STUDY ON THE ATTENUATION OF SEISMIC WAVES

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SUMMARY

This paper discusses the attenuation coefficient of seismic waves between two observation places by using the maximum acceleration amplitudes of the strong motion seismograph records obtained in Japan and the U.S.A.. The results of the present investigation tell us more certainly that the large number of buildings in the area of a big city has a great influence on the attenuation coefficient of seismic waves. The present result seems to be a considerably important part of the engineering seismology problem. By the way, this result may be utilized for disaster prevention measures in city planning.

INTRODUCTION

In the present investigation, the attenuation coefficient of seismic waves between two observation places is dealt with more quantitatively by using the maximum acceleration amplitudes of the strong motion seismograph records obtained in Japan and the U.S.A.. The number of the sets of two observation places in Japan and the U.S.A. are 68 and 11 respectively. Moreover, we discuss the attenuation coefficient of seismic waves in the metropolitan areas of Japan on the basis of the transmission theory of Love-type waves on the ground on which masses are distributed.

METHOD OF ANALYSIS

Generally speaking, the relation between the amplitude of seismic waves a, and its transmission distance x, can be expressed as follows:

$$a \propto x^{-n} e^{-kx} \tag{1}$$

in which n represents a constant by types of seismic waves and is often given a value of 1.0 and 0 and 0.5 for spherical and plane body waves and surface waves respectively. The attenuation coefficient k, is represented by the function of viscosity coefficient, period of seismic waves, and transmission velocity. Here, as a first approximation, we assume that viscosities and velocities of the surrounding ground of two observation places do not differ, and the periods of the maximum acceleration waves at their places are about the same. If the maximum amplitudes at two observation places are represented by a_1 and a_2 respectively then the ratio of a_1 and a_2 may be expressed as follows:

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$$\frac{a_1}{a_2} = \frac{a_0\, \alpha_1\, \Delta_1^{-n}\, e^{-k\Delta_1}}{a_0\, \alpha_2\, \Delta_2^{-n}\, e^{-k\Delta_2}} = \frac{\alpha_1}{\alpha_2} \left(\frac{\Delta_1}{\Delta_2}\right)^{-n} e^{-k(\Delta_1 - \Delta_2)} \tag{2}$$

where a_0, Δ and α are amplitude at origin, transmission distance (here, we use epicentral distance) and constant relating to the ground and installation conditions of seismograph respectively. Next, Eq.(2) may be rewritten as follows:

$$\log_e \left\{ \frac{a_1}{a_2} \left(\frac{\Delta_1}{\Delta_2}\right)^n \right\} = \log_e \left(\frac{\alpha_1}{\alpha_2}\right) - k(\Delta_1 - \Delta_2) \tag{3}$$

As an example, result calculated by Eq.(3) is shown in Fig.1. From Fig.1, we can obtain the values of k and α_1/α_2. That is, in Fig.1, k is inclination of straight line obtained by the method of least squares, and α_1/α_2 is the value of ordinate corresponding to abscissa zero.

RESULTS

Attenuation Coefficients of Seismic Waves in Japan.

Firstly, with Eq.(3), we try to get the attenuation coefficients of seismic waves k, between two observation places by using the maximum acceleration of the strong motion seismograph records (SMAC type, natural period of pendulum is 0.1 sec) obtained in Japan during 1963~1979. The maximum acceleration of the strong motion seismograph records obtained at two observation places treated in this case are greater than 10 gals and where more than three earthquakes are recorded. The relation between the attenuation coefficients of seismic waves and the number of the sets of two observation places in Japan are shown in Fig.2 and Table 3 (case of n is 0.5).

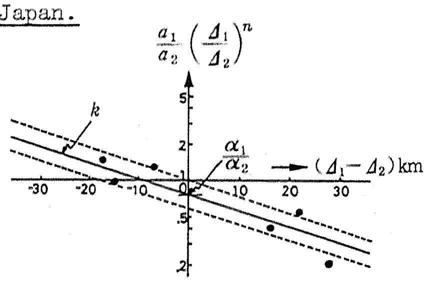

Fig.1 An example of calculation used Eq.(3)

Next, with Eq.(3), we try to get the attenuation coefficients of seismic waves between two observation places by using the maximum acceleration records (Ishimoto Seismograph, natural period of pendulum is 0.1 sec) obtained in the metropolitan areas of Japan in 1932~1934. On the other hand, with Eq.(3), the attenuation coefficients of seismic waves between two observation places by using the maximum amplitude records (1 Second Seismograph) in the same areas in 1954~1955 has already been obtained by one of the authors (Ref. 1). The number of the sets of two observation places in 1963~1979, 1954~1955 and 1932~1934 are 68, 9 and 8 respectively.

Table 1. Attenuation coefficients of seismic waves in Japan (R: ratio of no. of buildings)

Region	Observation period	R	k (km^{-1})
Metropolitan areas	1963 ~ 1979	10	0.034
	1954 ~ 1955	3	0.029
	1932 ~ 1934	1	0.020
Other regions	1963 ~ 1979	-	0.013

The mean values of the attenuation coefficients in the metropolitan areas of Japan in three periods and regions other than the metropolitan areas in 1963∿1979 are shown in Table 1. From Table 1, it may be said that the attenuation coefficients in the metropolitan areas showed a tendency to increase with period, and also it is larger than that in the other regions during the same period. In other words, from Table 1, it may be explained that the large number of buildings in the big city areas has a great influence on the attenuation coefficient of seismic waves.

Attenuation Coefficients of Seismic Waves in the U.S.A.

With Eq.(3), we try to get the attenuation coefficients of seismic waves k, between two observation places by using the maximum acceleration of the strong motion seismograph records (USCGS standard type, natural period of pendulum are 0.063∿0.080 sec) obtained in the U.S.A. in 1933∿1971 (Ref. 2). The number of the sets of two observation places is 11. The results of calculation are shown in Table 4. Comparison between the mean values of the attenuation coefficients of seismic waves in Japan and the U.S.A. are shown in Table 2. As shown in Table 2, the attenuation coefficients in the big city areas are larger than those in the other regions, not only in Japan but also in the U.S.A.. From Table 2, it seems that the attenuation coefficients in Japan are larger than that in the U.S.A..

Table 2. Comparison of the attenuation coefficients of seismic waves in Japan and the U.S.A.

Region	Japan	U.S.A.
Big city areas	0.034	0.015
Other regions	0.013	0.005

Consideration of the Attenuation Coefficient of the Metropolitan Areas of Japan on the Basis of the Theory Underlying the Transmission of Surface-Type Waves.

In the present investigation, we dealt with applied problem of theory (Ref. 3) underlying the transmission of Love-type waves on the ground on which one mass systems are distributed on a semi-infinite elastic body (see Fig.3).

If we assume that U and u be the horizontal displacements of the mass systems and the ground respectively; and for convenience, the surface layer is considered non-viscid, then the equation of the vibratory motion of mass system and surface layer become as follows:

$$M\frac{\partial^2 U}{\partial t^2} + \tau\frac{\partial}{\partial t}(U-u) + c(U-u) = 0 \tag{4}$$

$$\rho\frac{\partial^2 u}{\partial t^2} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{5}$$

in which M, τ and c are the mass per unit area in g/cm^2, damping force in g·sec/cm and elastic modulus in g/cm of the mass system respectively; and ρ and μ are the density in g/cm^3 and rigidity in g/cm·sec^2 of the ground respectively ($\sqrt{\mu/\rho}\equiv V_s$, V_s is velocity of S-waves in m/sec). The boundary condition in $z=0$ becomes as follows:

$$\mu\frac{\partial u}{\partial z} + \tau\frac{\partial}{\partial t}(U-u) + c(U-u) = 0 \tag{6}$$

The solution of the form U and Eq.(5) are expressed by

$$U = Ae^{i(pt-fx)} \tag{7}$$

$$u = Be^{i(pt-fx)-sz} \tag{8}$$

where

$$s^2 = f^2 - K^2, \quad K^2 = \frac{\rho p^2}{\mu} \tag{9}$$

From Eq.(4)

$$c = M\left(\frac{2\pi}{T_0}\right)^2, \quad \tau = 2Mh\frac{2\pi}{T_0} \tag{10}$$

where h and T_0 are damping coefficient and natural period of mass systems respectively, and $p=2\pi/T$, T is period of waves.
By substituting Eqs.(7) and (8) for Eqs.(4) and (5), we have

$$\{\mu sc - Mp^2(\mu s + c)\} + ip\tau(\mu s - Mp^2) = 0 \tag{11}$$

In order to get the velocity of transmission and the attenuation coefficient of the waves, we put

$$f = f_1 - if_2 \tag{12}$$

in which $f_1=2\pi/VT$, V is velocity of transmission, f_2 is the attenuation coefficient for the wave transmission distance, and f_2 has significant meaning for k in Eq.(1). By substituting Eq.(12) into Eq.(11), it can be expressed as follows:

$$\phi\{\mu(c - Mp^2)\cos q + p\tau\mu\sin q\} - Mp^2c = 0 \tag{13}$$

$$\phi\{p\tau\mu\cos q - \mu(c - Mp^2)\sin q\} - Mp^3\tau = 0 \tag{14}$$

where

$$\phi = \{(f_1^2 - f_2^2 - K^2)^2 + (2f_1f_2)^2\}^{1/4} \tag{15}$$

$$q = \frac{1}{2}\tan^{-1}\left(\frac{2f_1f_2}{f_1^2 - f_2^2 - K^2}\right) \tag{16}$$

Eqs.(13) through (16) may be rewritten as follows:

$$\Psi\left[\{(1 - \left(\frac{T_0}{T}\right)^2\}\cos Q + 2h\sin Q\left(\frac{T_0}{T}\right)\right] - 4\pi^2\frac{M}{\mu K}\left(\frac{1}{T}\right)^2 = 0 \tag{13'}$$

$$\Psi\left[\{\left(\frac{T_0}{T}\right)^2 - 1\}\sin Q + 2h\cos Q\left(\frac{T_0}{T}\right)\right] - 8\pi^2\frac{M}{\mu K}h\left(\frac{1}{T}\right)^2\left(\frac{T_0}{T}\right) = 0 \tag{14'}$$

where

$$\Psi = \{(F_1^2 - F_2^2 - 1)^2 + (2F_1F_2)^2\}^{1/4} \tag{15'}$$

$$Q = \frac{1}{2}\tan^{-1}\left(\frac{2F_1F_2}{F_1^2 - F_2^2 - 1}\right) \tag{16'}$$

$$F_1 = \frac{f_1}{K}, \quad F_2 = \frac{f_2}{K} \tag{17}$$

If we assume the value of damping coefficient h, of the mass systems (buildings), $(M/\mu K)(1/T)^2$ is a variable and T_0/T is parameter in Eqs.(13)' and (14)', their equations are expressed as the dual simultaneous equations concerning to F_1 and F_2. We shall try to get values of F_1 and F_2, which satisfies Eq.(13)' and Eq.(14)' at the same time by the trial and error method. Their results are shown in Figs.4 and 5.

Next, we assume that damping coefficient h, of the mass systems (buildings) is 0.05, and the attenuation coefficient $k(\equiv f_2)$ is 0.034 (see second line of Table 1). The values of the velocity of S-waves V_s; density ρ; and predominant period of the ground T; adopted in this case are the results (Ref. 4) of soil survey of several sites in Tokyo area. The mass per unit area M, of mass systems (buildings) are calculated for natural periods T_0, of mass systems: 0.20 sec and 0.25 sec by using the results shown in Figs.4 and 5. The mean values of M calculated for T_0=0.20 sec and 0.25 sec are 168 g/cm^2 and 88 g/cm^2 respectively. It can be considered that the values of the former case corresponded approximately to not only the natural period but also the mass per unit area of two storied reinforced-concrete building distributed on the whole ground. That is, it may be said that the attenuation coefficient of seismic waves in a big city can be explained by the theory of the transmission of the Love-type waves on the ground on which the masses are distributed.

Comparison of an Empirical Formula concerning to the Attenuation Coefficient of Seismic Waves obtained in Japan (Excluding the Metropolitan Areas) and Those obtained by Different Processes.

Comparison of an emperical formula obtained here and those by other researchers (Ref. 5) are shown in Fig.6 (In some formulae, epicentral distance x, are used. Here, for simplicity, x is assumed to be the same as the hypocentral distance Δ). From Fig.6, it will be found that the attenuation coefficient of seismic waves in Japan (excluding the metropolitan areas), (k= 0.013km^{-1}), is in considerably good correspondence with the empirical formula obtained by a different process, i.e., 10^{-K} where K is $(1.66+3.60/x)\log_{10}x-1.83/x$.

CONCLUSIONS

The conclusion of the present paper consists of four parts:

(1) The values of the attenuation coefficients of seismic waves in big city areas are larger than those in the other regions, and in big city areas, the values in recent years are larger than those in several ten years ago. That is, the values of the attenuation coefficients of seismic waves are seemed to be affected by the existence of large number of buildings.

(2) In general, the attenuation coefficients of seismic waves in Japan are larger than those in the U.S.A..

(3) The attenuation coefficient of seismic waves in a big city is explained by the theory of the transmission of the Love-type waves on the ground on which masses (buildings) are distributed.

(4) The mean value of the attenuation coefficient of seismic waves in Japan (excluding the metropolitan areas), (k=0.013km^{-1}), is in considerably good correspondence with the empirical formula obtained by a different process, i.e., 10^{-K} where K is $(1.66+3.60/x)\log_{10}x-1.83/x$.

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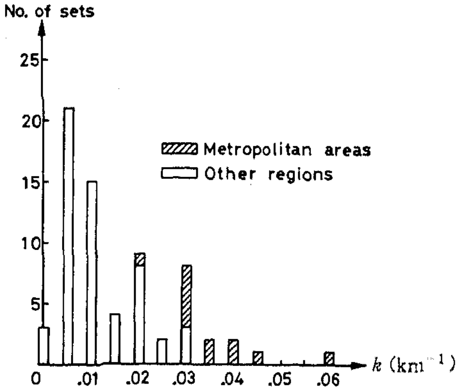

Fig.2 Relation between the attenuation coefficients of seismic waves and the number of the sets of two observation places in Japan

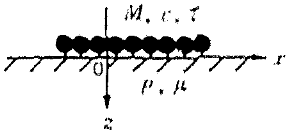

Fig.3 Model of the theory

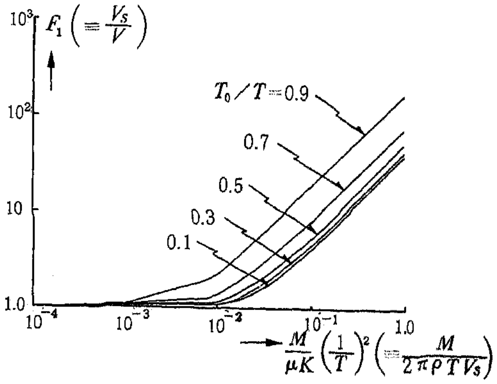

Fig.4 Reciprocal of velocity of Love-type waves

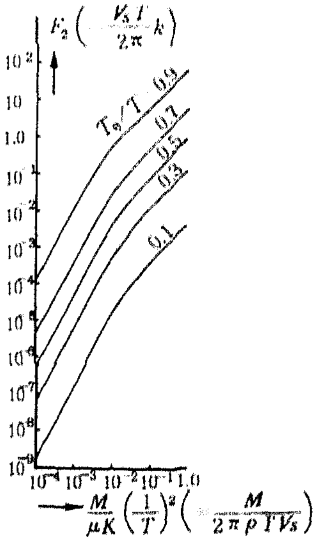

Fig.5 Attenuation coefficient of Love-type waves

Table 3. Attenuation coefficients of seismic waves in Japan (Mark:site number of the strong motion earthquake observation council)

(a) Metropolitan areas

Set of two places	k(km^{-1})
TK004:KT008	0.030
TK012:TK024	0.031
TK012:TK035	0.030
TK020:TK040	0.044
TK035:KT008	0.028
TK040:TK024	0.033
KT004:TK020	0.032
KT004:TK024	0.033
KT004:TK040	0.058
KT008:TK024	0.021
KT011:KT004	0.038
KT026:TK040	0.038

(b) Kanto (Excluding the metropolitan areas)

Set of two places	k(km^{-1})
TK012:KT008	0.021
KT014:KT026	0.005
KT018:KT021	0.009
KT021:KT014	0.019
KT901:KT017	0.010

(c) Hokkaido and Tohoku

Set of two places	k(km^{-1})
HK003:HK004	0.006
HK003:HK005	0.001
HK004:HK006	0.006
HK008:HK010	0.01
HK010:HK003	0.01
HK012:HK003	0.02
HK003:TH014	0.005
HK003:TH029	0.004
HK004:TH029	0.005
HK006:TH029	0.004
TH006:TH029	0.003
TH012:TH006	0.01
TH012:TH014	0.006
TH014:TH015	0.01
TH014:TH016	0.01
TH014:TH019	0.002
TH014:TH029	0.002
TH015:TH016	0.01
TH020:TH006	0.01
TH029:TH032	0.004
TH032:TH016	0.01

(d) Chubu and Kinki

Set of two places	k(km^{-1})
CB031:CB033	0.02
CB031:CB037	0.01
CB034:CB033	0.02
CB034:CB037	0.01
CB047:CB033	0.03
AC009:CB032	0.024
AC009:CB040	0.005
AC012:CB038	0.005
AC002:KK002	0.007
AC002:OS004	0.003
AC002:KK019	0.016
AC008:KK019	0.016
AC009:KK021	0.006
AC011:KK024	0.006
AC012:KK021	0.010
AC017:KK019	0.022
KK019:KK024	0.010
KK020:KK023	0.007
KK021:KK022	0.02
KK021:KK023	0.024
KK023:KK022	0.03
KK024:OS017	0.007
KK026:CB039	0.028
OS016:KK022	0.01

(e) Chugoku, Shikoku and Kyushu

Set of two places	k(km^{-1})
CG005:CG008	0.007
CG002:SK010	0.016
SK005:SK010	0.015
SK005:KS003	0.019
SK006:KS002	0.006
SK010:KS002	0.007

Table 4. Attenuation coefficients of seismic waves in the U.S.A.

Region	Set of two places	k(km^{-1})
Big city areas	Hollywood, L.A.:Long Beach	0.009
	Hollywood, L.A.:Vernon	0.008
	Hollywood, L.A.:Colton	0.010
	Subway, L.A. :Long Beach	0.019
	Subway, L.A. :Vernon	0.040
	Subway, L.A. :Colton	0.010
	Long Beach :Vernon	0.018
	Vernon :Colton	0.009
Other regions	EL Centro :San Diego	0.008
	San Diego :Vernon	0.004
	San Diego :Colton	0.004

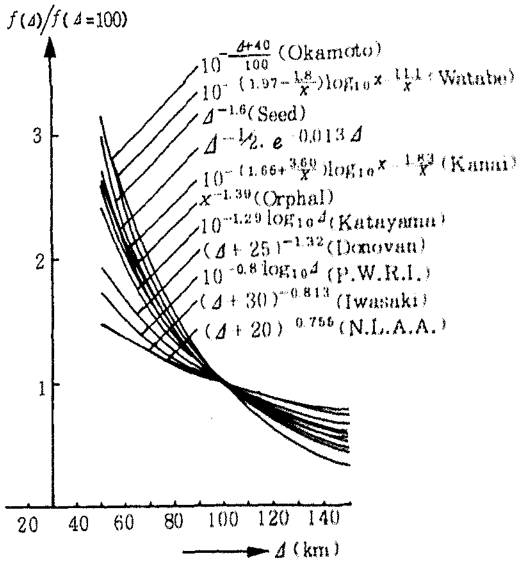

Fig.6 Comparison of an empirical formula obtained here and those by other researchers

APPENDIX

One of the authors has already carried out the measurement of the ground vibrations caused by air compressors in a site of about 20 hectares under two different conditions*. In one of them, no structure was existing on the site, and in the other, 160 numbers of 2∿4 storied reinforced-concrete buildings (apartment houses) were constructed on the same site. The results are shown in Table 5, from which, it can be said that the attenuation coefficient of the ground after the buildings were built was larger than that of the before, and it is found that the result agrees qualitatively with the conclusion of the present paper. By the way, the reason for the difference between the order of the attenuation coefficients of the two results, that is, Table 1 and Table 5, may be explained by the problems of the distance from origin and the others.

Table 5. Attenuation coefficients of elastic waves $k(\mathrm{km}^{-1})$, on the ground before and after more than one hundred R.C. buildings were built

	Before	After
Radial	2.3	4.7
Transverse	2.8	5.8

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