DESCRIPTION AND REPRESENTATION OF EARTHQUAKE GROUND MOTION RECORDS

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SUMMARY

A relatively simple and straightforward procedure is given for representing analytically defined or data-based covariance kernels of arbitrary random processes in a compact form that allows its convenient use in later analytical random vibration response studies. The method is based on the spectral decomposition of the random process by the orthogonal Karhunen-Loeve expansion and the subsequent use of least-squares approaches to develop an approximating analytical fit for the data-based eigenvectors of the underlying random process. The resulting compact analytical representation of the random process is then used to derive a closed-form solution for the nonstationary response of a damped SDOF harmonic oscillator. The utility of the method for representing the excitation and calculating the mean square response is illustrated by the use of an ensemble of acceleration records from the 1971 San Fernando earthquake.

INTRODUCTION

A state-of-the-art review of current probabilistic methods and their structural and geotechnical applications indicates an active interest and growing recognition by designers of the significance of probabilistic approaches in engineering applications, particularly in the areas of structural reliability and risk analysis. A major part of the impetus behind this strong interest in probabilistic methods is the need to quantify the safety margin in critical engineering facilities subjected to stochastic dynamic environments.

Among the major impediments to a more extensive use of probabilistic methods in structural dynamics applications, particularly those dealing with seismic excitations, are (1) the problems involved in the construction and representation of statistical models based on actual earthquake records, and (2) the computational difficulties encountered in subsequent analytical random vibration analyses to determine closed-form solutions for the response covariance.

This paper presents a procedure for the compact probabilistic representation of earthquake ground motion records in terms of their covariance kernel, while at the same time allowing the use of the resulting characterization to conveniently perform analytical random vibration studies.

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ORTHOGONAL DECOMPOSITION OF RANDOM PROCESSES

Consider a nonstationary random process $\{x(t)\}$ for which a statistically significant number of member function records $x_i(t)$ are available. The covariance kernel for $x(t)$ can be found from

$$K_{xx}(t_1,t_2) = E\{[x(t_1) - \mu_x(t_1)][x(t_2) - \mu_x(t_2)]\} \quad (1)$$

where $E\{.\}$ denotes the ensemble averaged value of the quantity within the braces and $\mu_x(t) = E[x(t)]$.

It is shown from functional analysis that according to Mercer's theorem (Ref. 1), a function $K_{ff}(t_1,t_2)$ which is symmetric, continuous, and non-negative definite in the square region $0 \leq t_1 \leq t_{max}$ and $0 \leq t_2 \leq t_{max}$ may be expanded in an absolutely and uniformly convergent series:

$$K_{ff}(t_1,t_2) = \sum_{k=1}^{\infty} \lambda_k \phi_k(t_1)\phi_k(t_2) \ , \quad (2)$$

where the ϕ's and the λ's are eigenfunctions and eigenvalues, respectively, of the integral equation

$$\phi(t_1) = \frac{1}{\lambda} \int_0^{t_{max}} K_{ff}(t_1,t_2)\phi(t_2)dt_2 \ . \quad (3)$$

A direct consequence of Mercer's theorem is the orthogonal decomposition of a random process $f(t)$ with a covariance function $K_{ff}(t_1,t_2)$ by the orthogonal Karhunen-Loeve expansion (Ref. 2):

$$f(t) = \sum_{k=1}^{\infty} \xi_k \phi_k(t) \ , \quad (4)$$

where

$$\xi_k = \int_0^{t_{max}} f(t)\phi_k^*(t)dt \ ,$$

and the $\phi_k(t)$ are a complete orthogonal set satisfying

$$\int_0^{t_{max}} \phi_i(t)\phi_j^*(t)dt = \delta_{ij} \ , \quad (5)$$

where $\phi_j^*(t)$ = the complex conjugate of $\phi_j(t)$, and δ_{ij} is the Kronecker delta.

SPECTRAL DECOMPOSITION OF EARTHQUAKE RECORDS

Applying the results of the previous section to the case of earthquake records, let the discretized ground acceleration measurements be processed as indicated in Eq. (1) to generate the covariance matrix [C], which is a symmetric square matrix of order n.

The spectral representation for such a matrix is

$$[C] = \sum_{i=1}^{k} \lambda_i \, p_i \, p_i^T + [E_k] \qquad (6)$$

where $\lambda_1, \lambda_2, \ldots, \lambda_k$ are the real and positive eigenvalues of $[C]$, and $p_1, p_2, \ldots, p_k$ are the corresponding normalized eigenvectors of $[C]$ such that $p_i^T \, p = \delta_{ij}$, $i = 1,2,\ldots k$, and $[E_k]$ is a matrix of residual error corresponding to truncating the series in Eq. (6) after term k.

Notice from (6) that if the number k of eigenvectors needed to reconstruct [C] with adequate accuracy is much less than the order n of [C], a significant reduction is obtained in the amount of data needed to represent [C].

ERROR ANALYSIS

A convenient scalar-valued measure of the truncation error in Eq. (6) is given by the normalized "energy" error defined as

$$\varepsilon_k = J_k / J_o \; , \qquad (7)$$

where J_k and J_o are the sum of the squares of the elements of $[E_k]$ and $[C]$, respectively.

It can be shown (Ref. 3) that ε_k is related to the eigenvalues of [C] by

$$\varepsilon_k = 1 - \left(\sum_{j=1}^{k} \lambda_j^2 \right) \Big/ \left(\sum_{j=1}^{n} \lambda_j^2 \right) \; ; \; k = 1,2,\ldots n. \qquad (8)$$

In practical cases, only a small number k of λ's is likely to be computed. It then becomes important to establish upper and lower bounds on the extent of the normalized error corresponding to the use of a different number m of λ's to reconstruct [C]. Then bounds for the case where $m < k$ are

$$(\varepsilon_m)_{min} = 1 - \frac{\lambda_1^2 + \lambda_2^2 + \ldots + \lambda_m^2}{\lambda_1^2 + \lambda_2^2 + \ldots + (n-k+1)\lambda_k^2} \; , \qquad (9)$$

and

$$(\varepsilon_m)_{max} = 1 - \frac{\lambda_1^2 + \lambda_2^2 + \ldots + \lambda_m^2}{\lambda_1^2 + \lambda_2^2 + \ldots + \lambda_k^2} \; . \qquad (10)$$

ANALYTICAL REPRESENTATION OF COMPUTED EIGENVECTORS

As a further step in the data condensation procedure for representing the characteristics of recorded earthquakes in a probabilistic format, it is convenient to determine an approximate analytical expression for each of the eigenvectors to be used for reconstructing [C].

Among the various functions that can be employed to approximate the eigenvectors, Chebyshev polynomials are most suitable due to their orthogonal nature and the fact that their form leads to convenient analytical expressions for the covariance kernels of the dynamic system's response.

Let an estimate of [C] based on using k eigenvectors in Eq. (6) be denoted by $[C_k]$:

$$[C_k] = \sum_{i=1}^{k} \lambda_i \, \underset{\sim}{p}_i \, \underset{\sim}{p}_i^T \, . \tag{11}$$

Using least-squares techniques to fit each $\underset{\sim}{p}_i$,

$$\underset{\sim}{p}_i(t) \approx \hat{\underset{\sim}{p}}_i(t) = \sum_{j=0}^{m_i-1} H_{ij} T_j(t') \, , \tag{12}$$

where

$$t' = 2(t/t_{max}) - 1 \tag{13}$$

the T's are Chebyshev polynomials defined as

$$T_n(\xi) = \cos(n \cos^{-1}\xi) \; ; \; -1 \leq \xi \leq 1 \, , \tag{14}$$

and H_{ij} is the coefficient of the Chebyshev polynomial of order j associated with eigenvector $\underset{\sim}{p}_i$. The H_{ij} can be found by using the orthogonality property of the T's. Making use of Eq. (12) then

$$[\hat{C}_k(t_1,t_2)] = \sum_{i=1}^{k} \lambda_i \sum_{j=0}^{m_i-1} \sum_{\ell=0}^{m_i-1} H_{ij} H_{i\ell} T_j(t_1') T_\ell(t_2') \tag{15}$$

where $0 \leq t_i \leq t_{max}$; i = 1,2.

NONSTATIONARY RESPONSE OF LINEAR SYSTEMS

Consider the nonstationary response y(t) of a linear single-degree-of freedom (SDOF) system, characterized by an impulse function h(t), under the action of a nonstationary stochastic process {x(t)} whose covariance kernel is expressed as $K_{xx}(t_1,t_2) = [\hat{C}_k(t_1,t_2)]$.

For convenience, assume that the input is a zero-mean process. Then the covariance of the system output is

$$E[y(t_1)y(t_2)] = \int_0^{t_1} \int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)K_{xx}(\tau_1,\tau_2)d\tau_2 \, d\tau_1 \, , \tag{16}$$

with

$$h(t) = -\frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t \, , \tag{17}$$

where ζ = ratio of critical damping , ω_n = natural frequency, $\omega_d = \omega_n\sqrt{1-\zeta^2}$. Replacing K_{xx} in equation (16) by its representation in equation (15), the covariance of the response will then be given by

$$E[y(t_1)y(t_2)] = \sum_{i=1}^{k} \lambda_i \sum_{j=0}^{m_i-1} \sum_{\ell=0}^{m_i-1} H_{ij}\, H_{i\ell}\, Y_{j\ell}(t_1,t_2) \tag{18}$$

where

$$Y_{j\ell}(t_1,t_2) \equiv \frac{t_{max}^2}{4\omega_d} \exp\,[\zeta\omega_n(t_{max} - t_1 - t_2)]F_j(t_1)F_\ell(t_2) \;, \tag{19}$$

and $F_j(t)$ are algebraic expressions (Ref. 4) dependent on the dynamic system parameters.

The mean-square response $E[y^2(t)]$ is obtained from (18) by setting $t_1 = t_2 = t$.

Note from (18) that the covariance of the response is made up of two non-interacting groups of terms: (1) the $H_{j\ell}$'s which depend only on the excitation characteristics, and (2) the $Y_{j\ell}$'s which depend only on the dynamic system characteristics.

APPLICATIONS

A covariance matrix [C] of order (501 x 501) representing the first 20 seconds of an ensemble of ground acceleration records corresponding to the 1971 San Fernando earthquake (Ref. 5) was constructed and is shown in Fig. 1.

The eigenvectors of [C] were computed and a typical one is shown in Fig. 2. The least-squares fit $\hat{p}_i(t)$ of a typical eigenvector is shown in Fig. 3 and the corresponding error between p_i and $\hat{p}_i$ is shown in Fig. 4. Note from Fig. 4 that the main contribution to the mismatch between p_i and $\hat{p}_i$ is in the high frequency components.

Using the procedure indicated in Eq. (11) to represent $[C_k]$, one obtains the results shown in Fig. (5) corresponding to the surface associated with the approximate covariance matrix $[C_{25}]$ based on using k = 25 exact eigenvectors.

Following the steps indicated in Eq. (15), least-squares techniques are used to obtain analytical expressions for the needed exact eigenvectors and the corresponding surface $[\hat{C}_{25}]$ based on using k = 25 approximate eigenvectors $\hat{p}_i$ is shown in Fig. 6.

A measure of the rate of convergence of $[C_k]$ which is based on a subset k of the n eigenvectors to [C] which corresponds to using all n eigenvectors is shown in Fig. 7, where the indicated bounds on ε_k are based on the expressions in Eq. (9) and (10). It is obvious that within ≈ 25 terms, the reconstructed $[C_k]$ is a very good estimate of the exact $[C]$; in other words, the first 25 (out of 501) eigenvectors embody practically all of the statistical information contained in $[C]$.

To illustrate the validity of the method under discussion for probabilistic structural dynamics, the procedure indicated in Eq. (18) is used to analytically determine the transient m.s. response of a linear, damped SDOF harmonic oscillator to the stochastic excitation represented by $[C]$. Figure 8 shows typical analytical results (Fig. 8c) and comparable numerical integration results based on using the exact $[C]$ (Fig. 8a) and the approximating $[C_k]$ (Fig. 8b).

The use of the results of this study to generate probabilistic response spectra is illustrated by the results in Fig. 9 where the extreme values $S_V \equiv \sqrt{\max_t E[\dot{y}^2(t)]}$ are plotted as a function of the oscillator period and damping parameter.

ACKNOWLEDGMENT

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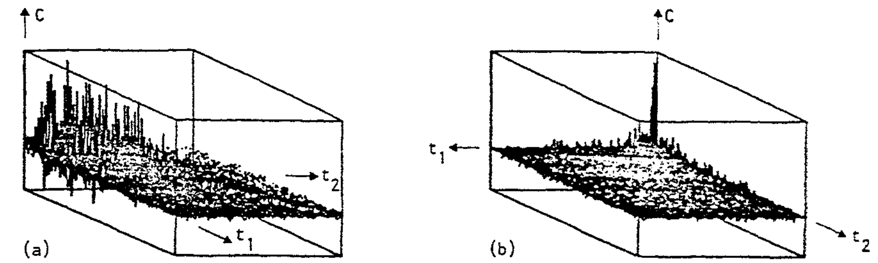

FIGURE 1. COVARIANCE MATRIX [C] SURFACE FOR THE SAN FERNANDO EARTHQUAKE

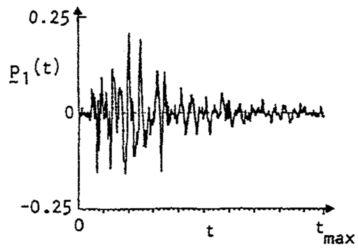

FIGURE 2. TYPICAL EIGENVECTOR $p_1(t)$ OF [C]

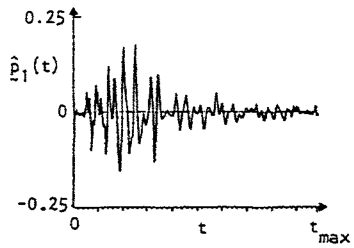

FIGURE 3. LEAST-SQUARES FIT $\hat{p}_1(t)$ OF A TYPICAL EIGENVECTOR

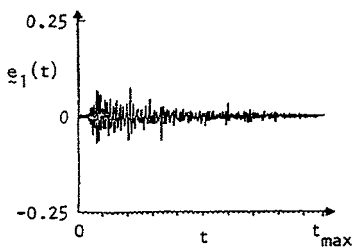

FIGURE 4. LEAST-SQUARES ERROR $e_1(t)$ BETWEEN $\hat{p}_1(t)$ AND $p_1(t)$

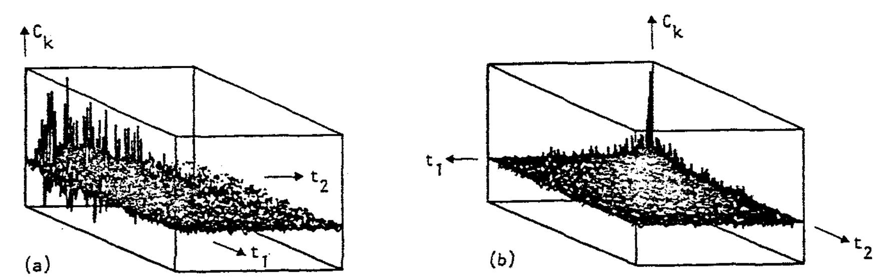

FIGURE 5. APPROXIMATE COVARIANCE $[C_k]$ SURFACE BASED ON USING k = 25 EXACT EIGENVECTORS

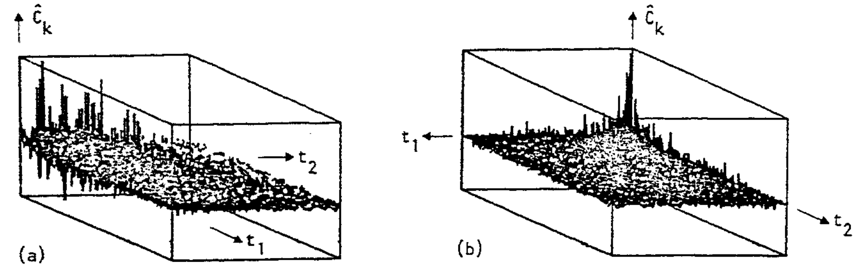

FIGURE 6. APPROXIMATE COVARIANCE $[\hat{C}_k]$ SURFACE BASED ON USING APPROXIMATING ANALYTICAL EXPRESSIONS FOR $k = 25$ EIGENVECTORS

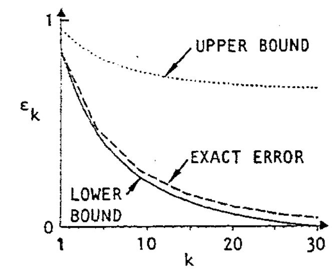

FIGURE 7. RATE OF CONVERGENCE OF $[C_k]$ TO $[C]$ AND ERROR BOUNDS ON ε_k

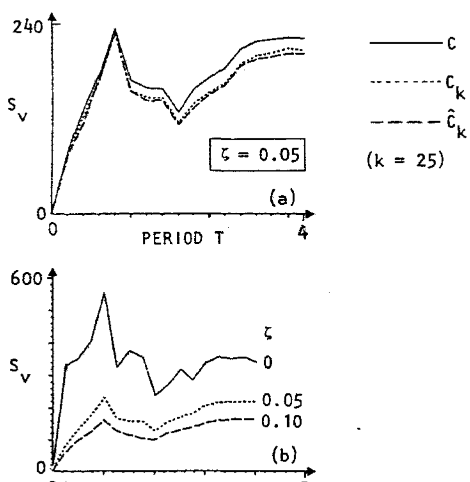

FIGURE 9. EXTREME VALUES OF THE NONSTATIONARY MEAN-SQUARE VELOCITY RESPONSE OF A SDOF OSCILLATOR

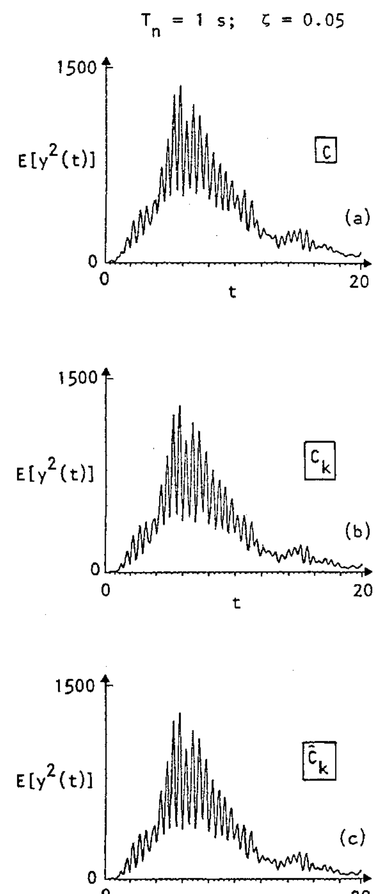

FIGURE 8. NONSTATIONARY RESPONSE OF A SDOF HARMONIC OSCILLATOR