NORMAL MODE SYNTHETICS OF STRONG GROUND MOTION

C. C. Schoof (I)
R. N. Haar (II)
R. J. Geller (III)
Presenting Author: C. C. Schoof

SUMMARY

The normal mode method is used to compute the elastic response of a layered, spherical earth model to a given source rupture process. A suite of synthetic acceleration time histories is obtained as a weighted superposition of the earth's normal modes corresponding to P-SV and SH wave motion. These can be used to study the random variables related to the site response, given an event size and location. This synthetic information can then be combined with empirical information on strong ground motion response parameters to update seismic hazard estimates via Bayes' theorem.

INTRODUCTION

Engineers have many techniques available to evaluate the dynamic effects that a particular realization of ground shaking at the base of a structure may have on that structure. Yet we remain uncertain as to how future seismic loading should be characterized for purposes of seismic design. Physically, strong ground motion is the result of many complex processes including non-uniform (spatial and temporal) stress accumulation along a fault; stress release (rupture initiation, propagation, redirection, and stopping); wave propagation through an inhomogeneous, anelastic medium; site amplification or attenuation due to local soil conditions; and soil-structure interaction. In order to provide more detailed design criteria for seismically-active regions, empirical models have been developed for estimating the levels of future ground shaking in terms of the most simple random variables (e.g. Ref. 1).

The characterization of seismic response has always been hindered by both the complexity of the fault rupture/wave propagation processes, and the lack of observations with which to adequately evaluate the important random variables. In this work, we use a theoretical model to synthesize the ground motion at a specified point as a function of the earth structure for the region, and a (random or deterministic) source rupture function. Specifically, the normal mode method (Ref. 2) is used to simulate earthquake acceleration records representing P-SV and SH wave motion in the frequency range of engineering interest (0 to 5 hz). These records serve as synthetic representations of the seismic response, and are an excellent tool for evaluating the importance of the random variables that affect site response.

By generating a large number of synthetics for a particular source function, we can approach an ideal sampling of the random variables related to site response. For example, a single-station recording of an earthquake gives little information on pga attenuation for a region, because we cannot evaluate the variation of peak amplitude as a function of distance for this event. On the other hand, a seismic array recording the event at many distances and azimuths would give an excellent account of the

---

(I) Ph.D. Candidate, Dept. of Civil Engineering, Stanford University, California, USA
(II) Ph.D. Candidate, Dept. of Geophysics, Stanford University, California, USA
(III) Assistant Professor of Geophysics, Stanford University, California, USA
Present address: Geophysical Institute, Tokyo University, Japan.

behavior of attenuation in that region. If there is not a well-positioned array recording the particular event of interest, then this information will be unavailable through direct observations. However, the loss due to the lack, or sparsity, of direct observations can be mitigated by simulating acceleration records at appropriate locations using the synthetic technique presented in this paper.

In this paper, we present an outline of the normal mode method used to compute synthetic acceleration records. This includes a brief discussion of the free oscillations of the earth and a description of the source model (rupture processes) which we use. Next, we show examples of an experiment designed to study the attenuation of peak horizontal ground acceleration. Then we indicate how this synthetic information can be used together with existing empirical information in seismic hazard studies. Finally, we discuss several potential applications of this theoretical model in terms of strong ground motion characterization.

NORMAL MODE MODELING OF STRONG GROUND MOTION ACCELEROGRAMS

The normal mode method conveniently partitions the problem into two parts:

(i) computation of normal modes and
(ii) the subsequent combination of these modes.

The partition is more than a convenience; it separates the information pertaining to the earth structure from the information pertaining to the source.

The normal modes consist of eigenvalues and eigenfunctions which represent the free oscillations of the earth. The earth, simply described, is considered a spherically symmetric, layered, elastic body. This yields two types of modes, toroidal and spheroidal, which correspond to SH and P-SV motion, respectively. The important point to note is that the toroidal and spheroidal normal modes provide a complete description of the dynamic behavior of the earth, and, as such, contain all the information pertaining to earth structure. The practical implication of this is that the lengthy calculation of modes need be performed only once for a particular earth structure and stored. These modes can be used repeatedly for a variety of source configurations. The procedure for computing normal mode synthetics is shown in Figure 1.

Seismic sources are represented as a collection of point sources. The effect of each point source can be simply represented by a linear combination of the normal modes. The weight, or excitation coefficient, is primarily a function of source depth and the shape of the particular eigenfunction (Ref. 3). There are several other factors which influence the seismic synthetics, such as dislocation rise time, directivity, and attenuation. These factors are represented by linear filters; that is, the excitation spectrum is multiplied by the spectrum of each factor.

The point source response is smoothed to represent finite rupture by means of the directivity function (Ref. 4). In terms of the finite source, seismic waves are generated from various locations on the fault plane. The amplitude and phase of waves associated with each ruptured segment depends on the local values of stress drop, rupture velocity, rise time, trigger time, and depth. This information is available in varying degrees from a myriad of geophysical studies. For example, the fault setting is defined in geologic and plate tectonic studies, and much of the local character of the fault can be obtained through studies of microseisms. A major point of this work is that *all* available geophysical information should be incorporated in a seismic hazard analysis.

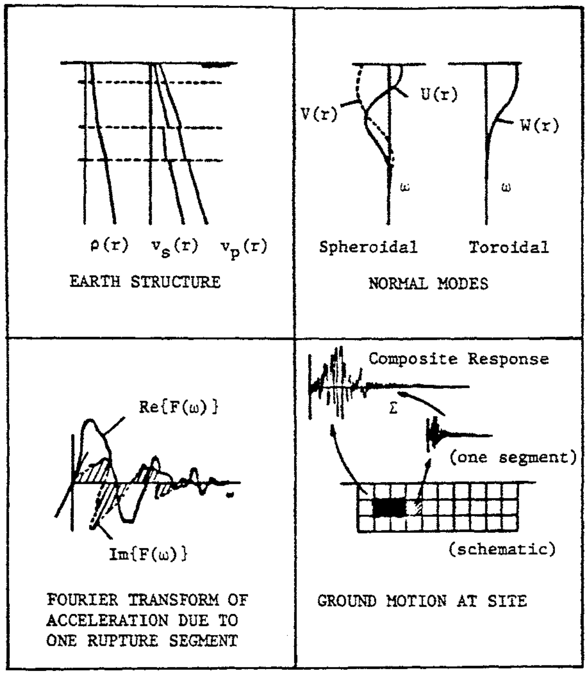


Figure 1. procedure for computing normal mode synthetics.

1. Define earth structure for the region.
2. Compute normal modes (these depend on the earth structure only).
3. The Fourier transform of displacement or acceleration due to a finite rupture segment (or point source) is obtained as a linear combination of modal responses.
4. The site response is computed as a summation of responses from many rupture segments representing the fault zone.

This highlights the difference between our analysis and the analyses that are commonly done in earthquake engineering, where it is common practice to simulate an acceleration time history by scaling a random waveform by an envelope function. Because our computed synthetics are based on physical models of the fault rupture and wave propagation processes, the waves from each segment exhibit a coherence that distinguishes these waves from scaled white-noise waveforms.

An example of a normal modes synthetic accelerogram is shown in Figure 2. The top trace is the north-south component of the observed waveform from the 1968 Borrego Mountain earthquake, and the second trace is the normal modes synthetic. The remaining traces represent contributions from the fundamental mode and first 19 overtones, each consisting of numerous modes. The excellent match between observed and synthetic waveforms is self-evident. Major arrivals are well represented both in amplitude and phase. It should be emphasized that the composite waves include both body wave and surface wave phases, as well as all other reflections and re-

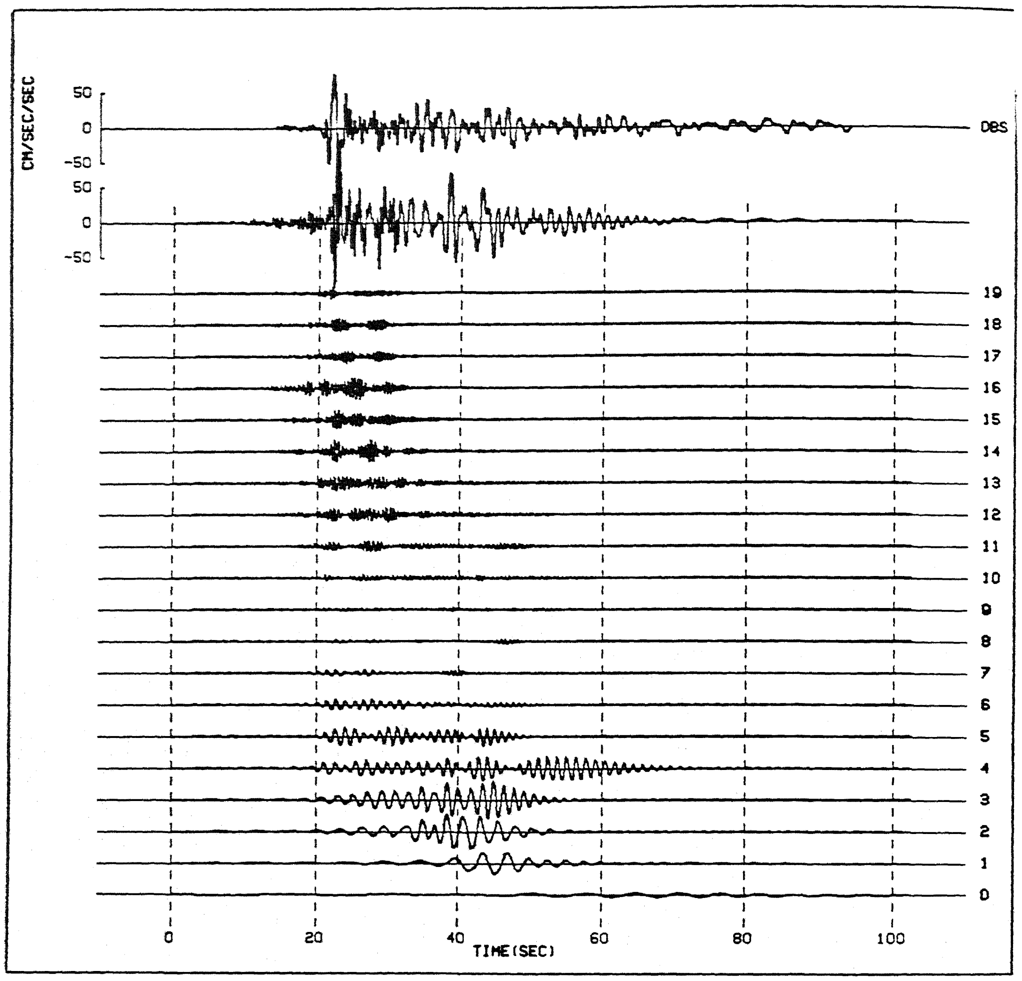


Figure 3. Example of modal superposition. The Fundamental mode and the first 19 overtones are summed to synthesize the acceleration response at El Centro for the 1968 Borrego Mountain earthquake. The top trace is the observed acceleration record (south component); the second trace is a summation of the first 20 overtones. The fundamental (N=0) and first few overtones represent surface waves, and the higher overtones correspond to body waves. The source was assumed to be vertical strike slip, with a seismic moment of $6.9e^{25}$ dyne-cm, a mean depth of 6.5km, rupture velocity of 2.4km/sec, rise time of 0.75sec, and a rupture that propagated 5km toward El Centro, and 4km away.

fractions (which may not be identifiable as distinct phases). For the generation of this synthetic, the local earth structure in the upper crust given by Hamilton (Ref. 5) is used. A frequency range of 0-5 hz is spanned by the modes, although there is no inherent limitation in our method to extending the frequency range to 10 hz, or higher.

ATTENUATION

The variation in amplitude and frequency content of waves associated with a moving fault rupture has been observed to produce strong focusing of waves in the direction of rupture propagation (Ref. 6,7). This effect is referred to as directivity focusing, and is highly dependent on the ratio of rupture velocity to phase velocity at the source. In the direction of rupture propagation, high-frequency response is enhanced, and the resulting amplitudes can be an order of magnitude larger or more than amplitudes in the back-azimuth. In this section, we look at a suite of synthetic accelerograms, and evaluate the peak horizontal acceleration as a function of the station distance and azimuth[1].

A series of 240 synthetic acceleration records are computed using the same Imperial Valley earth structure as above. The recording stations are uniformly distributed in a polar coordinate system centered at the epicenter in the quadrant from 0 to $\pi/2$. Figure 3. shows a series of five synthetic accelerograms computed for the case of constant azimuth, φ, and variable distance, **R**. The characteristics of the source function are described in Table 1. We note that the response is primarily body wave motion at shorter distances, with more surface wave response at greater distances.

Source Parameter	Value
Seismic Moment	10^{25}
Rupture Velocity	2.4 km/sec
Rise Time	1.25 sec
Depth	5.0 km
Attenuation	Q=150
Rupture Extent:	10 km toward site
(strike slip)	5 km away from site

Table 1. Source parameters for event in Figure 3.

Using the peak horizontal accelerations obtained from this simulated event, a regression analysis was performed which includes the station azimuth φ as an independent variable in the model. A quadratic function is chosen to represent the increase in amplitudes as φ goes to zero; the regression model used is

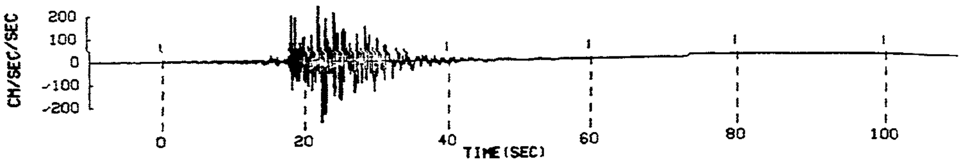
$$\log_{10}[y_m] = c_0 + c_1\mathbf{R} + c_2\ln\mathbf{R} + c_3\varphi + c_4\varphi^2$$

where y_m is the peak horizontal acceleration (g), **R** is the closest distance to fault rupture (km), φ is the station azimuth (radians) measured from the direction of major fault rupture, and $c_0 - c_4$ are coefficients obtained from a linear regression analysis.

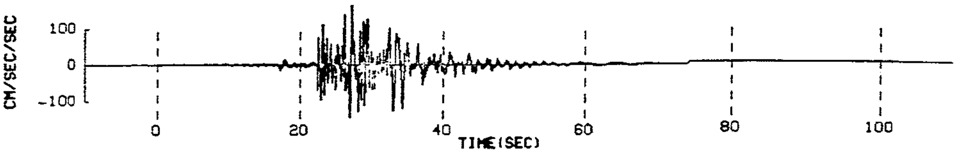
---

[1] The azimuth is defined as the angle between a horizontal line in the direction of rupture propagation and a line from the ruptured segment to the site.

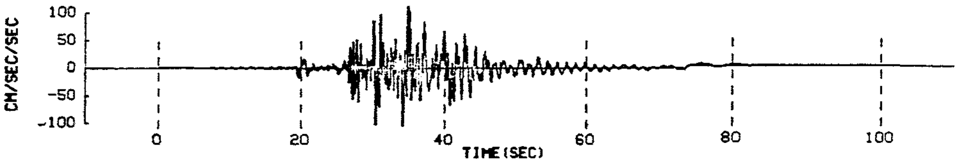
R=30 km



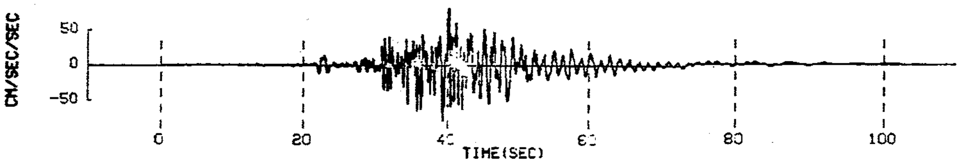
R=40 km



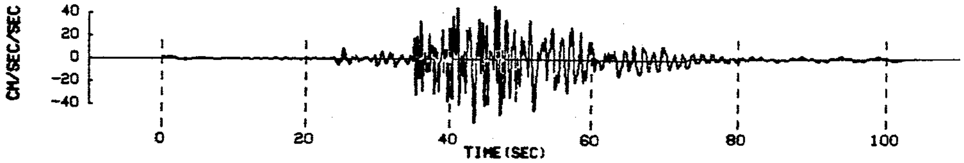
R=50 km



R=60 km



R=70 km



Figures 3. Synthetic acceleration records for a simulated event in the Imperial Valley. A description of the source is given in Table 1. The rupture propagates in the $\varphi=0$ direction.

Model	c_0	c_1	c_2	c_3	c_4	$\sigma_{\ln y}$
Synthetics: (omitting φ)	0.40	-.0011	-1.23	0.	0.	0.34
Synthetics: (incl. φ)	1.13	-.0015	-1.66	-1.99	0.95	0.13
Joyner & Boore (1981)	0.60	-.0023	-1.00	0.	0.	0.26

Table 2. Comparison of regression coefficients with and without a φ-dependence; Joyner & Boore's model is presented for comparison.

The regression analysis is done with and without a φ-dependence for a particular source. Comparing the results of the φ-dependent analysis to the results of the φ-independent analysis, we see that the inclusion of the azimuthal variable sharply decreases the standard error of the residuals with respect to the mean regression line. Also for comparison, Table 2 shows the regression coefficients of Joyner and Boore (Ref. 8) for a collection of real events. These results show that synthetics enable us to generate detailed information about a particular variable as it relates to attenuation. We note that for this suite of accelerograms, a highly-coherent source model was used, which tends to accentuate the directivity effects. The impact of φ in the regression model will be much less when the source is less coherent.

COMBINATION OF SYNTHETIC AND EMPIRICAL INFORMATION

The evaluation of seismic hazard at a site is based on 1) the probability that an event of a given size occurs at a given location, and 2) the probability of exceeding some response parameter (pga, rms acceleration, s_a, etc.) when the event size and location are known. Synthetics give us another source of information on the probability of exceeding the response parameter. These two sources of information, observations and synthetics, are combined using Bayes' theorem.

In this study, we obtain from the synthetics a mean value and variance of the peak ground acceleration as a function of moment magnitude (in this case, $\mathbf{M}$=6.5), closest distance to fault rupture and station azimuth (pga is assumed to be log-normally distributed). This serves as the prior distribution on the mean pga. Information from observed accelerograms or empirical relationships is used in the likelihood function, and the two (prior and likelihood) are combined to give an updated estimate of the distribution on the expected value of pga, given an event size and location (the location now includes the station azimuth). From this posterior distribution, the Bayesian distribution on peak ground acceleration is obtained which includes both synthetic and observed (empirical) information, and is used in the seismic hazard analysis.

In the Bayesian framework, the relative importance of the prior information can be weighted, depending on the degree of confidence in this information on the part of the analyst. If the observed data for a region is considered to give a very good estimate of the expected pga, then the variance of the prior may be assumed to be very large, resulting in a data-based estimate of the posterior. In cases where little observed data exist, the posterior may be heavily influenced by the prior (synthetics).

DISCUSSION

Normal mode synthetic accelerograms can be useful for studying many facets of earthquakes and related effects. For example, the normal mode synthetics can be used to study a single random variable (e.g. pga, rms acceleration, and duration). Another application is to estimate the statistics of the dynamic amplification factors relating peak ground acceleration at sites of varying stiffness to the spectral response as a function of frequency and damping. Finally, the synthetics can be used to study the complete history of any event. It is common practice to use observed acceleration records of other events for this purpose. Often, there is little physical reason for selecting one record over another. We suggest the use of synthetic accelerograms because they explicitly include our knowledge of the *specific earth structure* for the region, the *types of faults* (e.g. thrust, strike-slip) and *fault rupture characteristics* (e.g. multiple fault ruptures). The normal mode method is not limited to computing the surface response. The response can be computed at any depth, and used directly in soil-structure interaction programs, or as foundation excitation for dynamic analysis.

We suggest that normal mode synthetic acceleration records can be a useful tool in understanding important features of the fault rupture and wave propagation processes that lead to ground shaking at a site. The synthetic seismograms supplement sparse data sets to refine the seismic hazard analysis. The objective application of synthetics will be enhanced as we learn more about source mechanisms. This underscores the need for more work to be done on source theory, particularly the related near-field effects. In this paper, simple source models are input to our synthetic programs. But, conversely, the synthetic techniques can be used to yield valuable information about particular source mechanisms. The study of seismic sources, as well as the study of other engineering parameters (in addition to attenuation), remains as future work.

Acknowledgements: This research was supported by NSF grant EAR 8218961, NSF 77-17834, NSF 80-17533, and the John A. Blume Earthquake Engineering Center, Stanford University.

REFERENCES

1. Shah, H.C., et al (1975). "A Study of Seismic Risk for Nicaragua, Part I", Blume Center Report No. 11, Stanford University.
2. Takeuchi, H., and Sato (1972). "Seismic Surface Waves", Methods in Computational Physics, Bruce A. Bolt ed.,II.
3. Kanamori, H., "Synthesis of Long-Period Surface Waves and its Applicatioin to Earthquake Source Studies - Kurile Islands Earthquake of October 13, 1963", J. Geoph. Res. 75, 1970.
4. Ben-Menahem, A., and Singh, S.J. (1981). "Seismic Waves and Sources", Springer-Verlag, New York.
5. Hamilton, R.M. (1970). "Time-term Analysis of Explosion Data from the Vicinity of the Borrego Mountain, California, Earthquake of 9 April 1968", Bull. Seis. Soc. Am., Vol 60, pp 367-381.
6. Aki, K., and Richards, P.G. (1980), "Quantitative Seismology, Theory and Methods", W. H. Freeman and Company.
7. Bolt, B.A. (1983). "The Contribution of Directivity Focusing to Earthquake Intensities",U.S. Army Corps of Engineers Miscellaneous Paper S-73-1.
8. Joyner, W.B., and Boore, D.M. (1981). "Peak Horizontal Acceleration and Velocity from Strong Motion Records Including Records from the 1979 Imperial Valley, California, Earthquake", Bull. Seis. Soc. Am., Vol. 71.