A STUDY ON THE FOURIER ANALYSIS OF NONSTATIONARY SEISMIC WAVES

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SUMMARY

The relationship between the nonstationary character of seismic waves and information obtained from the Fourier analysis of the waves, including information on phases, is presented. Several applications of the phase information to earthquake engineering problems are also shown.

INTRODUCTION

For one function of a given seismic wave, two complex functions, the Fourier transform in the frequency domain and the complex envelope in the time domain, can be introduced to present its properties. Although in wave analyses, information on the amplitudes of the functions is generally utilized in order to describe the nature of waves, information on the phases is not used sufficiently. This may be due to the complicated characteristics of phase spectra. For such phases, the authors have shown that their meanings are made clearer if the derivatives $2\pi fgr(t)=d\psi(t)/dt$ and $tgr(\omega)=d\phi(\omega)/d\omega$ are considered, where $\psi(t)$ and $\phi(\omega)$ are the phases of the complex envelope and the Fourier transform respectively, and that the nonstationary nature of seismic waves can be efficiently explained by means of $fgr(t)$ and $tgr(\omega)$.[1,2,3]

In this paper, the meanings of phases $fgr(t)$ and $tgr(\omega)$, the relation between the nonstationary nature of seismic waves and the phases, and the applications of the phase information to earthquake engineering problems are summarized.

TWO COMPLEX FUNCTIONS $\bar{f}(t)$ AND $\bar{F}(\omega)$, AND THEIR PHASES $fgr(t)$ AND $tgr(\omega)$

For a given time function $f(t)$, we can introduce two complex functions $\bar{f}(t)$ and $\bar{F}(\omega)$. The method to calculate the complex functions $\bar{f}(t)$ and $\bar{F}(\omega)$ is shown in Fig.1. From $F(\omega)$, the Fourier transform of $f(t)$, $\bar{F}(\omega)$ is determined to be a causal function in the frequency domain. Accordingly, the real and imaginary parts of $\bar{f}(t)$, which is obtained by the inverse Fourier transform of $\bar{F}(\omega)$, must satisfy the Hilbert transform relation. Therefore, imaginary part of $\bar{f}(t)$ is the Hilbert transform of $f(t)$, and thus Fig.1 becomes the method to calculate both the complex

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envelope $\bar{f}(t)$ and the Hilbert transform of $f(t)$.

Functions $fgr(t)$ and $tgr(\omega)$ are defined as the derivatives of phases $\psi(t)$ and $\phi(\omega)$, as shown in Fig.2. The meanings of these functions are also shown in the figure. As shown in the figure, the first and second moments of $|\bar{F}(\omega)|^2$, which are the center of gravity and the width of $|\bar{F}(\omega)|^2$ respectively, can be determined by the values of $fgr(t)$ and $|\bar{f}(t)|^2$ in the time domain. Therefore, the contribution of $|\bar{f}(t)|^2$ to $|\bar{F}(\omega)|^2$, that is the time-dependent character of $|\bar{F}(\omega)|^2$, can be understood in terms of $fgr(t)$. Similarly, the first and second moments of the envelope $|\bar{f}(t)|^2$, which are the center of gravity and the width of $|\bar{f}(t)|^2$ respectively, can be determined by the values of $tgr(\omega)$ and $|\bar{F}(\omega)|^2$ in the frequency domain. Accordingly, the contribution of $|\bar{F}(\omega)|^2$ to $|\bar{f}(t)|^2$, that is the frequency-dependent character of $|\bar{f}(t)|^2$, can be understood in terms of $tgr(\omega)$.

In general, $fgr(t)$ and $tgr(\omega)$ fluctuate in each domain, as shown in Fig.3. However, the fluctuations of $fgr(t)$ and $tgr(\omega)$, whose ordinates are frequency and time respectively, look meaningful. Namely, there seems to be a certain relation between the fluctuation of $fgr(t)$ and the change of the frequency contents of $f(t)$. The rough displacement on the time axis of the wave components within a certain frequency range can be understood by the value of $tgr(\omega)$. Therefore, $tgr(\omega)$ and $fgr(t)$ become measures by which the nonstationary nature of seismic waves can be evaluated numerically.

It is important to note that these meaningful fluctuations of phases $fgr(t)$ and $tgr(\omega)$ can be obtained if and only if the time function $f(t)$ possesses clear nonstationary characteristics. As is shown in Fig.4, for most strong ground motions, it is difficult to determine such characteristics, because, generally, there is no such clear nature involved in the motions.

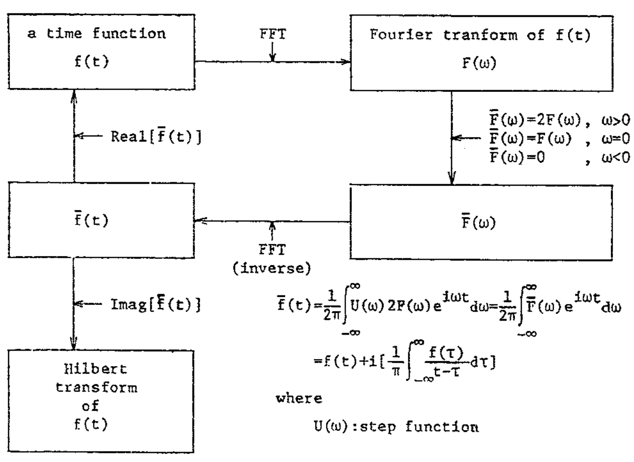

Fig.1 FLOW CHART to Calculate $\bar{f}(t)$ and/or the Hilbert Transform of $f(t)$

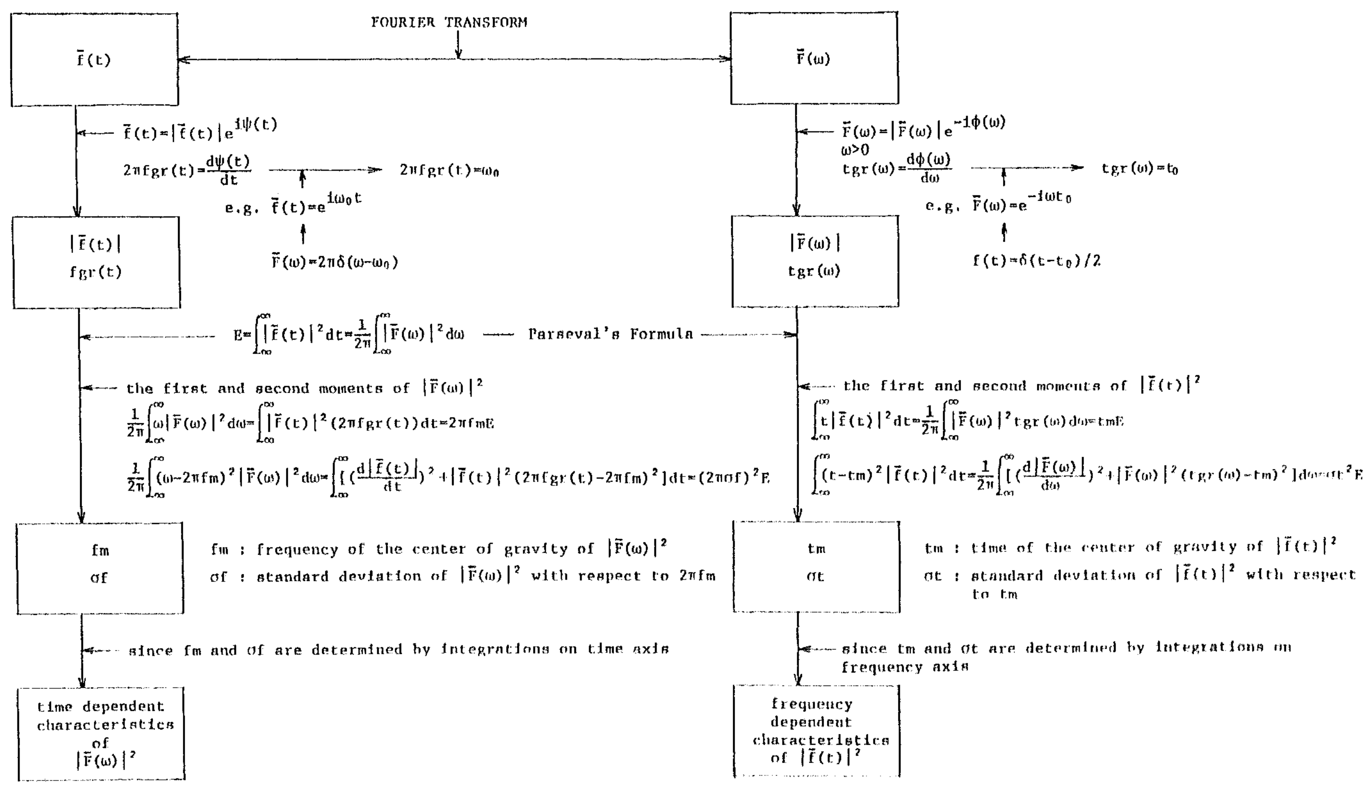

Fig.2 Phase information obtained from fgr(t) and tgr(ω)

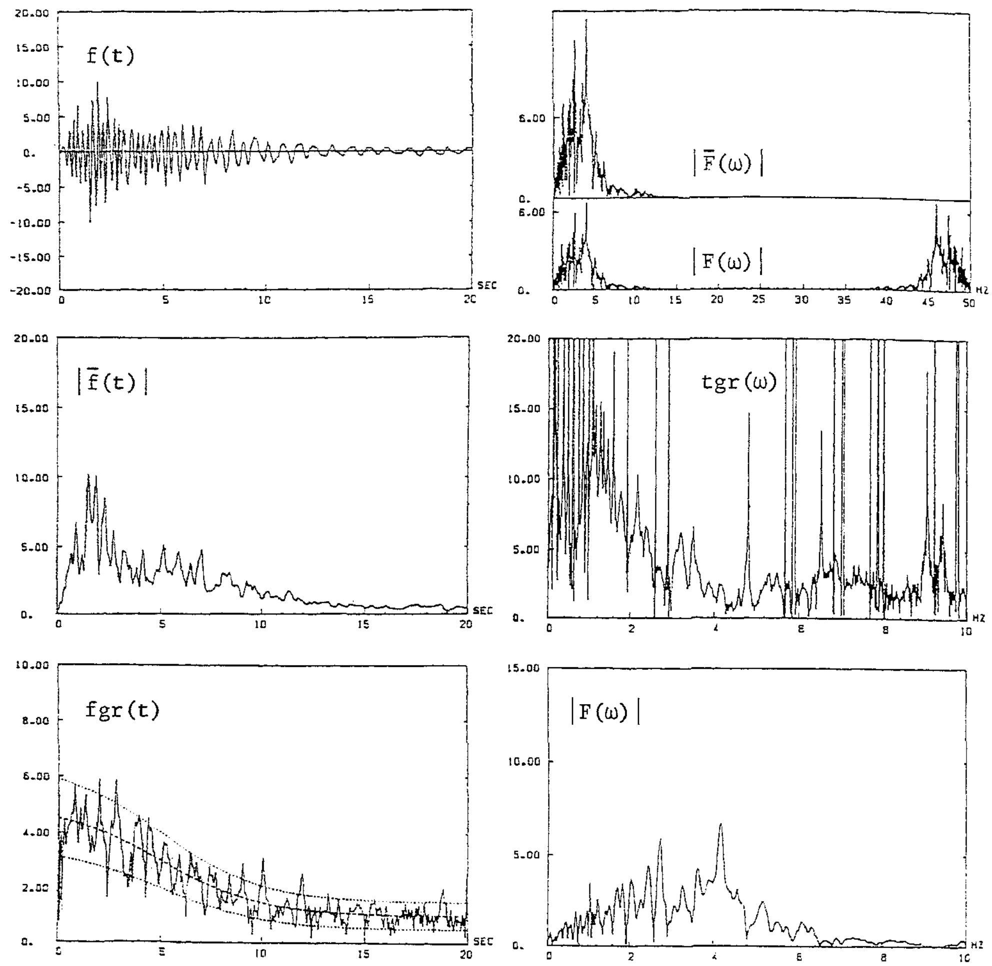

Fig.3 Simple Example of Fluctuations fgr(t) and tgr(ω)
The shape of $|\bar{f}(t)|$ can be deduced roughly from the fluctuation of tgr(ω) weighted by $|\bar{F}(\omega)|$. Similarly, the shape of $|F(\omega)|$ can be deduced roughly from the fluctuation of fgr(t) weighted by $|\bar{f}(t)|$.

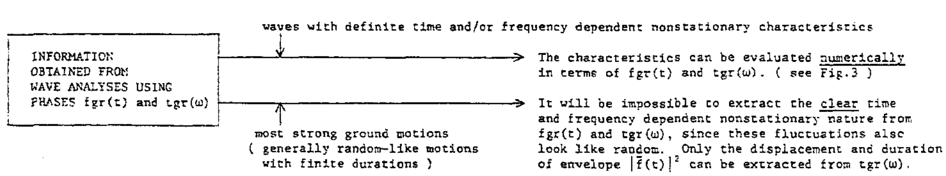

Fig.4 Information extracted from wave analyses using phases fgr(t) and tgr(ω)

APPLICATIONS OF PHASE INFORMATION

Some applications of the phase information to earthquake engineering problems are summarized in Fig.5.

For the wave analyses of seismic waves, fgr(t) gives information on a nonstationary nature similar to the time axis of a running spectrum, because fgr(t) represents the contribution of $|\bar{f}(t)|^2$ to $|\bar{F}(\omega)|^2$. Therefore, fgr(t) becomes a measure which numerically presents the time-dependent structure of the spectrum $|\bar{F}(\omega)|^2$.

Because the change of frequency contents can be examined by fgr(t), it may be possible to observe the stiffness degradation of structures subjected to strong ground motions in terms of fgr(t). An example is shown in Fig.6. The acceleration was recorded on the 9th floor of a 9-story SRC building during the Miyagi-oki Earthquake in Japan in 1978. In the figure of fgr(t), the value of fgr(t) fluctuation becomes smaller with the lapse of time. This change of fgr(t) seems to give information on the stiffness degradation of the building during the earthquake.

In general, frequency components interfere with each other, and the Fourier spectrum is deformed by the interferences. As fgr(t) is determined for each time, it will be possible to evaluate the frequency contents which are not affected by the interferences among different frequency components and/or among the same frequency components with different phase angles. An example is shown in Fig.6. In the figure, the trough seen in the Fourier spectrum around 1.1 Hz disappears in the density of fgr(t). The difference of the densities of fgr(t) for two intervals, from 0 to 10 s and from 10 to 20 s, well explains the change of the acceleration with the lapse of time. It will be useful particularly for the analyses of microtremors with considerably long durations to utilize fgr(t), which expresses the transient nature of frequency contents.

As mentioned above, the time-dependent character of frequency contents can be determined in terms of fgr(t). It is also possible to simulate a nonstationary seismic wave by assuming the fluctuation of fgr(t).[3] In fact, the wave in Fig.3 is a simulated one to produce the fgr(t) characters which are shown by the broken and dotted lines in the figure of fgr(t). As the simulated wave possesses a definite nonstationary nature, and as it is possible to simulate many such waves, a set of random waves can be obtained which will be useful for random vibration and nonlinear response problems.

For the wave analyses of seismic waves, tgr(ω) gives information on a nonstationary nature similar to the frequency axis of a running spectrum, because tgr(ω) represents the contribution of $|\bar{F}(\omega)|^2$ to $|\bar{f}(t)|^2$. Therefore, tgr(ω) becomes a measure which numerically presents the frequency-dependent structure of the envelope $|\bar{f}(t)|^2$.

The dispersion of $|\bar{f}(t)|^2$ becomes a measure of the duration of f(t). Therefore, from $\sigma t(\omega)$, information on the frequency-dependent duration can be obtained. In Fig.7, the frequency-dependent durations of a seismic wave (El-Centro NS) and a simulated wave to possess the same velocity response spectrum as that of the seismic wave are shown. In the figure, two horizontal lines show the durations determined by the maximum values of the waves. The great difference between both durations of the seismic wave, which is not observed in the simulated wave, may possibly express the character of the earthquake motion well.

The meanings of the phase of transfer functions are also made clearer if their tgr(ω) are considered. In Fig.8, the tgr(ω) of SDOF vibratory systems are shown. The phase transfer functions have peaks at natural frequencies similar to the absolute values of transfer functions. The total time shift of a response wave to a corresponding input wave is determined by the phase transfer function weighted by the amplitude of the transfer function. In the right hand figure, where the abscissa represents the natural frequencies of the SDOF systems, broken lines show the time shifts calculated by the transfer functions assuming that the input wave is a white noise, and solid lines show the time shifts for the seismic wave input shown in Fig.7.

Similar to the simulation of nonstationary waves based on fgr(t), it is possible to simulate nonstationary seismic waves assuming the fluctuation of tgr(ω) in the frequency domain. Similarly, a set of random waves with a definite nonstationary character can be obtained which will be useful for random vibration and nonlinear response problems.

CONCLUSIONS

In this paper, the Fourier analyses of nonstationary seismic waves, the relation between the nonstationary character and phases introduced in the Fourier analyses, and applications of the phase information to earthquake engineering problems are summarized.

It is explained in particular detail that the nonstationary nature of seismic waves can be evaluated numerically by phases fgr(t) and tgr(ω). Since fgr(t) and tgr(ω) can easily be calculated by FFT as shown in Fig.1, these quantities will become simple but sufficiently good measures expressing the nonstationary nature of seismic waves.

The considered ways to apply the phase information to earthquake engineering problems are summarized in Fig.5.

REFERENCES

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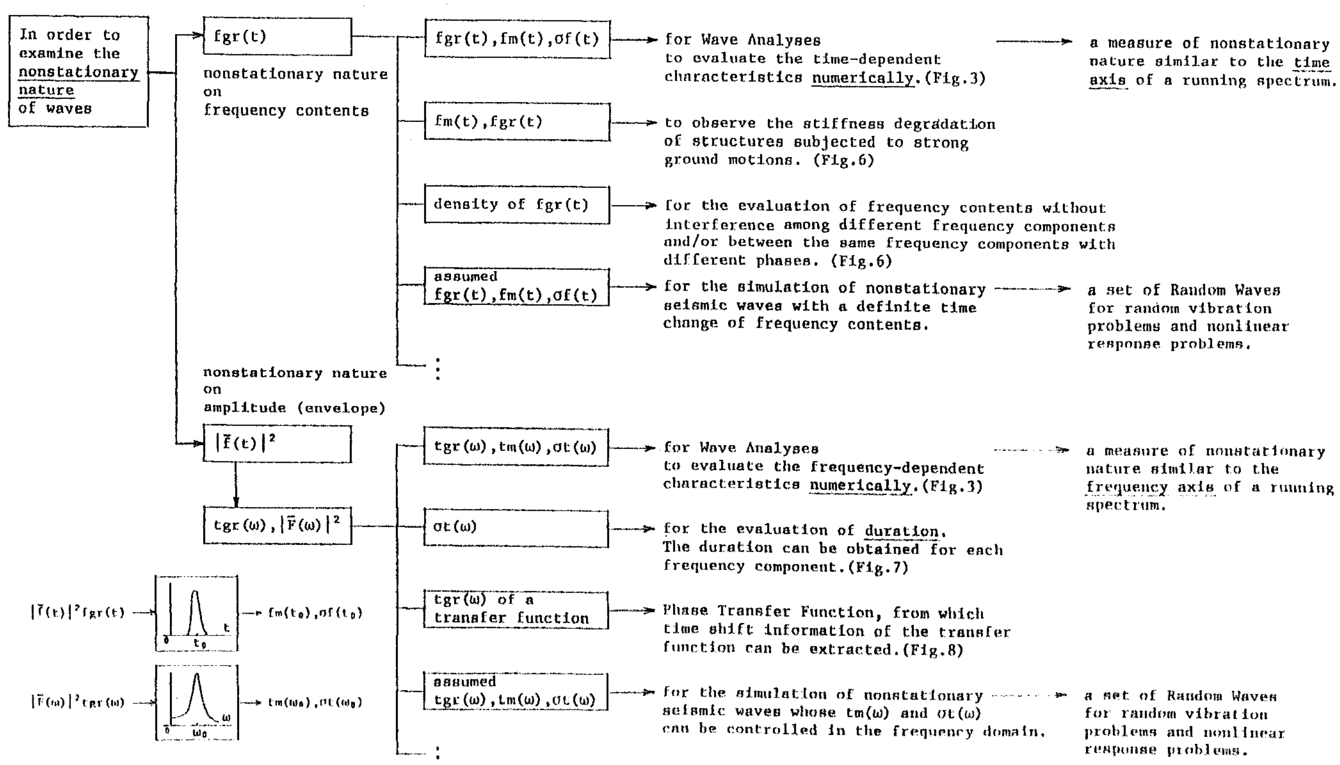

Fig.5 Applications of Phase Information to Earthquake Engineering Problems

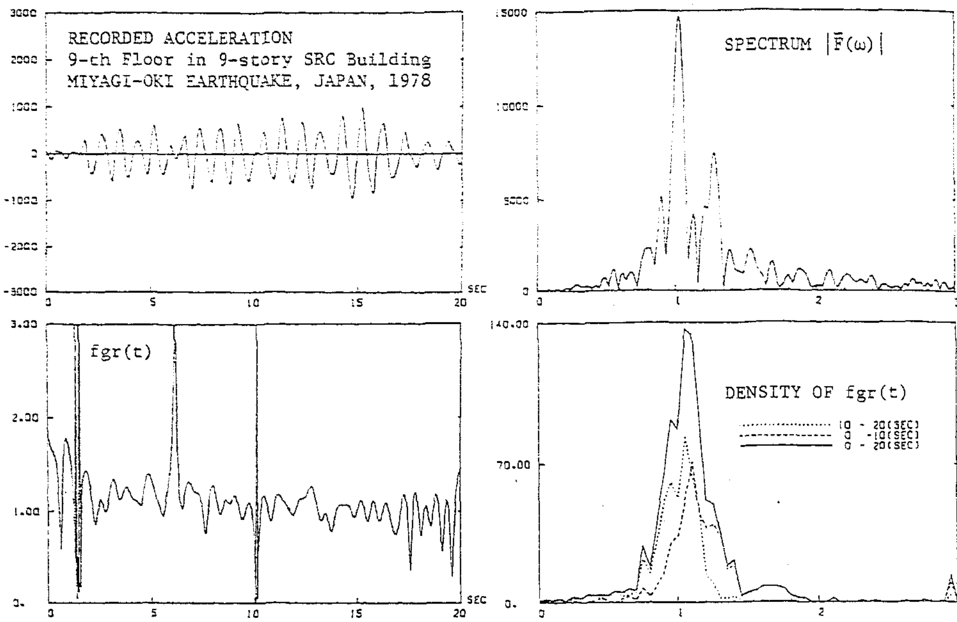

Fig.6 Change of fgr(t) due to the stiffness degradation of a structure subjected to a strong ground motion and the comparison of $|F(\omega)|$ with the density of fgr(t).

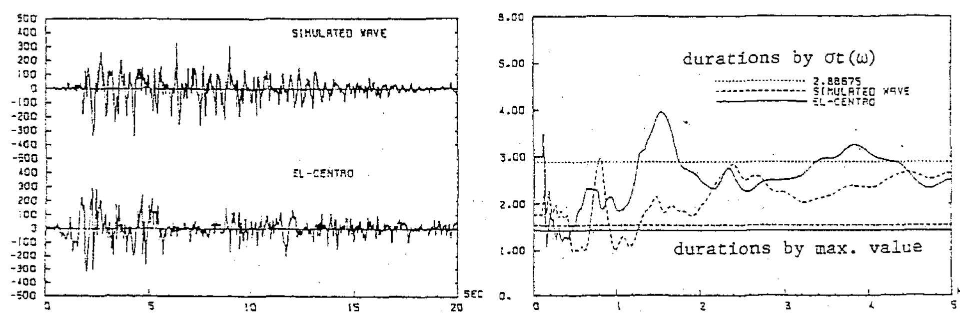

Fig.7 $\sigma t(\omega)$ and Duration of wave

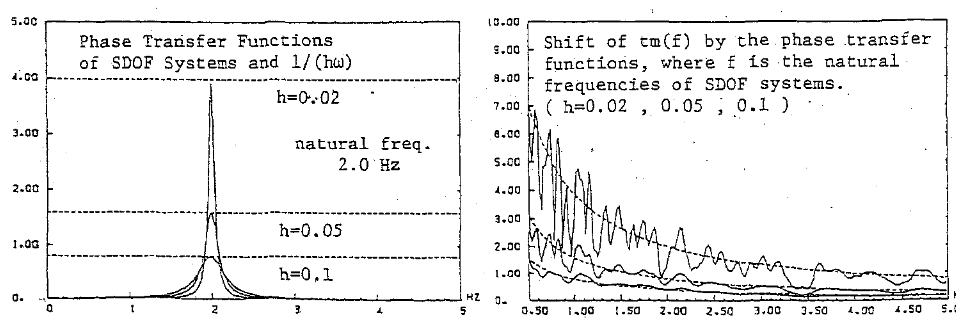

Fig.8 PHASE TRANSFER FUNCTION