows

$$-2\pi(f_j-f_n)t_n + \Psi_n = \phi_j , \qquad j = k-l, \cdots\cdots, k+l \qquad (8)$$

Eq.8 consists of $(2l+1)$ equation including unknown parameters t_n and Ψ_n. Then, estimated values of the unknown parameters, denoted by $\hat{t}_n$ and $\hat{\Psi}_n$, can be determined by making use of the maximum likelihood method, as follows

$$\hat{\Psi}_n = \overline{\phi}_n \qquad (9)$$

$$\hat{t}_n = -\sum_{j=k-l}^{k+l} (f_j-f_n)(\phi_j-\overline{\phi}_n) / 2\pi \sum_{j=k-l}^{k+l} (f_j-f_n)^2$$

$$= -\left[\sum_{j=1}^{l} j\,(\Delta\phi_{k+j-1} + \cdots\cdots + \Delta\phi_{k-j}) / 2 \sum_{j=1}^{l} j^2 \right] / \Delta\omega \qquad (10)$$

in which $\Delta\omega = 2\pi\Delta f$ and $\overline{\phi}$ is represented by

$$\overline{\phi}_n = \sum_{j=k-l}^{k+l} \phi_j / (2l+1) \qquad (11)$$

It should be noted that $\hat{t}_n$ in Eq.10 is the peak appearing time of the envelope of the group of harmonic waves which is an approximation of the discrete spectra of earthquake motion, while the right hand side of the same equation consists of the weighted average of $(2l+1)$ phase differences around the center frequency $f_n = f_k$. It is found from Eq.10 that the phase differences nearby the frequency f_k are closely related to the arrival time of the peak of the power of the wave components nearby that frequency, since the envelope function represents the time variation of the power of the wave. If we may divide the frequency range into a number of subdivided regions and approximate the discrete spectra of earthquake motion in each region by that of a group of harmonic waves, it can be verified that the probability distribution of phase differences is closely related to the envelope of time history.

NONSTATIONARY SPECTRAL ANALYSIS USING PHASE DIFFERENCES

The strong motion accelerograms obtained from the 1968 Tokachioki earthquake and its aftershock are used for this study. The accelerograms have been recorded on Hachinohe and Aomori sites located on alluvial grounds (Ref.7).

The arrival time, $\hat{t}_n$, derived in preceding chapter may be influenced by the number l of extracted discrete spectra. It is examined here how l influences the form of the time history. Fig.5 illustrates original time history of Hachinohe and approximated ones in each case of l = 1, 2, 4, and 8. It is found from these figures that even if l is considerably large, approximated time history may fairly reflect the characteristics of original record.

Using the arrival time $\hat{t}_n$, shown in Eq.10, the nonstationary frequency content for the strong motion accelerograms is evaluated. Fig.6 and 7 illustrate $\hat{t}_n$ for the accelerogram of Hachinohe, on the time-frequency plane, in the case for l = 1 and 8, respectively. From the figures, it is found that $\hat{t}_n$ for l = 1 fluctuates excessively while $\hat{t}_n$ for l = 8 varies much more smoothingly with frequency. It is suggested from this that large l is better to evaluate the nonstationary frequency content of earthquake motions. Thus, l = 8 will be used in the following.

In Figs.7, 8, and 9, $\hat{t}_n$ of Hachinohe, Aomori, and Aomori (aftershock), respectively, are plotted, together with the contour representation of evolutionary spectra by Kameda (Ref.1). The nonstationarity of frequency content does not appear too much in Fig.7, while it is recognized in Figs.8 and 9, by observing the variation of $\hat{t}_n$ with frequency. In each figure, $\hat{t}_n$ obtained from phase defferences shows a fairly good agreement with evolutionary spectra as a whole. It is concluded from this, that $\hat{t}_n$ obtained from phase differences is useful for an analysis of nonstationary earthquake motion.

CONCLUDING REMARKS

By making use of the approximation to discrete spectra by a group of harmonic waves, it has been verified that the probability distribution of phase differences is closely related to the envelope of earthquake motion. The relationship has been applied to analyze the nonstationary frequency content of recorded accelerograms, the results of which have been compared with evolutionary spectra. It is concluded that the phase differences are useful to evaluate the nonstationary frequency content, since they have agreed fairly well with evolutionary spectra.

It has been suggested that the concept of phase differences is useful for the simulation of earthquake motions as well as for estimation of dispersion properties of surface waves and for evaluation of the earthquake source mechanisms. There is a possibility that the concept may be applied to such a subject.

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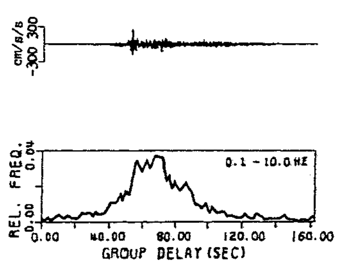

Fig.1 Probability Distribution of Phase Differences (1968 Tokachioki Earthquake,Hachinohe Site)

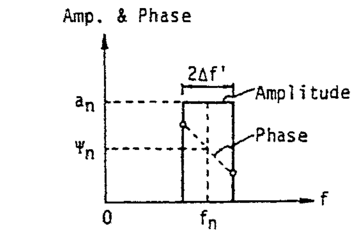

Fig.2 Fourier Spectrum of A Group of Harmonic Waves (Ref.6)

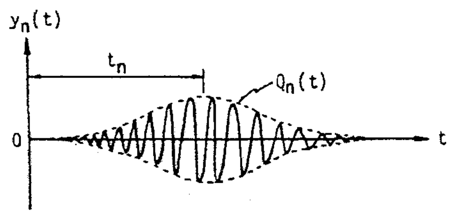

Fig.3 Time History of Eq.5 (Ref.6)

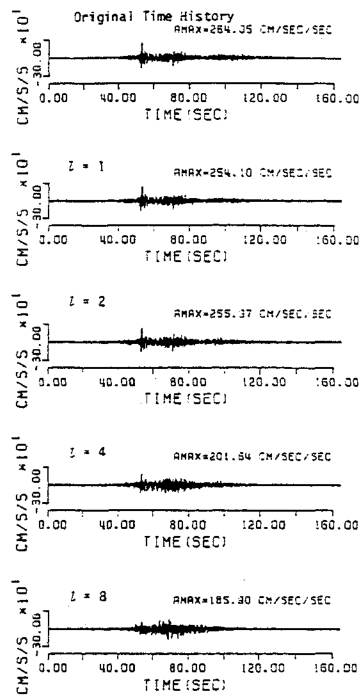

Fig.5 Original and Approximated Time Histories (1968 Tokachioki, Hachinohe Site)

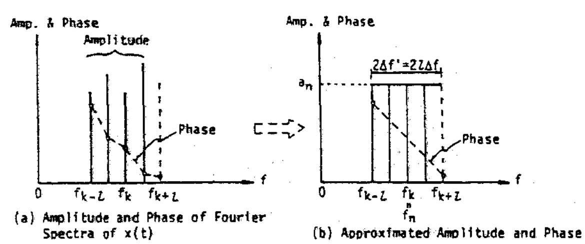

Fig.4 Relation between Discrete and Approximated Fourier Spectra

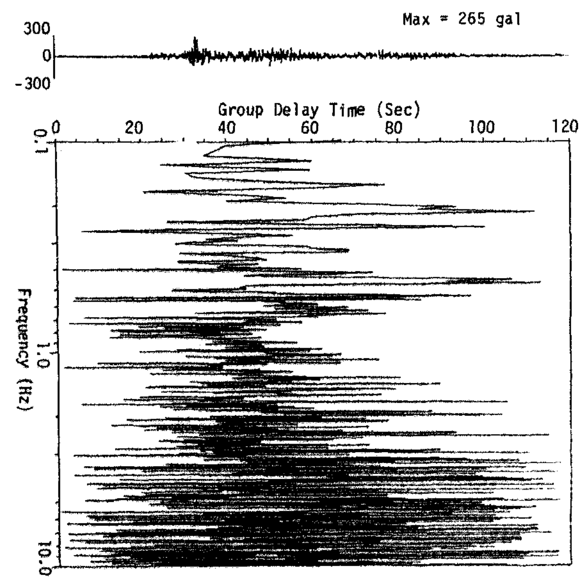

Fig.6 Group Delay Time for Accelerogram of Hachinohe from 1968 Tokachioki Earthquake (l=1)

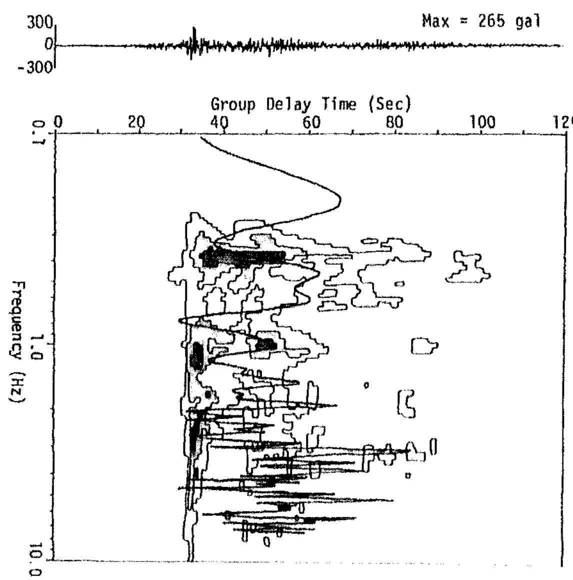

Fig.7 Group Delay Time and Evolutionary Spectrum for Accelerogram of Hachinohe from Tokachioki Earthquake (l=8)

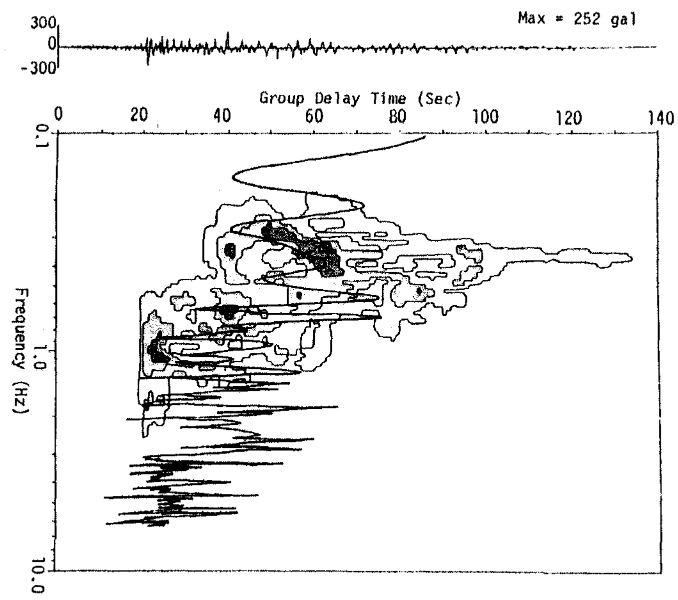

Fig. 8 Group Delay Time and Evolutionary Spectrum for Accelerogram of Aomori from 1968 Tokachioki Earthquake ($l=8$)

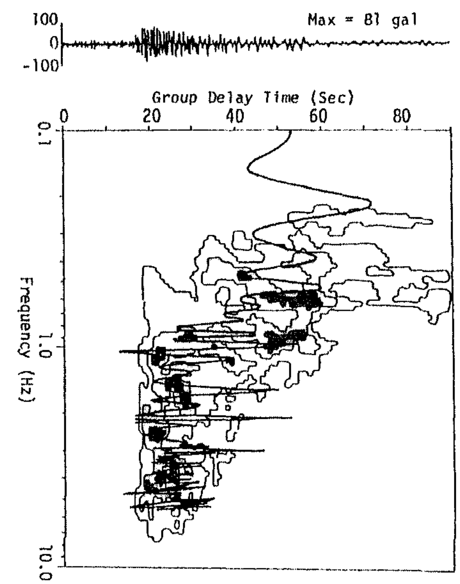

Fig.9 Group Delay Time and Evolutionary Spectrum for Accelerogram of Aomori from 1968 Tokachioki Aftershock ($l=8$)