NONSTATIONARY SPECTRAL ANALYSIS AND MODELING
OF THREE-DIMENSIONAL SEISMIC GROUND MOTION

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SUMMARY

Three models for combining earthquake records are presented. They are based on a nonstationary spectral resolution of the three recorded components at a site. The first model generates an acceleration record along directions that optimize in time and frequency the spectral characteristics of the recorded accelerograms. This nonstationary model represents the ground motion by a single component that fully incorporates the characteristics of the three original records. Accelerograms of the two remaining models are compatible with directions that optimize the instantaneous and total energies of the recorded components. The proposed models are particularly suitable for multi-component seismic design of structures.

INTRODUCTION

The earthquake ground motion at a particular location may have different characteristics depending on the instrument's orientation. The orientation usually is unrelated to any physical aspect of the earthquake mechanism and has little meaning to the designer. Also, the seismic parameters of each of the recorded components at a site are usually not identical; for example, peak values, durations, energies, and spectral contents are usually different. In addition, the three components always have some amount of cross-correlation. In multi-component design of critical structures the effects of cross-correlation may be significant and should not be disregarded a priori.

This paper proposes three different methods for solving for the influence of instrument orientation on the earthquake motion. The methods are based on a time-varying spectral resolution of the components.

BACKGROUND

Multi-dimensional ground motion has been the subject of many recent publications. Hadjian (Ref. 1) defined a time domain axes rotation that makes the correlation coefficient between two components of ground motion zero. Penzien and Watabe (Ref. 2) introduced the concept of principal axes of the ground motion, which they defined as those axes along which the variances of the three components of an earthquake have stationary values (or, equivalently, the covariances are zero). Chrostowski and Lee (Ref. 3) argued that such rotations of axes do not actually solve for the effect of component cross-correlation on the response of a structure. From another approach, Shoja-Taheri and Bolt (Ref. 4) proposed a method for generating a strong

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motion accelerogram along a direction that maximizes the Fourier spectra of two recorded horizontal components.

THE BASIC SOLUTION

Our basic solution for the modeling of three-dimensional ground motion derives from the equation for the mean response of a linear system to multiple input exitations. For three ground motion inputs, that equation can be written in the form:

$$\overline{y}(t) = \{\overline{y}^2_{x_r x_r}(t) + \int_{-\infty}^{+\infty} |S_{x_r x_s}(\omega)| [H^*_{x_r}(\omega) H_{x_s}(\omega) e^{i\theta(\omega)} + H^*_{x_s}(\omega) H_{x_r}(\omega) e^{-i\theta(\omega)}] d\omega\}^{1/2} \qquad r,s = 1,2,3;\ r \neq s \tag{1}$$

where $H_{x_r}(\omega)$ is the frequency response function due to the ground motion input $x_r(t)$; $S_{x_r x_s}(\omega)$ with $r \neq s$ is the cross-spectral density function between the components $x_r(t)$ and $x_s(t)$; $\theta(\omega)$ is the phase angle; and ω is the angular frequency. The symbols $|.|$ and $*$ denote the modulus and complex conjugate, respectively. Also, the summation convention for repeated indices applies. It is clear from Eq. (1) that if $|S_{x_r x_s}(\omega)|$ is zero everywhere, the cross-integral on the right-hand side of Eq. (1) drops out. Accordingly, an axes rotation that makes the amplitude of the cross-spectral densities zero everywhere guarantees that the correlation between the input components will have no effects on the system response.

More importantly, the matrix $\underset{\sim}{S}$ whose elements are the moduli of the spectra and cross-spectra of the three ground motion inputs is singular and has a rank of one. This important property of the matrix $\underset{\sim}{S}$ can be proven by showing that any row of $\underset{\sim}{S}$ is actually a scalar multiple of any other row. It follows that the same axes rotation that makes the cross-spectra zero will also make two principal spectra zero, thus resulting in a representation of the ground motion with a unique spectrum (or corresponding time history).

This concept is the starting point of the proposed methodology. In addition, the nonstationarity of seismic records is recognized in this study; therefore, the preceding solution is generalized by the introduction of a time-dependent power spectrum that displays the nonstationarity in the spectral content of each of the recorded components and permits a combination of these records in the time and frequency domains simultaneously.

NONSTATIONARY SPECTRAL RESOLUTION OF THREE-DIMENSIONAL EARTHQUAKE MOTION

Consider the earthquake motion at a point as a vector with three mutually orthogonal components of acceleration $x_1(t)$, $x_2(t)$, and $x_3(t)$. A nonstationary spectral description of these components is obtained through the 3×3 physical spectrum matrix $\underset{\sim}{P}(f,t;w)$ whose generic elements are defined by

$$P_{rs}(f,t;w) = F_r(f,t;w)F_s^*(f,t;w) \qquad r,s = 1,2,3 \tag{2}$$

where $F_r(f,t;w)$ is the running Fourier transform of the earthquake component $x_r(t)$; that is,

$$F_r(f,t;w) = \int_{-\infty}^{+\infty} x_r(u)w(t-u)e^{-i2\pi fu}du \tag{3}$$

in which t is time, f is frequency, and w(t-u) is a time-window function that satisfies the normalization condition $\int_{-\infty}^{+\infty} |w(t)|^2 dt = 1$.

The concept of a physical spectrum is due to Mark (Ref. 5). The engineering significance of the physical spectrum for nonstationary phenomena is analogous to that of the power spectrum for stationary phenomena. Also, the physical spectrum concept applies to a single event x(t) as well as to the ground motion process {x(t)}. In essence, the physical spectra (i.e., the diagonal elements of $\underset{\sim}{P}$) represent a simultaneous decomposition in time and frequency of the instantaneous energy of each component $x_r(t)$; the cross-physical spectra (i.e., the off-diagonal elements of $\underset{\sim}{P}$) represent a decomposition in the f-t plane of the energy shared by the two components $x_r(t)$ and $x_s(t)$. Consequently, when related to the Cartesian set $O_{x_1x_2x_3}$, the instantaneous energy of the three-dimensional ground motion is described by the 3×3 matrix $\underset{\sim}{U}(t)$ whose elements are defined by

$$U_{rs}(t) = \int_{-\infty}^{+\infty} P_{rs}(f,t;w)df \qquad r,s = 1,2,3 \tag{4}$$

Also, the total energy is defined by the matrix $\underset{\sim}{V}$ of elements

$$V_{rs} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P_{rs}(f,t;w)dfdt = \int_{-\infty}^{+\infty} U_{rs}(t)dt \qquad r,s = 1,2,3 \tag{5}$$

The matrices $\underset{\sim}{U}(t)$ and $\underset{\sim}{V}$ are real and symmetric. Also, the diagonal elements of $\underset{\sim}{P}$ are real, but the off-diagonal elements are complex. A real, symmetric spectral matrix $\underset{\sim}{G}(f,t;w)$ can be formed of elements as follows:

$$G_{rs}(f,t;w) = |P_{rs}(f,t;w)| \qquad r,s = 1,2,3 \tag{6}$$

in which the symbol $|\cdot|$ denotes the modulus. Thus, a complete nonstationary spectral description of the three-dimensional earthquake motion is given by the elements of Eq. (6).

The spectral resolution is controlled by the shape and parameters of the window function. For the treatment of earthquake records the choice of a Gaussian window function seems logical because it offers a good combined frequency-time selectivity and causes minimum leakage. It is also a self-reciprocal, even, real function, which provides considerable computational advantage. The following Gaussian time-window function was used in this work:

$$w(t) = \left(\frac{\sqrt{2}}{T}\right)^{1/2} \exp\left\{\frac{-\pi}{T^2} t^2\right\} \qquad (7)$$

The parameter T, representing the length, is defined as $T = \int_{-\infty}^{+\infty} \omega(t)dt/\omega(0)$. We see from Eq. (7) that T completely defines the window function. A value of T = 2.725 secs was selected, because it insured good accuracy as well as stability of the spectral resolution algorithm.

The concept of a physical spectrum has been applied in simulation and wave studies of earthquake time histories (see, for example, Refs. 6 and 7).

MODELING OF THE RECORDED EARTHQUAKE COMPONENTS

With respect to the three orthogonal components of an earthquake, it can be proven that the 3×3 matrices $\underset{\sim}{G}(f,t;w)$, $\underset{\sim}{U}(t)$ and $\underset{\sim}{V}$ defined by Eqs. (6), (4) and (5), respectively, constitute second-order symmetric tensors. Hence, with each of these matrices one can associate a set of principal axes along which the matrices become diagonal. The values of the components along the diagonal are the eigenvalues of the given matrix, and the direction cosines of the principal axes are given by the eigenvectors associated with each eigenvalue. The matrices whose rows are the eigenvectors of each principal direction are the tensor transformation matrices. Note that the principal axes associated with G(f,t;w) are time and frequency dependent, the axes associated with $\underset{\sim}{U}(t)$ are time dependent, and the axes associated with $\underset{\sim}{V}$ are fixed.

As in the case for the matrix $\underset{\sim}{S}$ of the moduli of the power spectra, the matrix $\underset{\sim}{G}$ of the moduli of the physical spectra is singular with rank one. Therefore, it will have only one non-zero eigenvalue corresponding to the unique principal spectrum. That eigenvalue is equal to the trace (or first invariant) of the matrix $\underset{\sim}{G}$:

$$G_{x'x'}(f,t;w) = G_{xx}(f,t;w) + G_{yy}(f,t;w) + G_{zz}(f,t;w) \qquad (8)$$

where the indices x,y,z relate to the three recorded components of the earthquake and the primed indices relate to the principal directions. Knowing the principal spectrum, we can devise an algorithm to reconstitute its corresponding time history (see next section). The resulting accelerogram is referred to as a physical spectrum optimized accelerogram (PSOA).

The time histories along the principal directions of the matrices $\underset{\sim}{U}(t)$ and $\underset{\sim}{V}$ are generated by direct rotation of the recorded accelerograms using the corresponding tensor rotation matrix for each case. These time histories are referred to as instantaneous energy optimized accelerograms (IEOA) and total energy optimized accelerograms (TEOA); their components are referred to as MAX, INTER, and MIN according to the principal values to which they refer. The IEOA and TEOA have, in general, three mutually orthogonal components that are uncorrelated in the energy sense. However, it must be clear that the PSOA represents the earthquake motion with a unique component.

Algorithm for the PSOA

To generate the PSOA, we need a nonstationary representation of the earthquake acceleration in terms of the principal spectrum. Such a representation is available (Ref. 6). In fact, under the usual assumption of zero-mean value for the ground motion process, the amplitude of the ground motion acceleration is related to the modulus of the physical spectrum as follows:

$$A(f_k,t_i) = [2\Delta f G(f_k,t_i)]^{1/2} \tag{9}$$

in which Δf is the frequency sampling interval. The acceleration at time t_i is obtained by a summation over all frequency components:

$$x(t_i) = \sum_{k=1}^{N} A(f_k,t_i)\sin[2\pi f_k t_i + \theta(f_k,t_i)] \tag{10}$$

in which N is the Nyquist sampling number. The phase angle $\theta(f_k,t_i)$ is computed by direct rotation of the phase angles of the recorded components through the tensor transformation matrix of the physical spectrum matrix. With that representation of the ground acceleration, the algorithm for the PSOA proceeds as follows. Starting at time step one, we form the physical spectrum matrix $\underset{\sim}{G}(f_k,t_i;w)$ corresponding to the three recorded components. Then we perform the eigenvalue resolution, form the transformation matrix, and compute the value of the principal spectrum. Finally, we use Eqs. (9) and (10) to generate the value of the acceleration at that time. Then we proceed to the next time step and repeat the procedure.

DISCUSSION OF RESULTS AND CONCLUSIONS

The following conclusions are based upon studies of the three components of the 1966 Parkfield earthquake, station Chalome 8 (Ch. 8). Other earthquakes are currently being studied.

1. The time histories for the IEOA and TEOA are similar in shape to the recorded ones (Fig. 1). Also, plots of the SRSS combination of the response spectra to each component of the two optimized models and the recorded earthquake show almost no difference between the three plots, as illustrated by Fig. 3. This concurs with the argument advanced by many researchers that strictly temporal combinations of earthquake records should not be expected to reveal valuable information about the earthquake. However, the IEOA and TEOA have the advantage that their characteristic functions are distinct. Also, in view of the importance of energy as a measure of destructiveness of earthquakes, the IEOA and TEOA could be valuable in design.

2. As might be expected, it is the PSOA model that presents the most interesting properties. The most relevant is that the PSOA eliminates the need for a three-dimensional representation of earthquakes. The motion at the site is represented by a single component that fully incorporates the characteristics of the three recorded components. The study of some PSOA parameters, such as response spectra, Fourier spectra, and cumulative RMS function, supports the preceding qualitative result. First, it must be emphasized that the spectrum of the optimized record (Fig. 2), at any single time and for

every frequency component, is the sum of the spectra of each record (as expressed by Eq. 8). However, the time history of the PSOA is not a simple superposition of the recorded accelerograms. As expected, the PSOA shows much higher acceleration peaks along the time axis (Fig. 1). However, the normalized peak acceleration (NPA), defined as the ratio of maximum acceleration to the SRSS of the accelerations, is almost equal to the average of the NPA of the recorded components. Of particular interest is a comparison between the response spectrum of the unique component of the PSOA to the SRSS of the responses of the three recorded accelerograms. The plots of these two functions show good agreement (Fig. 4). This means that the use of the PSOA in design may well replace the need for a multicomponent input, and yet it results in an optimum (but not necessarily conservative) design.

Of course, additional earthquakes and many other aspects of the PSOA must be studied before the preceding conclusions can be made general. Such work is continuing. However, based upon the work to date, it appears likely that there will be many applications for the PSOA in earthquake engineering.

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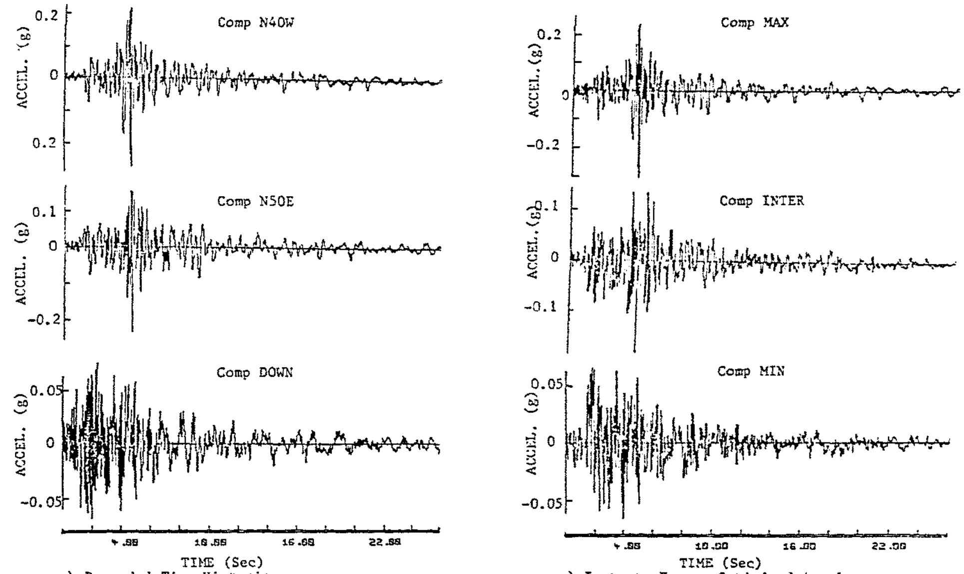
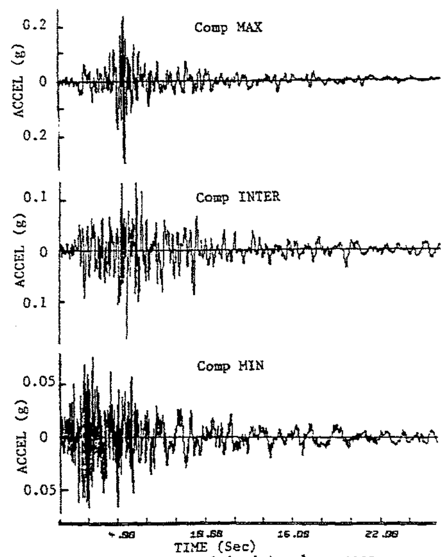
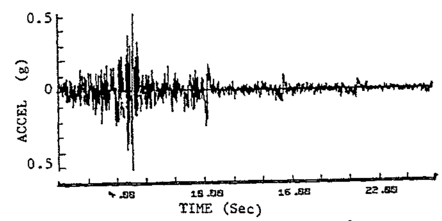


a) Recorded Time Histories

c) Instant. Energy Optimized Accelerograms



d) Total Energy Optimized Accelerograms



b) Physical Spectrum OPtimized Accelerogram

		Peak (gal)	Time (sec)	NPA	
RECORDED	N40W	-269.6	4.54	0.248	Average 0.217
	N50E	-232.6	4.64	0.238	
	DOWN	77.7	2.30	0.152	
PSOA		-515.2	5.10		0.227
IEOA	MAX	-307.2	4.54	0.252	Average 0.205
	INTER	-181.1	4.64	0.218	
	MIN	66.3	2.30	0.144	
TEOA	MAX	-307.9	4.54	0.269	Average 0.206
	INTER	-181.1	4.64	0.200	
	MIN	75.2	2.30	0.149	

FIG.1. 1966 Parkfield Eq. (Ch.8): Time Histories and Characteristics of Recorded and Optimized Accelerograms.

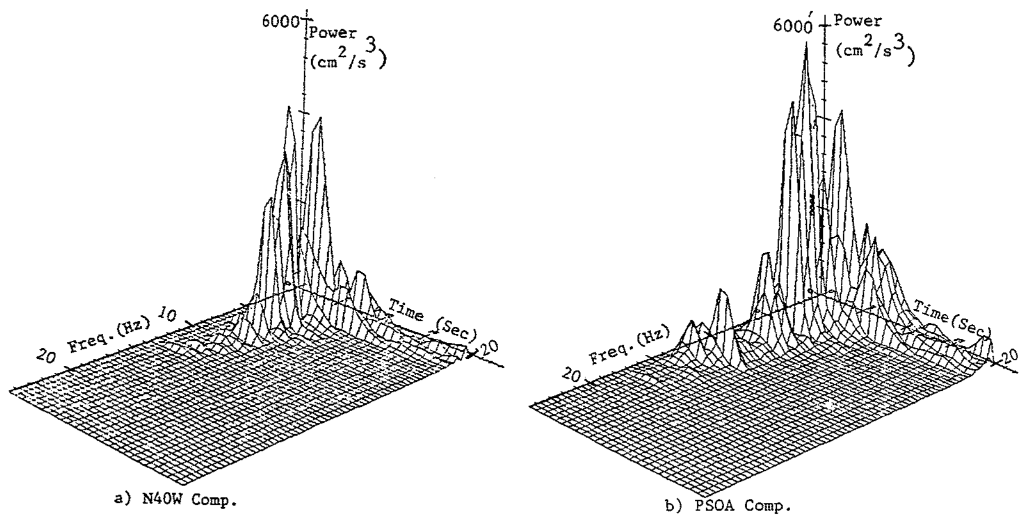


FIG.2. 1966 Parkfield Eq. (Ch.8): Physical Spectra of (a)N40W Component and (b) PSOA Component. The Maxima of these spectra are 4300 and 6140 cm^2/s^3,respectively.(Not shown in the figure are the spectra for the N50E component and the vertical component; these spectra have maxima of 2360 and 710 cm^2/s^3,respectively. Also not shown are the cross-spectra of the recorded components.)

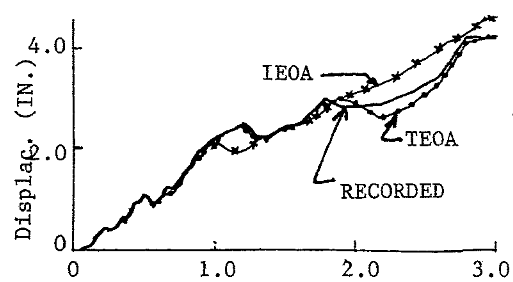


FIG.3. 1966 Parkfield Eq.: SRSS Combination of Response Spectra (Damping = 5%).

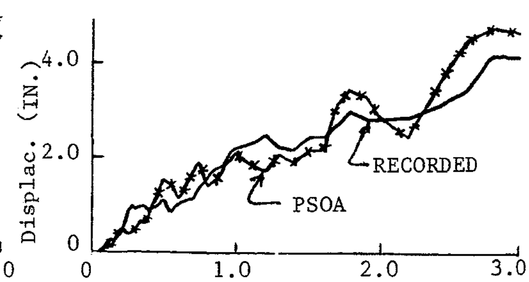


FIG.4. 1966 Parkfield Eq.: SRSS Combination of Response Spectra of Recorded Components Compared to Response Spectrum of PSOA (Damping 5%).