ESTIMATION OF RELATIVE GROUND MOTIONS INDUCED BY SURFACE WAVES USING KINEMATIC SOURCE MODEL

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SUMMARY

This paper presents an analytical method for estimating the relative ground motions caused by surface waves generated by earthquakes. The method is based on the fault dislocation and normal mode theory. Sensitivity of phase velocity to small perturbations in elastic parameters is also examined. The seismic surface waves are calculated with the fault parameters which are given by the 1971 San Fernando earthquake. They are compared with the longitudinally polarized ground motions observed at Vanowen Street, California. Moreover, the relationship between relative ground motions and separation distance is quantitatively discussed.

INTRODUCTION

Aseismic design of structures with long dimensions such as lifelines located on or near the ground surface is usually controlled by large strains and relative displacements caused by body and surface waves. Therefore, only a few observations from dense instrument arrays have been carried out for earthquake ground motions to obtain basic data for lifeline earthquake engineering (for example, Ref. 1).

In relation to the seismic response of underground lifeline systems, it is the most important to estimate the waveform of the motion induced in the ground in which the physical properties of the ground shall be reflected. Paying attention to the horizontal transmission of surface waves, it is of interest to investigate the relationship between the phase velocity and the ground strain (Refs. 2 and 3). From these point of view, the influence of ground motions developed in the subsurface is an area of research deserving of more attention.

In the present paper, sensitivity analysis is developed for the variation in the phase velocity with the perturbation of a specific elastic parameter in layered medium, and an analytical procedure for predicting relative ground motions at an arbitrary depth is proposed by using the fault dislocation and normal mode theory.

A METHOD TO ESTIMATE THEORETICAL SPECTRA OF RELATIVE GROUND MOTIONS

Using a double couple focal mechanism at a point source, theoretical surface-wave amplitude spectra will be generated for a realistic multi-layered medium through the normal mode theory (Refs. 4 and 5). The double couple mechanism is defined in terms of strike $N\phi^{\circ}E$, dip angle δ, slip angle λ and focal depth h of a fault plane. The cylindrical coordinate system (r, θ, z) is used to analyze waves, where r, θ and z are epicentral distance, azimuthal angle measured clockwise from the north, and ground depth, respectively.

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In order to estimate the vertical distribution of underground motions, the far-field displacement spectra of Rayleigh wave for radial direction can be given by the following equation:

$$S_R(\omega, r, \theta, z) = W(\omega)\sum_{j=1}^{N_R(\omega)} \frac{M(\omega) V_{jRH}(\omega, z) \cdot \chi_{jR}(\omega, h, \theta) k_{jR}(\omega)^{-1/2}}{2 C_{jR}(\omega) U_{jR}(\omega) A_{jR}(\omega) (2\pi r)^{1/2}} \cdot \exp\left[-i\left\{k_{jR}(\omega) r + \frac{3}{4}\pi\right\} - \frac{\omega r}{2 Q_{jR}(\omega) U_{jR}(\omega)}\right]$$

$$= \sum_{j=1}^{N_R(\omega)} S_{Rj}(\omega, r, \theta, z) \qquad \cdots\cdots\cdots\cdots\cdots (1)$$

in which ω =angular frequency, i=imaginary unit, $N_R(\omega)$=upper limit of modal superposition at a given frequency, j=number of modes, k_{jR}=wavenumber of j-th mode, C_{jR}=phase velocity, U_{jR}=group velocity, $(2C_{jR}U_{jR}A_{jR})^{-1}$=amplitude response factor in the medium, Q_{jR}=quality factor, V_{jRH}=radial component of Rayleigh-wave eigenfunction, χ_{jR}=complex radiation pattern function depending on $\theta, \phi, \lambda, \delta, h, \omega, C_{jR}$ and medium, $W(\omega)$=tapered band-pass Ormsby filter. The subscript R denotes a Rayleigh-wave quantity.

The effect of attenuation of propagating surface waves due to inelasticity of the medium is non-negligible, and can be taken into account by $\exp(-\omega r/2Q_{jR}(\omega)U_{jR}(\omega))$ in Eq.(1). For a simple dislocation with a modified ramp function having a rise time τ, the spectral source function $M(\omega)$ yields

$$M(\omega) = M_0\left\{\pi\delta(\omega) - \frac{i}{\omega}\,\frac{\sin(\frac{\omega\tau}{2})}{\frac{\omega\tau}{2}}\,\frac{\exp(-i\frac{\omega\tau}{2})}{1-(\omega/\omega_n)^2}\right\} \qquad \cdots\cdots\cdots\cdots\cdots (2)$$

in which $\delta(\omega)$ is Dirac's delta function, M_0 is the seismic moment and $\omega_n=2\pi/\tau$.

The relative ground motions (separation distance between two points : Δr) are strongly affected by geological conditions at the site. Using the ground displacements at two points which locate $\Delta r/2$ radially away from the point $P(r,\theta,z)$ in the ground, the spectra of relative ground displacement at the site P can be defined as follows:

$$\Delta u_R(\Delta r, \omega, r, \theta, z) = \sum_{j=1}^{N_R(\omega)}\left\{S_{Rj}\left(\omega, r+\frac{\Delta r}{2}, \theta, z\right) - S_{Rj}\left(\omega, r-\frac{\Delta r}{2}, \theta, z\right)\right\} \quad \cdots\cdots (3)$$

Substituting Eq.(1) into this equation, the following approximation becomes possible:

$$\Delta u_R(\Delta r, \omega, r, \theta, z) = \sum_{j=1}^{N_R(\omega)} S_{Rj}(\omega, r, \theta, z)\left[\cos\left(k_{jR}(\omega)\frac{\Delta r}{2}\right)\cdot\left\{\left(1-\frac{\Delta r}{4r}\right)e^{-\frac{\omega\Delta r}{4Q_{jR}(\omega)U_{jR}(\omega)}} - \left(1+\frac{\Delta r}{4r}\right)e^{\frac{\omega\Delta r}{4Q_{jR}(\omega)U_{jR}(\omega)}}\right\} - i\sin\left(k_{jR}(\omega)\frac{\Delta r}{2}\right)\left\{\left(1-\frac{\Delta r}{4r}\right)e^{-\frac{\omega\Delta r}{4Q_{jR}(\omega)U_{jR}(\omega)}} + \left(1+\frac{\Delta r}{4r}\right)e^{\frac{\omega\Delta r}{4Q_{jR}(\omega)U_{jR}(\omega)}}\right\}\right] \quad \cdots\cdots\cdots (4)$$

The spectra of average normal strain may be estimated from the relative ground displacement between two points:

$$\bar{\varepsilon}_r(\Delta r, \omega, r, \theta, z) = \Delta u_R(\Delta r, \omega, r, \theta, z)/\Delta r \qquad \cdots\cdots\cdots\cdots\cdots (5)$$

Following the relation " $\varepsilon_r(\omega,r,\theta,z)=\partial S_R(\omega,r,\theta,z)/\partial r$ ", the local strain spectra observed at the site P can be written as follws:

$$\varepsilon_r(\omega, r, \theta, z) = \sum_{j=1}^{N_R(\omega)} S_{Rj}(\omega, r, \theta, z)\cdot\left\{-\frac{1}{2r} - \frac{\omega}{2Q_{jR}(\omega)U_{jR}(\omega)} - ik_{jR}(\omega)\right\} \quad \cdots\cdots\cdots (6)$$

The local strain can be also estimated as the limiting value, $\varepsilon_r(\omega,r,\theta,z)=\lim_{\Delta r\to 0}\bar{\varepsilon}_r$

$(\Delta r, \omega, r, \theta, z)$. Since the real part in the brackets of Eq.(6) is usually negligible compared with the imaginary part, Eq.(6) can be rewritten in the following form,

$$\varepsilon_r(\omega, r, \theta, z) = \sum_{j=1}^{N_R(\omega)} -i\omega S_{Rj}(\omega, r, \theta, z)/C_{jR}(\omega) \quad \cdots\cdots\cdots \quad (7)$$

This relation is often written as the maximum ground strain ε_{max} =maximum ground velocity V_{max}/effective phase velocity $\overline{C}$, in consistent manner (Refs. 2 and 3).

According to Eqs.(1) to (7), inverse fast Fourier transform of the spectra gives the time history of surface-wave motion due to a point source. By use of the similar process, relative ground motions induced by Love waves can be also derived.

The contribution of both in-phase and out-of-phase input motions to the response of a lifeline is considered. Many previous studies on the behaviours of underground lifelins have pointed out that axial strains are more predominant than bending strains. For this reason, attention is mainly paid here to the relative ground motions induced by propagation of Rayleigh waves.

SENSITIVITY ANALYSIS OF PHASE VELOCITY

Adpoting formulas given in Ref. 5 to calculate partial derivatives of phase velocity with respect to medium parameters, sensitivity of phase velocity will be discussed. Examination of the partial derivative curves as a function of period gives insight into the relative importance of the various elastic parameters (P-wave velocity α , S-wave velocity β and density ρ) at different depths.

As a basic crustal model, the homogeneous layered model beneath the Vanowen Street, California is adopted as shown in Fig.1 (Ref. 6). Fig.2 shows the partial derivatives in Rayleigh-wave phase velocity. The values of fundamental and 3rd (2nd higher) modes at the depths 0.5km and 2km are considered. From Fig.2, the following characteristics can be seen: 1) The derivatives of the P-wave velocity are small, 2) The derivatives of density are also small, but they emphasize sensitivity to variations in S-wave velocity, although for certain period ranges the density is an important variable, 3) The density partials can be either positive or negative, and 4) Since the S-wave velocity is one of the controlling parameter, increases in any layer's S-wave velocity will produce greater increase in the phase velocity.

Next, the partial derivatives in the ground of phase velocity with respect to the S-wave velocity are shown in Fig.3 where in typical periods (T=0.5sec and 2sec) the derivatives for fundamental mode are plotted. From Fig.3, the following can be seen: 1) At the longer period, the deeper layer has a great influence on the phase velocities. $\partial C_j(\omega)/\partial\beta(z)$ shows distinct changes at the boundary of soil layer representing striking contrast of the S-wave velocity to adjacent layer, and 2) In general, fractional change $\delta C/C$ in the phase velocity is proportional to the perturbations in model parameters (Lame constants λ, μ and density ρ). The weight functions for $\delta\lambda/\lambda$ and $\delta\mu/\mu$ are proportional to the dilatational- and strain-energy densities, and that for $\delta\rho/\rho$ to the kinematic-energy density. From these point of view, perturbations in α , β and ρ at the depth where the energy densities are larger affect the phase velocity more strongly.

The crustal structure model used is shown in Fig.1. The incoherent (out-of-phase) motion sometimes may cause the failure of the lifeline system. Therefore, a potential earthquake is herewith considered, which occures on a left-lateral vertical strike slip fault with focal depth h=8km and seismic moment $M_0=0.78\times10^{25}$dyn.cm. Then, the rise time τ is 0.72sec and the quality factor Q is constantly 300. The period range cosidered is from 0.3sec to 7.5sec in the form of a cosine-tapered rectangular spectral window.

Calculations are carried out for the maximum ground displacement, velocity, acceleration and local strain for Love wave along a ground surface at the epicentral distance of 70km. Fig.4 shows the influence of modal superposition on ground motions of Love waves. The Love waves up to the fundamental, 3rd (2nd higher) and 8th (7th higher) modes are respectively included. From Fig.4, it is found that most of the displacement motion correspond to the fundamental normal mode solution and that other intensity parameters except for maximum acceleration can be adequately estimated by superposing up to the 3rd modes. The contribution of higher modes on the ground acceleration should be considered in the high frequency range. The surface-wave (ground velocity) syntheses are shown in Fig.5. In this case, the higher modes are essential in defining the shape of the waveform. The acausal surface-wave synthesis gives a poor fit to the direct arrival. Acausality is a characterstic of all modal contributions, but superposition of all flat structure modes cannot remove these fictitious contributions.

Therefore, the fundamental mode solution employed so far (Refs. 3 and 4) may give a serious error according to choice of a crustal structure and the period range of our interest. With confidence gained from the comparisons above, the surface-wave synthesis method is applied in the next chapter to superpose up to first eight modes, i.e., $\max_{\omega} N_R(\omega)=8$.

ESTIMATION OF RELATIVE GROUND MOTIONS OBSERVED DURING THE 1971 SAN FERANDO EARTHQUAKE

Fault Dislocation Model Fig.6 shows the fault geometry and fault parameters on the basis of the study by Heaton et al (Ref. 7). Although the earthquake consists of double event, the fault is modeled by only lower segment on fault planes shown in Fig.6. The source parameters used are as follows: strike=N105°E, slip angle=104°, dip angle=53°, fault length=16km, fault width=14km, focal depth=13km, rupture velocity=2.8km/sec, seismic moment=1.4×10^{26} dyne.cm, and rise time=0.82sec. The value of quality factor is assumed to be 300.

Effect of the propagating rupture along the fault length can be taken into account by multiplication of Eqs.(1) and (4) to (7) by the following factor

$$\frac{\sin X_j}{X_j}\frac{\sin Y_j}{Y_j}e^{-i(X_j+Y_j)}\bigg|_R \qquad \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

in which

$$\left.\begin{aligned} X_j&=\frac{L}{2}k_j(\omega)\cos\delta\sin(\varphi-\theta)\\ Y_j&=\frac{W}{2}\left\{\frac{\omega}{V_R}-k_j(\omega)\cos(\varphi-\theta)\right\}\end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

for the case of unilateral rupture to strike direction. L is the fault length, W is the fault width and V_R is the rupture velocity (unilateral).

Ground motions The synthetic ground motions of radial direction at the ground surface (azimuthal angle=N193°E) are shown in Fig.7(a). For reference, the seismograms recorded at the Vanowen Street during the 1971 San Fernando earthquake are shown in Fig.7(b). With respect to waveform and duration, the observed earthquake motions are well-simulated by the synthetic Rayleigh waves except for the time longer than about 16sec. The disagreement in the record may be due to the presence of leaking modes. It is found from Fig.8 that the local strain calculated at Vanowen Street provides good agreement with the maximum value inferred from recorded waves ,$1.9x10^{-4}$ (Ref.2), and that the maximum strain in a pipeline may not necessarily occur during the maximum acceleration of the ground. The waveform of ground strain resembles one of the ground velocity and the time to the maximum amplitude is almost equivalent. A precise coinsidence of attenuation between synthetic peak ground motions and recorded ones (Ref.8) is quite dramatic as shown in Fig.9. A simple rupture-propagation approximation is adequate but a slight improvement may be obtained by evaluating a more complex fault model (for example, barrier model) with many localized sources to predict reliable ground motions.

Relative Displacement and Ground Strain versus Separation Distance

Fig.10 shows the normalized maximum relative displacement $\Delta u_{Rmax}(\Delta r)/\Delta u_0$ for Rayleigh waves as a function of separation distance Δr. Here, a normalizing factor Δu_0 is an average stroke displacemnt ,i.e., the absolute value of the half difference between the maximum and minimum values of the displacement at each site of epicentral distances 25, 50 and 100km in azimuthal angle N193°E. The influence of time and spatial variation of surface waves on realistic ground motions is investigated. The phase velocity will be the only source of the non-coherent motion. It is also clear that the relative ground displacement is sensitive to phase difference as the wave propagates through the soil along the lifeline's axis. When separation distance between two points is greater than the half wavelength, Δu_{Rmax} is almost equal to or smaller than the absolute value of two times Δu_0. On the other hand, the normalized average strains are shown in Fig.11 where the sensitivity of longitudinal strains to earthquakes with changing epicentral distance is found. A normalized factor is an average strain ε_{r0} estimated at the separation distance Δr=0.1km . From Figs.10 and 11, the following characteristics can be seen:

It is reasonable to assume that the relative displacement increases with the separation distance. However, distinct differences are recognized in the values of relative displacements for different sites. According to the relationship between wavelength and separation distance, $\Delta u_{Rmax}(\Delta r) / \Delta u_0$ varies irregularly such as maximum at 4km, minimum at 7km and maximum at 9km, etc. The maximum strain decreases rapidly as the separation increases over distance of 1km. The shape of $\bar{\varepsilon}(\Delta r) - \Delta r$ curves does not change much from site to site. The difference of relative displacement and average normal strain along the epicentral distance r depends on the attenuation of propagating velocity waves. However, numerical evaluation of the parameters shows the attenuation of waves is less important than the loss of coherence as waves propagate from the source to the site.

In reality, even uniform plane waves involve scattering due to the imhomogeneity of the medium between the sites. Consequently, the loss of correlation of the waves is to be expected. The loss of coherence for surface waves that propagate along the epicentral direction is faster than that of shear wave propagating along the same direction. This is because the surface waves propagate on or near the ground surface and the shear waves comeing directly from the soil layer beneath the site, so the site condition may have greater influence on the coherence of surface waves than that of the shear waves.

Fig.12 shows the change of time trace of average strain waves with respect to separation distance. With respect to waveform and duration, the strain traces for different separation distances appear to be somewhat different. As the time trace of average strain loses higher frequency component with increasing separation distance, for example, for Δr =7km, strain wave resembles displacement wave shown in Fig.7(a). The reason for this is that the relative displacement between two points functions as one kind of space filter.

CONCLUSIONS

The major results obtained in this study may be summarized as follows:

(1) An analytical method is developed for estimating the relative displacement, local strain and average strain spectra, using the fault dislocation and normal mode theory.
(2) It is found that the S-wave velocity plays a particularly important role in estimating the phase velocity. This fact encourages us to use only the S-wave velocity in the ground as independent variable in numerical inversion of surface waves.
(3) The usual method based on wave propagation of the fundamental mode may underestimate true ground motion amplitudes. Modal superposition should be essentially taken into account in compliance with choice of ground motion parameters and crustal structures.
(4) The adequancy of present procedure in modeling for relative ground motions induced by surface waves at Vanowen Street was examined with the fault parameters which are given by 1971 San Fernando earthquake. The fit to data is fairly good for the synthetic ground motions and strains induced by Rayleigh waves.

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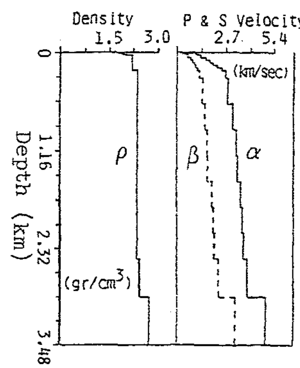

Fig.1

Crustal Structure Model (Vanowen Street,CA,USA)

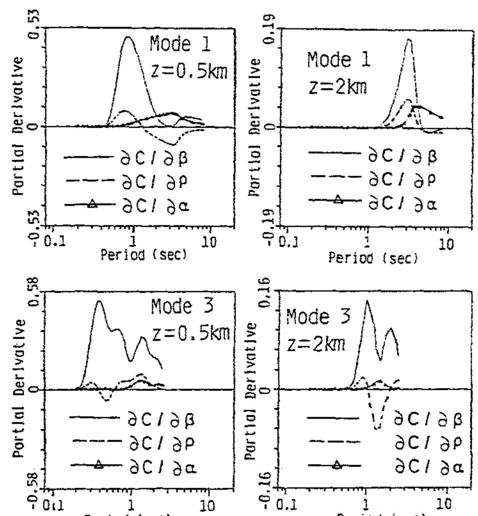

Fig.2

Partial Derivatives of Rayleigh-Wave Phase Velocities relative to Elastic Parameters for Fixed Depth

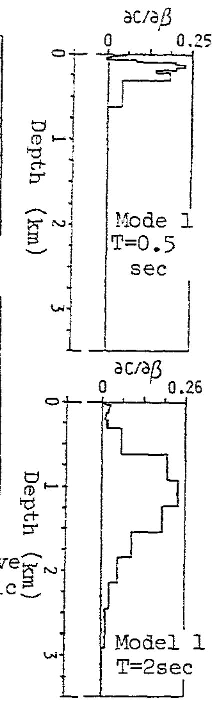

Fig.3

Partial Derivative -Depth Functions of Rayleigh-Wave Phase Velocities Induced by Changes of Elastic Parameters for Fundamental Mode and Fixed Periods

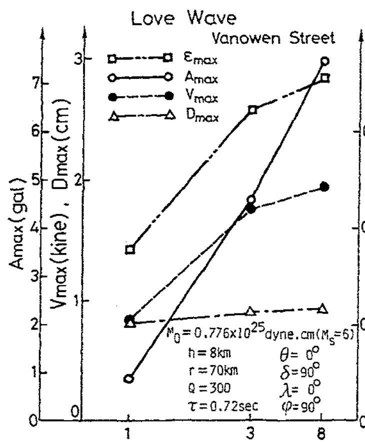

Fig.4

Effect of Modal Superposition on Earthquake Intensity Parameters of Love Wave (Left-Lateral Strike Slip Fault striking East)

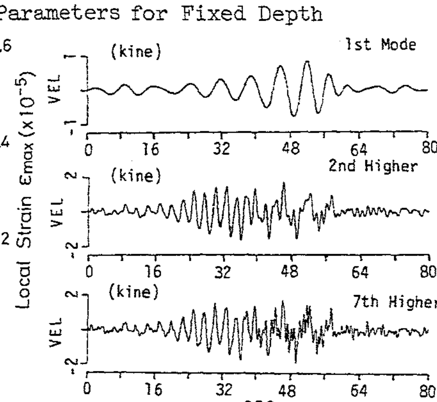

Fig.5 Effect of Modal Superposition on Love-Wave Synthetic Ground Velocities (Left-Lateral Strike Slip Fault striking East)

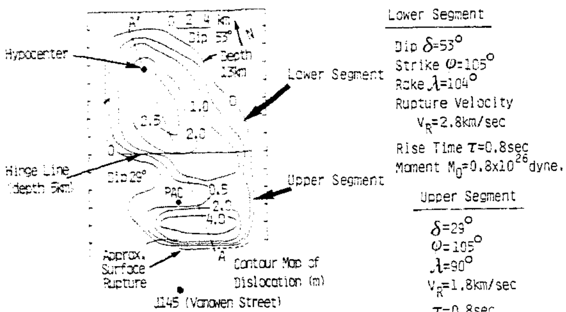

Fig.6 Assumed Fault Trace for the 1971 San Fernando Earthquake

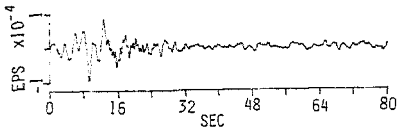

Fig.8 Local Strain Induced by Synthetic Rayleigh Wave at Vanowen Street

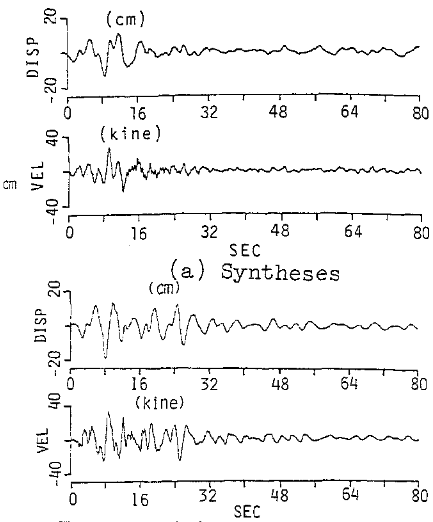

Fig.7 Rayleigh Waves at Vanowen Street

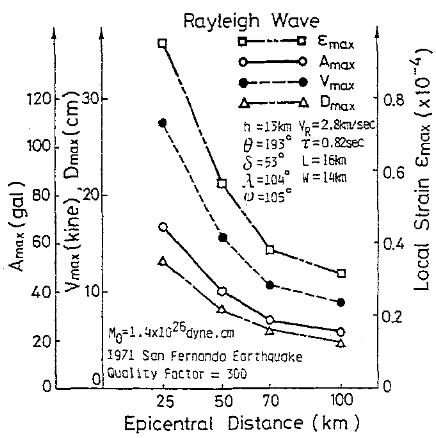

Fig.9 Attenuation of Earthquake Intensity Parameters for Rayleigh Waves

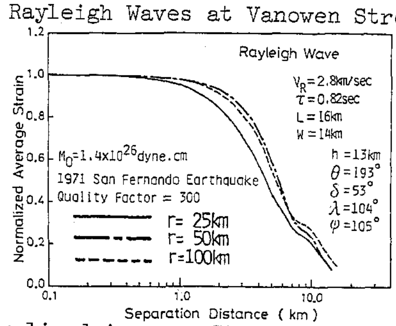

Fig.11 Normalized Average Strain versus Separation Distance for Rayleigh Waves

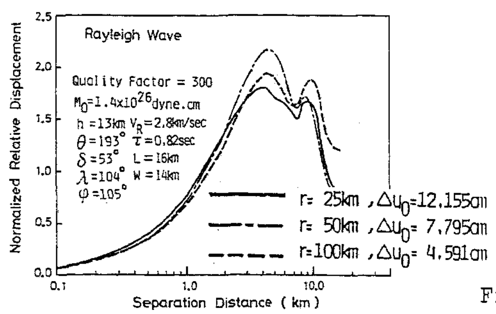

Fig.10 Normalized Relative Displacement versus Separation Distance for Rayleigh Waves

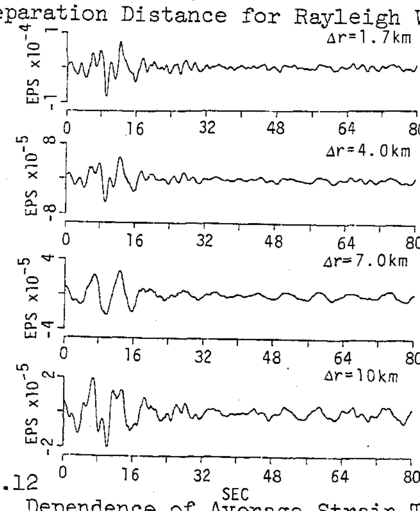

Fig.12 Dependence of Average Strain Time Trace on Separation Distance for Rayleigh Wave at Vanowen Street