

## INVESTIGATING THE UNCERTAINTY IN GROUND MOTION PREDICTION

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### SUMMARY

A study was conducted to develop attenuation relationships for the root mean square acceleration ( $RMS_a$ ) and peak ground acceleration (PGA). To describe the attenuation of each parameter, four mathematical relationships were investigated. Estimates of the  $RMS_a$  were obtained using three measures of strong motion duration for a data set of worldwide strong motion records that remained constant throughout the analysis. The results of the study provided a basis to consider the influence of functional form, ground motion measure (i.e., PGA or  $RMS_a$ ) and  $RMS_a$  measure on predicted ground motions. Significant observations are that the peak accelerations have lower variability than the  $RMS_a$ , and the greatest variation in median prediction curves (e.g., PGA and  $RMS_a$ ) are at short distances, where the data set is the most sparse.

### INTRODUCTION

A study was conducted to develop empirical attenuation relationships for the root mean square acceleration ( $RMS_a$ ) and peak ground acceleration. Although considerable attention has been paid to the potential of the  $RMS_a$  and duration as engineering and seismological measures of ground motion intensity (Ref. 1-4), there has not been a comprehensive, empirical study of  $RMS_a$  attenuation.

In this work we consider a number of questions related to  $RMS_a$  attenuation. Among them, is the matter of the relative variability between the  $RMS_a$  and peak ground acceleration (PGA); the dependence of  $RMS_a$  estimates on the method of defining the strong motion duration, and the influence of the assumed mathematical relationship used to predict ground motion attenuation as a function of magnitude and distance. The results of a study that has considered these issues are summarized. A complete description of the investigation is provided in Reference 5.

### METHOD OF ANALYSIS

Most empirical ground motion attenuation relationships have been developed for peak amplitudes of ground shaking; acceleration, velocity or displacement. Although considerable attention has been paid to the potential of  $RMS_a$  and duration as engineering and seismological measures of ground motion intensity (Ref. 1-4), there has not been a comprehensive, empirical study of  $RMS_a$  attenuation. To develop  $RMS_a$  and PGA attenuation relationships the following steps were taken:

- Four mathematical functions were selected to describe the attenuation with magnitude and distance.

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- An up to date strong motion data base was developed.
- $RMS_a$  and duration values were calculated for each strong motion record for three definitions of duration.
- Regression analyses were conducted for each mathematical model, for PGA and each measure of the  $RMS_a$ .

Critical steps in the analysis involved the selection of the independent model parameters and the form of the mathematical relationships. Also, when evaluating the  $RMS_a$ , a method to determine the duration of strong ground motion was an important step. In this work, to avoid possible bias, three measures of duration were evaluated and the data base was kept constant throughout the analysis. In subsequent paragraphs each step in the attenuation study is described.

#### MATHEMATICAL MODEL

Typically, studies to develop empirical ground motion models select a single, preferred mathematical relationship to describe the attenuation of strong motion amplitudes. Models are selected on the basis of simplicity and fundamental physical arguments that lead to the basic functional dependence of ground motion on the independent variables, usually magnitude and distance. The results of recent studies (Ref. 6-8) (and this work) show that for the range of magnitudes and distances where adequate data exists, differences in median ground motion estimates differ by very little. However, in regions of the magnitude-distance space where sampling has been less complete (i.e., for large magnitudes and at short distances), the greatest discrepancies appear. Furthermore, statistical analysis (Ref. 8) shows that models with inherently different functional forms cannot be rejected by the data.

To describe the attenuation of the  $RMS_a$  with magnitude and distance, four mathematical relationships were used. The functional forms were selected to provide some diversity in the characterization of ground motion attenuation. For reference the models are referred to hereafter as Models I, II, III, and IV.

Model I, which is similar to the relationship recently used by Joyner and Boore (Ref. 7) for PGA, is expressed by the following equation:

$$\log_{10} Y = a + bM + d \log_{10} [(R^2 + h^2)^c] \quad (1)$$

where Y is the ground motion parameter, M is moment magnitude, R is the source-site distance as defined in the next section, h is a parameter derived in the regression analysis which can be considered a pseudo-depth measure, c is taken as 1/2, and a, b, and d are empirical constants. Equation 1 differs slightly from the relation used by Joyner and Boore in two respects; the coefficient on the geometric spreading term, d, is allowed to vary, whereas Joyner and Boore set d = -1; and their anelastic attenuation term is not included here.

Model II corresponds to the functional form described by Campbell (Ref. 6), and is expressed as:

$$\log_{10} Y = a + bM + d \log_{10} [R + c_1 \exp(c_2 M)] \quad (2)$$

The significant feature of Campbell's equation is the term,  $c_1 e^{c_2 M}$ , which describes a magnitude dependence of the transition from near-field to far-field attenuation. Implied here, and in contrast to equation 1, is a distance saturation that depends on the extent of the fault rupture zone.

Model III was developed for this study from consideration of the expression for the near-field response of an elastic whole space (Ref. 9). Based on this, the attenuation of ground motion amplitudes was inferred to assume the general form:

$$\log_{10} Y = a + bM + d \log_{10} \left[ \frac{c_1}{R^2} + \frac{c_2}{R} \right] + eR \quad (3)$$

The significant feature of this model is the incorporation of the first and second order geometric spreading terms,  $R^{-1}$  and  $R^{-2}$ , respectively. Also, the term  $eR$  accounts for anelastic attenuation. With respect to the objectives of this study, Model III provides a unique variation in functional form that will further test the impact of modeling assumptions.

The fourth model is a linear relationship of the form,

$$\log_{10} Y = a + bM + d \log_{10} [R + h] \quad (4)$$

Equation 4 is a relationship that has been used in a number of studies in the last twenty years (Ref. 10). The constant term  $h$  is arbitrarily taken as 25, a value typically used. Clearly, a best fit of the data would be better achieved had  $h$  been derived in the regression analysis. Recall this was the case in equation 1. However, in order to investigate the potential variability in prediction equations, the simplicity of equation 4 and its historical perspective provide the principal motivation for its selection.

In an approach similar to that used by Campbell (Ref. 6), weights were assigned to each record in the regression analysis in order to control the influence of those few events having multiple records. In this study the 1979 Imperial Valley main event, the Imperial Valley aftershock, 1979, and the 1971 San Fernando earthquake combined to represent 62% of the records in the data base. In an unweighted analysis, all the records have the same weight and the marginal contribution of the other 15 poorly recorded events would be small. The selection of the weights was based on distance. In this way poorly or singly recorded events will enhance, at their recording distances, the information on spatial attenuation given by the multiply recorded events.

#### DATA BASE

Digitized strong ground motion records from eighteen earthquakes were used in the statistical analysis. The data base was composed of 83 North American and foreign records with magnitudes greater than 5.0. Although the tectonics and travel path conditions during foreign events may be different from those in western North America, the additional information provided by the foreign events with regard to understanding near-field strong ground motion was considered more relevant.

The data base was established according to criteria that is a composite of the selection processes used by Campbell (Ref. 6) and Joyner and Boore

(Ref. 7). Reference 5 describes the basis on which strong motion records were included in the data base.

The significant distance was measured from the recording site to the closest point of rupture on the fault. The characterization of earthquake magnitude was expressed in terms of moment-magnitude as defined by Hanks and Kanamori(Ref. 11).

No distinction was made in the analysis between geologic conditions at the recording stations. Records obtained in buildings with more than two stories were not included in the analysis based on evidence reported in past studies (Ref. 12, 13). These studies indicate that tall structures can significantly influence the character of ground motion and contribute to the overall variability in attenuation.

#### ESTIMATING THE ROOT MEAN SQUARE ACCELERATION

The  $RMS_a$  of strong motion accelerograms is usually estimated by first identifying the significant duration of the record. In recent years, a variety of methods have been proposed to determine duration and thus the root mean square of strong motion accelerograms. To ensure that the  $RMS_a$  attenuation relations derived here are not biased by the method of determining the strong motion duration,  $T_D$ , three methods were used.

Several definitions of strong ground motion duration have been introduced in the literature. Three of these definitions proposed by, Trifunac-Brady (Ref. 14), Vanmarcke-Lai (Ref. 15), and McCann (Ref. 3) were used in this study. The  $RMS_a$  values estimated by these definitions are denoted,  $RMS_a(TB)$ ,  $RMS_a(VL)$ , and  $RMS_a(M)$ , respectively. Each of the three definitions of duration are summarized in the references cited above.

#### REGRESSION ANALYSIS RESULTS

From the set of PGA and three  $RMS_a$  estimates and four functional forms of the attenuation equation, a total of twelve  $RMS_a$  attenuation relationships and four PGA attenuation curves were developed. For the four functional forms (Models I-IV), regression analysis using the method of least squares was performed. Table 1 summarizes the results for PGA and each  $RMS_a$  measure and mathematical model. Included in the table are estimates of the regression coefficients, the goodness-of-fit,  $r^2$ , the logarithmic standard deviation,  $\sigma$ , and the factor  $10^\sigma$  corresponding to the one standard deviation level of the lognormal distribution.

From a comparison of the results for PGA and  $RMS_a$ , we find that  $RMS_a$  is not a more stable measure of ground motion, at least within the context of the attenuation functions considered here. A comparison of four PGA attenuation and twelve  $RMS_a$  functions resulted in PGA having a higher goodness-of-fit and a lower standard deviation than the  $RMS_a$  in all cases. Estimates of the variability of PGA ranged from a factor of 1.43 to 1.50, while the  $RMS_a$  had results that ranged from 1.52 to 1.82.

A comparison of the results for each  $RMS_a$  measure suggests that the  $RMS_a(TB)$  and  $RMS_a(VL)$  estimates have nearly equivalent statistics for all four attenuation models. Estimate of  $r^2$  and  $\sigma$  for the  $RMS_a(M)$  measure are in all cases not as good as those for the other two. The range of  $r^2$  is from 0.58 to 0.81, with Models I and II having nearly equivalent  $r^2$  values in all cases, indicating the best fit to the data.

The  $RMS_a$  residuals (variation) about the best fit curve are typified by estimates of the logarithmic standard deviation that ranges from 0.181 to 0.261. As noted above, the variability in  $RMS_a$  the data is, in all cases higher than that estimated for PGA.

With respect to the model dependence of the median  $RMS_a$  equations, Figure 1 presents a comparison of the median  $RMS_a$  (VL) curves are for magnitudes 5.5, 6.5, and 7.5 for all four attenuation models. The figure shows that the largest discrepancies occur at short distances, less than 10 km, where the data sample is the most scarce. For distances greater than 10 km, these differences are not great ( $\pm 20\%$ ), even for large magnitude events ( $M = 7.5$ ). A similar comparison of the plus one standard deviation curves suggests greater differences in the magnitude 5.0-5.9 range, while at the higher magnitudes discrepancies remain insignificant.

The influence of the definition of duration on the derived  $RMS_a$  attenuation relations is illustrated in Figure 2. Attenuation curves for each definition are compared using the same functional form (Model II). The variation is,  $\pm 20\%$  in the worst case.

As part of the regression analysis the distribution of the residuals about the best fit curve was investigated. Specifically, the validity of the generally used assumption that the log residuals are normally distributed was tested. Also, the possibility of bias in the regression was investigated by studying the correlation between the normalized weighted residuals with magnitude, distance, and the predicted  $RMS_a$ .

The test statistics performed on the normalized weighted residuals indicated that the data cannot disprove the hypothesis that the residuals are normally distributed. These results also indicate that the residuals are not linearly correlated with either magnitude, distance or predicted  $RMS_a$  values.

#### CONCLUSIONS

Peak ground acceleration and  $RMS_a$  attenuation relationships were obtained by performing a weighted regression analysis of 83 strong motion records from 18 earthquakes. Four mathematical expressions were used to model the dependence on magnitude and distance and three definitions of strong ground motion duration were considered to evaluate the  $RMS_a$ .

From a study of PGA and  $RMS_a$  attenuation, we find that  $RMS_a$  is not a more stable measure of ground motion, at least within the context of the attenuation functions considered here. A comparison of four PGA attenuations and twelve  $RMS_a$  functions resulted in PGA having a higher goodness-of-fit and lower standard deviation than the  $RMS_a$  in all cases.

From the four attenuation models, variability in the median PGA curves did not exceed  $\pm 20\%$  at distances greater than 10 km. At distances less than 10 km, the variation was greater, as much as  $\pm 50\%$ . For  $RMS_a$ , similar observations were made. A comparison of  $RMS_a$  attenuation for different measures of duration indicated that the largest variation did not exceed  $\pm 35\%$ , again at short distances. Beyond 10 km this difference was less.

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TABLE 1 REGRESSION ANALYSIS RESULTS FOR PGA AND RMS<sub>a</sub>

MODEL	a	b	c1	c2	d	e	h	r <sup>2</sup>	σ	10 <sup>σ</sup>
<u>PGA</u>										
I	-1.320	0.262	-	-	-0.913	-	3.852	0.84	0.158	1.44
II	-1.115	0.341	1.000	0.333	-1.270	-	-	0.85	0.154	1.43
III	-2.000	0.270	0.968	0.312	0.160	-0.0105	-	0.81	0.175	1.50
IV	1.009	0.222	-	-	-1.915	-	25*	0.76	0.174	1.49
<u>RMS<sub>a</sub>(TB)</u>										
I	1.095	0.270	-	-	-1.019	-	4.656	0.81	0.184	1.53
II	1.458	0.334	2.071	0.238	-1.423	-	-	0.81	0.185	1.53
III	0.246	0.276	0.998	-0.968	0.167	0.0115	-	0.74	0.220	1.66
IV	3.304	0.244	-	-	-1.992	-	25*	0.74	0.195	1.57
<u>RMS<sub>a</sub>(VL)</u>										
I	1.310	0.269	-	-	-1.050	-	5.779	0.80	0.182	1.52
II	1.789	0.347	2.675	0.243	-1.545	-	-	0.80	0.181	1.52
III	0.472	0.268	0.966	0.255	0.129	0.01167	-	0.75	0.207	1.61
IV	3.673	0.217	-	-	-2.003	-	25*	0.75	0.188	1.54
<u>RMS<sub>a</sub>(M)</u>										
I	1.589	0.201	-	-	-0.979	-	4.073	0.72	0.227	1.69
II	1.660	0.336	0.464	0.481	-1.476	-	-	0.73	0.223	1.67
III	0.698	0.225	1.000	9.996	0.175	0.0113	-	0.67	0.250	1.78
IV	4.011	0.132	-	-	-1.908	-	25*	0.58	0.261	1.82

\*assigned value

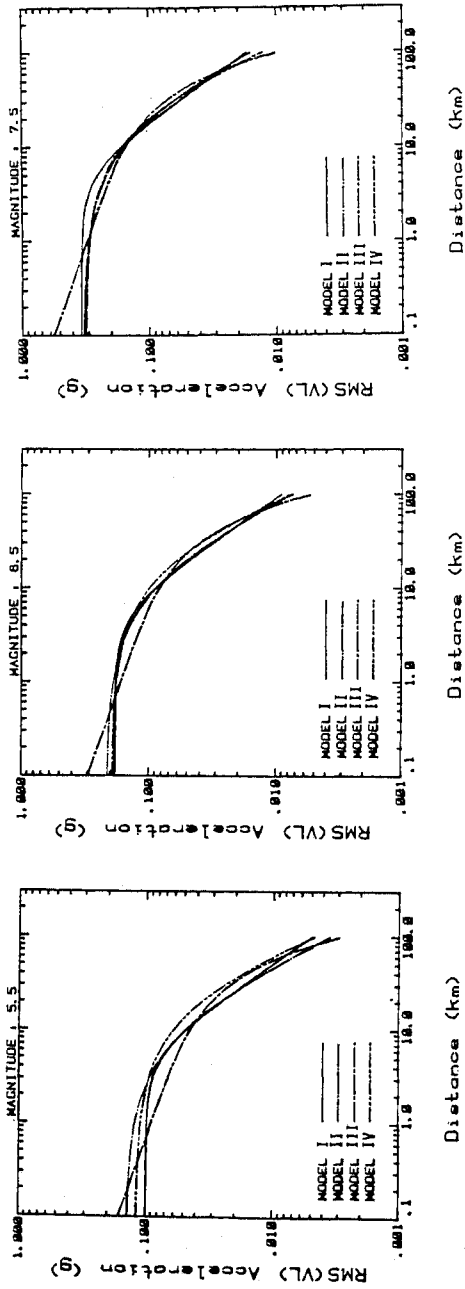


Figure 1.  $RMS_a$  (VL) median attenuation curves for all four models and magnitudes of 5.5, 6.5, and 7.5.

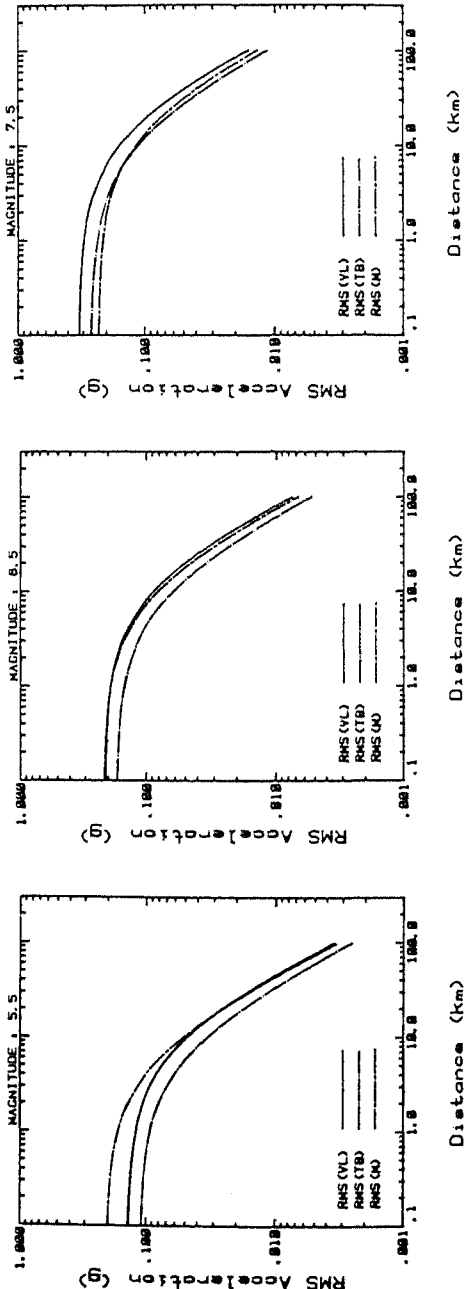


Figure 2. Comparison of the median  $RMS_a$  attenuation curves using each definition of duration for Model II and magnitudes of 5.5, 6.5, and 7.5.