PHASE PROPERTIES OF EARTHQUAKE GROUND ACCELERATION RECORDS

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SUMMARY

The ground acceleration records of strong-motion earthquakes at moderate distances from the epicentre and on firm ground are modelled as a uniformly modulated white noise random process. It is shown that Fourier transform of the model is a homogeneous random process with frequency as the indexing parameter, and the probability density function of the derivative of the phase of the transformed random process depends upon the first three 'intensity moments'. The earthquake accelerograms of three strong-motion earthquakes are analysed and the probability density functions of the derivative of the phase of the transformed random process are computed and compared with the analytical distribution.

INTRODUCTION

The nature of ground motion at a site during an earthquake is quite complex. The temporal properties of ground motion are generally described by measured, or simulated, acceleration-time records. The Fourier transform of the acceleration-time record, which is a complex function of the frequency, reflects the frequency characterstics of the ground acceleration. In earthquake engineering, most of the attention has been focussed on Fourier amplitude in view of the fact that it closely approximates the undamped response spectra (Refs. 1 and 2). The Fourier phase has generally been ignored as characterless. Recently the characterastics of the phase and phase derivative of the Fourier transform of acceleration-time records of earthquakes have been examined (Refs.3, 4 and 5). By computing the relative frequency functions of the phase and phase derivative of the Fourier transform of several acceleration-time records Ohsaki (Ref.3) came to the following qualitative conclusions:-

(i) the probability distribution of the Fourier phase angle seems to be uniform; and

(ii) the probability distribution of the Fourier Phase differences appears to be normal, or normal like, and resembles the shape of the accelerogram envelope function.

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The theoretical basis of these conclusions was examined by the author (Ref.5). It was shown that if the earthquake accelerograms are modelled as a uniformly modulated, gaussian white noise, the exact probability density of phase and phase derivative of the Fourier transform of the accelerograms can be derived in the framework of random process theory. The distribution of Fourier phase was shown to be uniform. A closed form expression was derived for the derivative of the phase and it was shown that it is not normal. Further it was shown that the probability density function of the phase derivative depends upon the first three moments of the intensity function, which was defined as the square of the envelope function.

In this paper, the probability density functions of the phase derivative of three major earthquake accelerograms are computed numerically, and compared with the theoretical distribution. It is concluded that for strong-motion accelerograms recorded on firm ground and at moderate distances from the epicentre, a uniformly modulated white noise represents a satisfactory stochastic model.

PHASE PROPERTIES OF EARTHQUAKE ACCELEROGRAMS

The statistical analysis of earthquake accelerograms (Ref.6), and the physical models of the source-mechanisms, travel-path and local-soil suggest that the ground motion may be treated as gaussian (Ref.7). Further at moderate distances from the epicentre and on firm ground, the acceleration-time records can be modelled by a uniformly modulated white noise process of the form

$$Y(t) = \zeta(t)\, X(t), \qquad 0 \leq t \leq T \tag{1}$$

where $Y(t)$ is the ground acceleration during an earthquake; $\zeta(t)$ is the deterministic modulating function; $X(t)$ is a white noise random process with power spectral density Φ_0; and T is the duration of the record.

The Fourier transform of $Y(t)$ can be expressed as

$$\tilde{Y}(\omega) = A(\omega) \exp[i\, V(\omega)] \tag{2}$$

where $\tilde{Y}(\omega)$ is a complex random process with ω as the indexing parameter; $A(\omega)$ is the Fourier amplitude; and $V(\omega)$ is the Fourier phase; and $i = \sqrt{-1}$. It follows from Eqs,(1) and (2) that

$$E[\tilde{Y}(\omega)] = 0 \tag{3}$$

and

$$E[\tilde{Y}(\omega)\, \tilde{Y}^*(\omega)] = R(\omega_2-\omega_1) = \frac{\Phi_0}{2\pi} \int_0^T \zeta^2(t) \exp[-i(\omega_2-\omega_1)t]\, dt \tag{4}$$

Setting $\omega_1=\omega_2$ in Eq.(4), we get

$$R_{\tilde{Y}\tilde{Y}}(o) = \frac{\Phi_0}{2\pi}\int_0^T \hat{\zeta}^2(t)\,dt = \frac{\Phi_0}{2\pi}\int_0^T I(t)dt \tag{5}$$

It follows from Eq.(4) that $\tilde{Y}(\omega)$ is a homogeneous random process with p.s.d. - like function

$$\Phi_{\tilde{Y}\tilde{Y}}(\Omega) = \frac{\Phi_0}{2\pi} I(\Omega), \qquad 0 \leq \Omega \leq T,$$
$$= 0, \qquad \text{otherwise.} \tag{6}$$

where Ω is a 'frequency-like' parameter.

It is clear that $V(\omega)$ is also a homogeneous random process. It can be shown that $V(\omega)$ is uniformly distributed in the range $(0, 2\pi)$ (Ref.5). Let the derivative of $V(\omega)$ be defined as

$$V'(\omega) = \frac{dV}{d\omega} = \lim_{\Delta\omega\to 0} \frac{V(\omega+\Delta\omega)-V(\omega)}{\Delta\omega}, \text{ if } V(\omega+\Delta\omega) \geq V(\omega),$$
$$= \lim_{\Delta\omega\to 0} \frac{2\pi + V(\omega+\Delta\omega)-V(\omega)}{\Delta\omega}, \text{ if } V(\omega+\Delta\omega) < V(\omega). \tag{7}$$

It can be shown that the probability density of $V'(\omega)$ is given by (Ref.5)

$$P(v') = \frac{\rho^2}{2\gamma_0}\left\{\frac{1}{\left[(v'-t_1)^2 + \frac{\rho^2}{\gamma_0}\right]^{\frac{3}{2}}} + \frac{1}{\left[(v'+t_1)^2 + \frac{\rho^2}{\gamma_0}\right]^{\frac{3}{2}}}\right\} \tag{8}$$

where

$$\rho^2 = \gamma_2 - \frac{\gamma_1^2}{\gamma_0} \tag{9}$$

$$t_1 = \frac{\gamma_1}{\gamma_0} \tag{10}$$

and γ_j, called the 'intensity moments', in analogy with the 'spectral-moments', are given by

$$\gamma_j = \frac{\Phi_0}{4\pi}\int_0^T t^j\, I(t)\,dt\ , \ j = 0,1,2,\ldots.. \tag{11}$$

It is seen from Eq.(8) that the distribution of $V'(\omega)$ is not normal as suggested by Ohsaki (Ref.3). It is also seen that the probability density function of $V'(\omega)$ depends on the ratios of the first three 'intensity moments' which represent the gross properties of the envelope of the accelerograms.

NUMERICAL RESULTS AND CONCLUSIONS

To check the validity of the theoretical results derived in the preceding section, several strong-motion earthquake accelerograms have been analysed. The envelope of an accelerogram is computed using the relation

$$\zeta(t) = [\, Y^2(t) + \hat{Y}^2(t) \,]^{\frac{1}{2}} \tag{12}$$

where $\hat{Y}(t)$ is the Hilbert transform of Y(t) (Ref.8). The intensity moments γ_j, j=0,1,2 are then computed from Eq. (11). The theoretical probability density function of the phase derivative is computed from Eq.(8) using these intensity moments.

The Fourier phase of the accelerogram, $V(\omega)$, is computed numerically using FFT algorithm. The derivative of the phase, $V'(\omega)$, is computed from Eq.(7). Assuming $V'(\omega)$ to be an ergodic random process, the probability density function of $V'(\omega)$ is computed from the sample function.

The accelerogram, the computed envelope, and the theoretical and computed probability density functions of the phase derivative of the following earthquake records are shown in Figs. 1 to 3:-

1. El Centro , May 18, 1940, SOOE ;
2. Taft , July 21, 1952, N21E: and
3. Parkfield (Shandon) , June 27, 1966, N85E.

It is seen that for the first two accelerograms, which represent ground motion on firm ground at moderate distance from epicentre, the agreement between computed and theoretical probability density function is good. It is concluded that a uniformly modulated white noise model is adequate for such accelerograms. The third accelerogram exhibits narrow band characterstics, and therefore it can be modelled as a uniformly modulated white noise random process. The agreement between theoretical and computed probability density function is poor. It may also be noted that computed probability density functions exhibit bi-modal distribution. It is not possible to explain the second peak. It could be numerical.

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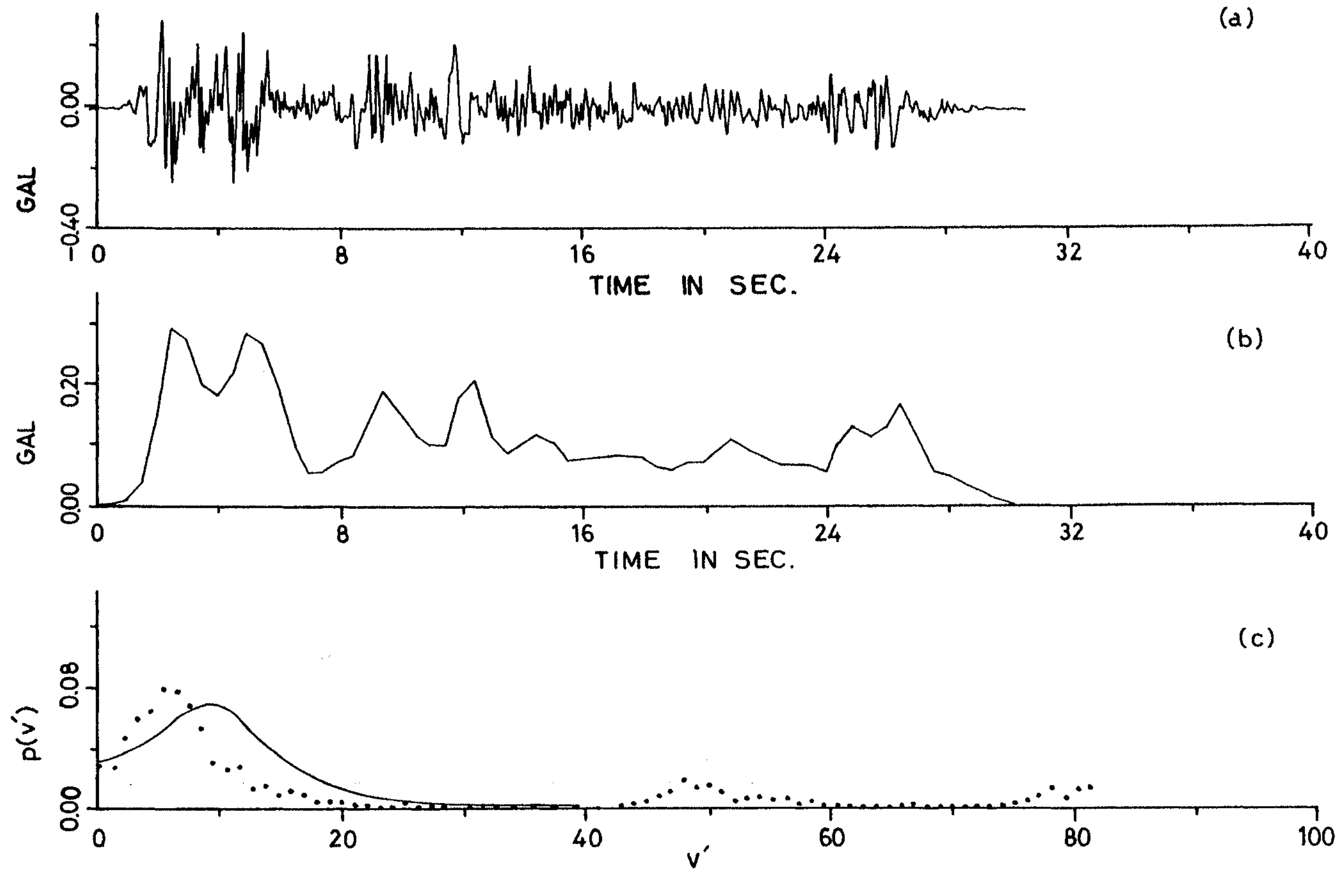

Fig.1. EL CENTRO MAY 18, 1940 S00E (a) ACCELEROGRAM; (b) ENVELOPE; (c) PROBABILITY DENSITY OF PHASE DERIVATIVE, THEORY (—), COMPUTED (···).

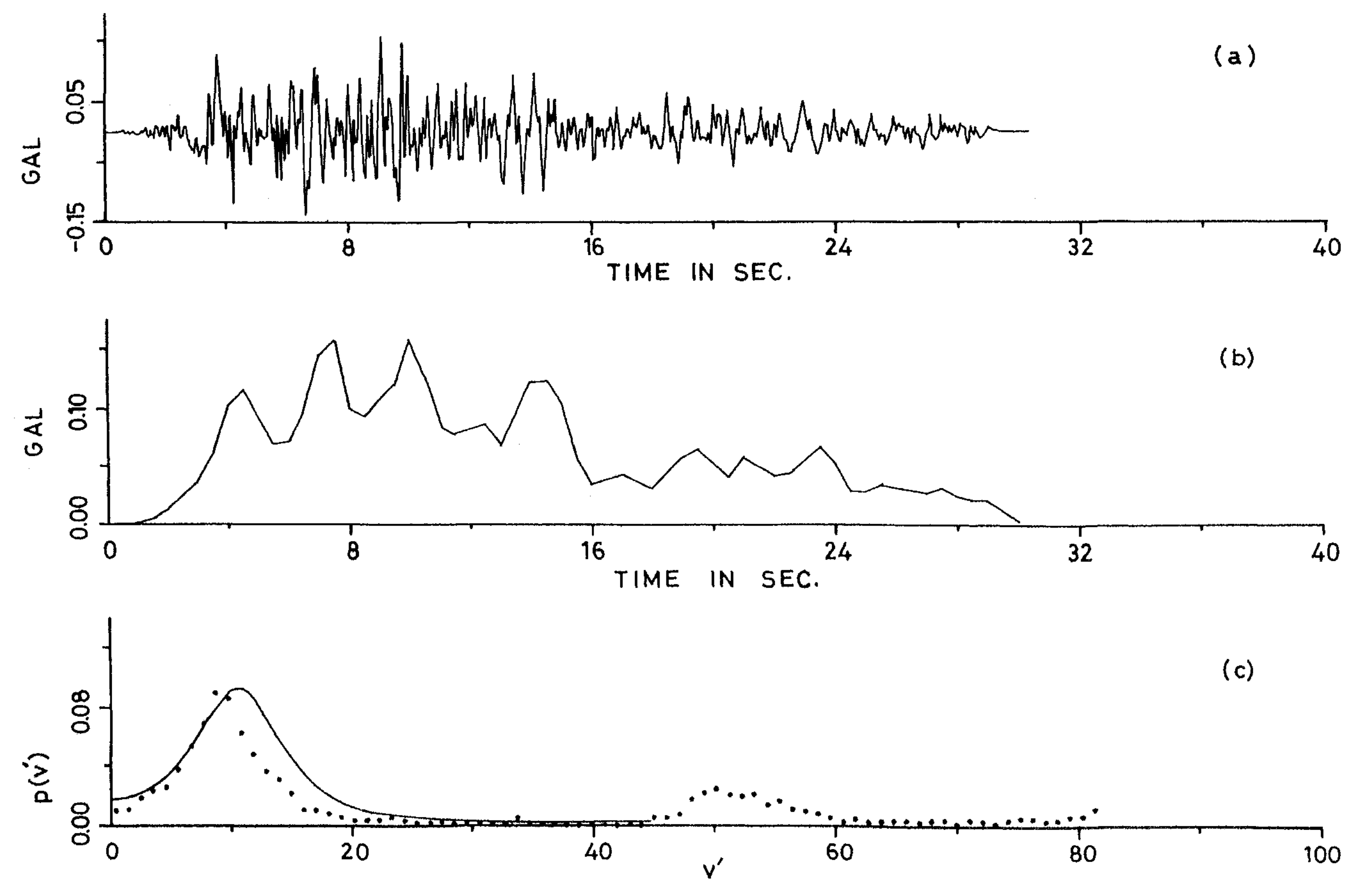

Fig. 2. TAFT JULY 21, 1952 S74W (a) ACCELEROGRAM; (b) ENVELOPE; (c) PROBABILITY DENSITY OF PHASE DERIVATIVE, THEORY (—), COMPUTED (···).

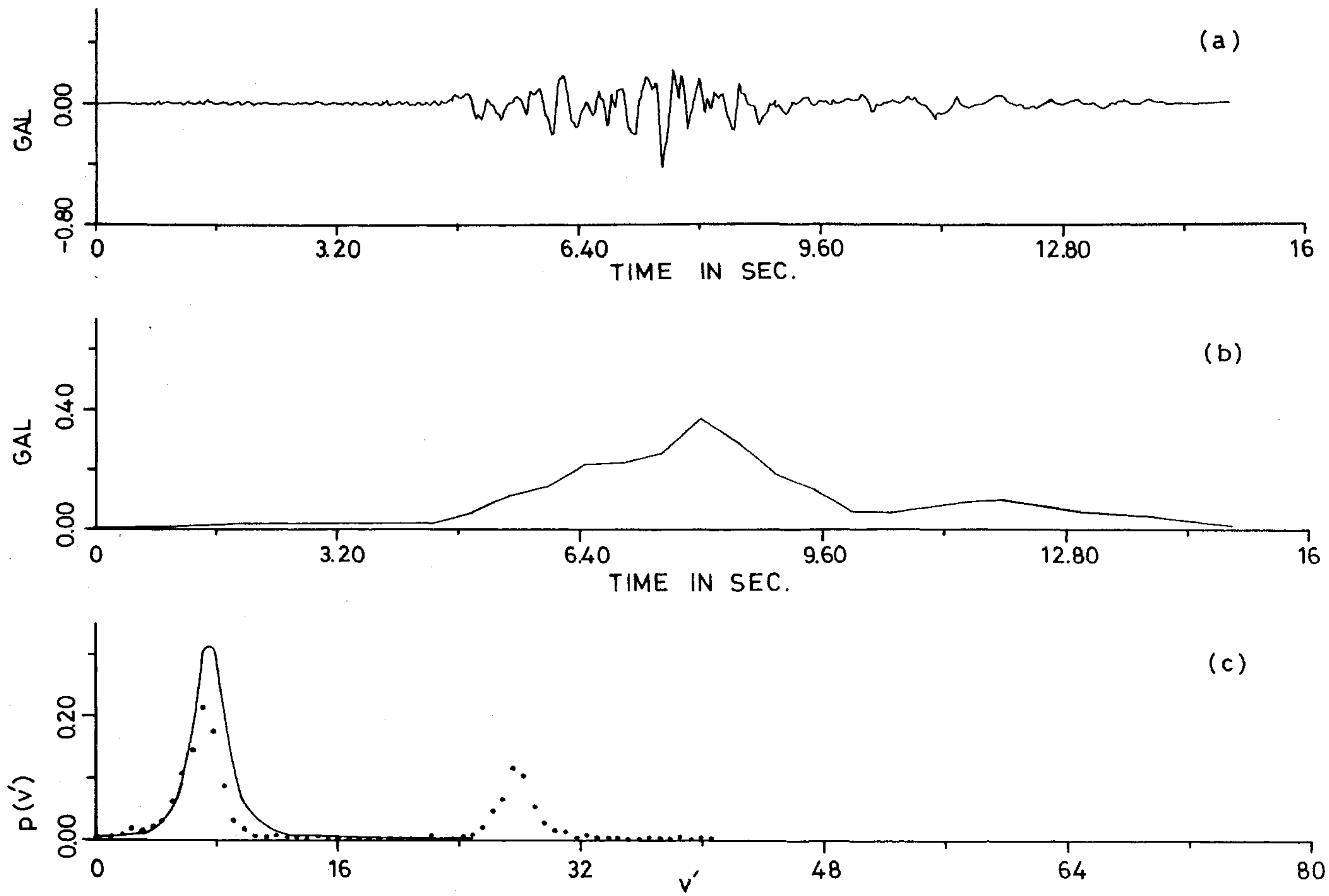

Fig. 3. PARKFIELD (SHANDON), JUNE 27, 1966 N85E (a) ACCELEROGRAM; (b) ENVELOPE; (c) PROBABILITY DENSITY OF PHASE DERIVATIVE THEORY (—), COMPUTED (···).