RECENT DEVELOPMENTS IN DATA PROCESSING OF STRONG-MOTION ACCELEROGRAMS

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SUMMARY

The current developments in data processing of strong motion accelerograms continue with the assumption that most of the acceleration data available will be on analog films. With this type of recording and hand digitization, some ten years ago, the data processing techniques were developed. More recently, we have completed the development of automatic digitization system (ARDS). This system together with other modern laser scanners is capable of producing data at 200 or more points/sec.; compared to manually digitized data which typically averaged only 20-30 points/sec. A new generation of data processing software is thus evolving to include data of frequencies up to 100 Hz and higher. Within this frequency range, a routine to determine the optimum band pass frequency limits that are free from digitization and processing noise is adopted. An accurate and efficient band pass filter is then developed to perform the band pass filtering of the data with the variable frequency limits. Within this frequency range, a new differentiation filter is used for instrument correction, and similarly an integration filter is developed to calculate the velocity and displacement data. Methods for estimating the unknown initial velocity and displacement are investigated.

INTRODUCTION

The main objective of the recent work in this subject area has been to improve and extend the existing data processing software for analyzing strong-motion accelerograms. The existing data processing method was developed more than ten years ago at California Institute of Technology, when only hand digitized accelerograph data were available. With the development of the automatic routine digitization system (ARDS) completed at University of Southern California (USC) in 1978, and with the recent advances in digital filter theory, a new and more refined data processing software can now be implemented.

The development of routine computer programs for processing strong motion earthquake accelerograms was initiated and completed in the early 1970's, and consists of the following steps:

(A) Volume I Processing: The timing marks are first checked for "evenness" of spacing, and then smoothed by the ¼, ½, ¼ running filter. The x coordinates of each trace are then scaled to units of time in seconds. Each fixed trace (baseline) is next smoothed and subtracted from the

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corresponding acceleration trace, where y coordinates are subsequently scaled to units of g/10.

(B) Volume II Processing: The scaled, uncorrected Vol. I accelerogram data are next corrected for instrument frequency response and baseline adjustment. The data are first low pass filtered with an Ormsby filter having a cut-off frequency f_c = 25 Hz and a roll-off termination frequency f_T = 27 Hz. Instrument correction is then performed using the instrument constants ω_o and ζ_o determined from calibration tests for each accelerograph component. The data are then baseline corrected by a high pass Ormsby filter. The cut-off and roll-off frequencies of the filter are usually determined from the signal-to-noise ratio of each component (Ref. 15). The accelerogram data are then integrated twice to get the velocity and displacement data. To avoid long period errors resulting from the uncertainties involved in estimating the initial values of velocity and displacement, the computed velocity and displacement data are high-pass filtered at each stage of integration, Using the Ormsby filter with the same cut-off and roll-off frequencies.

(C) Volume III Processing: Using an approach based on the exact analytical solution of the Duhamel integral for successive linear segments of excitation, Response and Fourier Spectra for 91 periods and 5 dampings from the Vol. II corrected accelerogram data are calculated. The times of maximum response for all periods and dampings are also recorded.

Following the development of ARDS, an automatic routine digitization system at USC, the acceleration data are now digitized automatically with at least 200 points/sec. using a Photodensitometer Photoscan P-1000 by Optronics International. A sampling period of .005 second means that data up to a Nyquist frequency of 100 Hz are now available.

RECENT DEVELOPMENTS

Following the recent improvements of the digitization systems, and the advances in the theory of digital filters, we upgraded and improved the accuracy and efficiency of the routine data processing software. These developments can be divided into 4 steps:

Step 1: Automatically determining the frequency limits for bandpass filtering

Before, for the data digitized manually at about 30 points/second, the band pass frequency limits of .05-.07 and 25.-27. Hz have been assumed implicitly in the Volume II processing programs. With the data now digitized automatically, accurate information up to about 100 Hz is available. The long period, low frequency high pass limit, which equals .05-.07 Hz, or .95-.125 Hz, in routine operation, is in fact a variable dependent upon the signal-to-noise ratio of the data at that frequency range (Ref. 15). Since the records digitized are from different locations and of different amplitudes, in the batch of 186 records (558 components) were refiltered with band pass frequencies predetermined by visual inspection to maximize the signal-to-noise ratio within the band (Ref. 15). Now we have developed a routine which for each input component of acceleration data, determines the appropriate limit frequencies for band pass filtering so that the resulting data in the band is as free from noise as possible. The determination of this frequency band for each component involves knowledge of the digitization and processing noise of the system used, to enable the comparison between the Fourier spectra of the actual data and that of the noise.

Step 2: Upgrading the design of Low, High and Band pass filters.

We have studied the availability, characteristics, properties and efficiencies of digital filters for low, high and band pass filtering. Much of the attention here will be paid to low-pass filters since they provide the essential relations for deriving either of the other two types.

An ideal low pass filter passes all low frequencies, $|\omega| \leq \omega_L$, without any change and blocks all high frequencies, $|\omega| > \omega_L$. For input data sampled at equally spaced intervals of δT seconds, the filtered output data will be given by

$$y_n = \sum_{k=-\infty}^{\infty} h_k x_{n-k} , \qquad (1)$$

where $\{h_k\}$ are the filter weights, which for the ideal filter are given by (for $k = -\infty$ to ∞)

$$h_k = \frac{\omega_L}{\pi} \frac{\sin(k\pi\omega_L/\omega_N)}{k\pi\omega_L/\omega_N} . \qquad (2)$$

An exact realization of an ideal low-pass filter thus requires an infinitely long sequence $\{h_k\}$ which extends in the direction of both positive and negative time. Truncation of this sequence to a finite number of terms introduces in the vicinity of the cut-off frequency, so called Gibbs effects, which cannot be eliminated even by increasing the length of the sequence. A common procedure for smoothing and diminishing this discontinuity at the cut-off is to introduce a transition frequency interval within which pass band amplitude (1), gradually rolls off to the stop band amplitude (0). The data processing software developed some 10 years ago uses an Ormsby-type filter which allows a first order roll off from pass-to-stop-band.

Sunder (Ref. 7) proposed the use of an elliptic infinite impulse response (IIR) filter. The input-output relationship is given by

$$y_n = \sum_{k=0}^{M} b_k x_{n-k} - \sum_{k=1}^{N} a_k y_{n-k} \qquad (3)$$

Sunder proposed its use for the reason that elliptic filters are optimal in the sense that for a given order and for given ripple specifications of the amplitudes in both the pass and stop bands, no other filter achieves a faster transition between the pass and stop bands. However, some optimal filters tend to be robust in their maintenance of performance standards when the quantities assumed for design purposes are not the same as the quantities encountered in operation. For the case of earthquake engineering with input earthquake data, signal distortions due to phase characteristics are not permissible and should not be present. The transfer function of the elliptic filter from (3) takes the form (elliptic)

$$H(\omega) = \left(\sum_{k=0}^{M} b_k \, e^{-ik\omega}\right) \Bigg/ \left(1 + \sum_{k=1}^{N} a_k \, e^{ik\omega}\right), \tag{4}$$

where ω is the frequency, normalized so that π corresponds to the Nyquist frequency of the sampled data ($1/2\delta T$). Since no phase-distortion requirement is specified in the design of the resulting IIR elliptic filter, the resulting "optimal" elliptic filter from (4) does give a phase-distorted output. This is why, to eliminate this, the original elliptic filter has to be replaced by the corresponding magnitude-squared filter, $H_{eq}(\omega)$, given by

$$H_{eq}(\omega) = |H(\omega)|^2 . \tag{5}$$

The optimality of the resulting filter is thus lost.

The Ormsby type filter used, on the other hand, has input-output relationship given by

$$y_n = \sum_{k=-1}^{N} h_k \, x_{n-k} \tag{6}$$

with $h_k=h_{-k}$ and the corresponding transfer function given by (Ormsby-type)

$$H(\omega) = h_o + 2 \sum_{k=1}^{N} h_k \cos(k\pi\omega/\omega_N) \quad , \tag{7}$$

a real transfer function as a result of the symmetry of the filter weights h_k. The symmetry thus assures that the filter performs a perfect phase-distortionless transmission.

Comparison of the performances of the two filters is an example of a comparison of the two general types of filters: recursive vs. non-recursive filters. The Ormsby type filter is a non-recursive filter that can respond only to the values of x_m in the range x_{n-N} to x_{n+N} (Equation 6). Non-recursive filters are thus also known as "Finite Impulse Response" (FIR) filters. The elliptic filter used is an example of a "one-sided" physically realizable recursive filter. From its ability to produce, from a single impulse, effects indefinitely far into the future, it is also called an "Infinite Impulse Response" (IIR) filter.

It is known that both types of filters have about the same flexibility to meet the various requirements. However, the transient character of the elliptic filter is evident from its form in Equation (3): the filter used only has memory terms ($k \geq o$) and no anticipation terms ($k < o$). This means that the output values are determined completely in terms of past and present inputs and recursively on the past outputs. Thus at the beginning of time, with a sudden change in input, the filter undergoes a transient state before settling down. The non-recursive Ormsby type filter, on the other hand, has both memory ($k \geq o$) and anticipation ($k < o$) terms, so that the outputs depend on both the past, present and future inputs. It thus shows a much less transient response. Because of this problem, as well as other problems like instabilities and phase shifts, recursive filters tend to be used only in systems where there are very long runs of data; non-recursive filters, on the other hand, are simpler to understand, design and use.

Step 3: Choosing the appropriate differentiation and integration filters

Numerical Differentiation is required for Instrument Correction. The transducer of the recording instrument typically records the relative displacement response of the instrument mass. Thus, for example the Instrument Correction represents the tranformation from the transducer relative displacement x(t) to the input ground acceleration a(t):

$$\ddot{x}(t) + 2\zeta\omega_o\, x(t) + \omega_o^2\, x(t) = - a(t) \tag{8}$$

where ω_o is the natural frequency of the transducer and ζ is the critical damping ratio. In the digital domain, this takes the form

$$a_n = - \ddot{x}_n - 2\zeta\omega_o\, \dot{x}_n - \omega_o^2\, x_n\ , \tag{9}$$

for n=0, 1, 2,.... The existing data processing software (Ref. 14) uses the central difference formulae for the first and second derivatives

$$\dot{x}_n = (x_{n+1} - x_{n-1})/2\delta T \quad , \quad \ddot{x}_n = (x_{n+1} - 2x_n + x_{n-1})/(\delta T)^2. \tag{10}$$

With data originally sampled at 100 points/sec., these formulae are good for frequencies up to 15 Hz to 20 Hz, the average sampling frequency of the hand digitized data.

Sunder (Ref. 7) proposed the use of 9 points' and 31 points' differentiating formulae. With data he proposed to be sampled at 100 point/sec, corresponding to a Nyquist frequency of 50 Hz, both formulae are accurate to frequencies beyond 25 Hz, but they both decrease to magnitude zero at 50 Hz.

Data are now available from the Automatic Routine Digitization System (ARDS) at USC, sampled at 200 points/sec., corresponding to a Nyquist frequency at ω_N = 100 Hz. With the proposed automatic determination of the limit frequencies for band pass filtering, we need a differentiation formula that is accurate for data of frequencies up to the variable cut-off frequency, ω_L, used in the program, with $o < \omega_L < \omega_N$.

Let $\lambda = \omega_L/\omega_N$, the ratio of the cut-offs frequency to that of the Nyquist frequency, with $o < \lambda < 1$. We propose the following for the design of the differentiation filter:

(1) that the filter be such to give the following input-output relationship:

$$\dot{x}_n = \sum_{-N}^{N} C_k\, x_{n-k} = \sum_{1}^{N} C_{-k}(x_{n+k} - x_{n-k})\ , \tag{11}$$

where $N = N(\lambda)$ is the order of the filter, $C_k = C_k(\lambda)$, k = -N,N will be the coefficients of the filter, all of which will depend on λ. The closer is λ to 1, the larger will the order be. The coefficients, C_k, will be chosen to be antisymmetric ($C_o = o$, $C_{-k} = -C_k$). Such an Nth order central difference type formula will give a transfer function, $H(\omega)$, that is purely imaginary

$$H(\omega) = 2i \sum_{k=1}^{N} C_{-k} \sin(k\pi\omega/\omega_N) \tag{12}$$

as in the case of the ideal differentiator, so that no phase errors exist in the filter.

(2) that the coefficients $C_k = C_k(\lambda)$ be chosen to satify Chebyshev minimax criterion within the frequency band $o \leq \omega \leq \omega_L$: to minimize the maximum error in approximating the ideal differentiator

(Ideal)

$$H(\omega) = \begin{cases} i\pi\omega/\omega_N & |\omega| < \omega_L \\ o & |\omega| > \omega_L \end{cases} , \tag{13}$$

which is the ideal filter cut-off at the same cut-off frequencies used in band pass filtering. This filter has the corresponding Fourier sine series given by

$$H(\omega) = 2i \sum_{k=1}^{\infty} b_k \sin(k\pi\omega/\omega_N) ,$$

with

$$b_k = \frac{1}{\pi} \left(\frac{\sin(k\pi\omega_L/\omega_N}{k^2} - \frac{\pi\omega_L \cos(k\pi\omega_L/\omega_N)}{k\omega_N} \right) \tag{14}$$

and finally,

(3) that the ratio |design filter/ideal filter| → 1 as $\omega \to o$, so that they have the same tangency at $\omega = o$.

With the digital acceleration data bandpass filtered and instrument corrected, digital integration is next performed twice to get the velocity and the displacement data. The data processing program developed originally for the manually digitized data used the trapezoid rule of integration,

$$y_{n+1} = y_n + .5\delta T (x_n + x_{n+1}) , \tag{15}$$

with $\{x_n\}$ the input and $\{y_n\}$ the output sequence, δT is the equally spaced sampling time of the sequence in seconds. Since y occurs on both sides of this equation, this is a recursive filter. Again, as in the case of differentiation formulae, with data originally sampled at 100 points/sec., the trapezoidal rule is good for data up to frequencies of ~ 15 Hz. Sunder (Ref. 7) proposed the use of an integration formula (Schussler-Iber) of the form

$$y_{n+1} = y_n + (\delta T/3) \sum_{k=0}^{7} b_k(n) x_{n+1-k} , \tag{16}$$

with $b_k = b_{7-k}$. The phase of the corresponding filter, is not $-\pi/2$, unlike the case of the ideal integrator, and hence the use of the filter will produce a phase distorted output, which is strongly objectionable in the case of earthquake engineering.

As in the case of differentiation, we use an integration filter that is accurate for data of frequencies up to the variable cut-off frequency, ω_L. Again let $\lambda = \omega_L/\omega_N$, the ratio of the cut-off frequency to that of the Nyquist frequency, with $o < \lambda < 1$. Then we use the following criteria for the design of the integration filter:

(1) that the filter be of the form

$$y_n = y_{n-N} + \sum_{k=o}^{N} C_k \, x_{n-k} \; , \tag{17}$$

where $N = N(\lambda)$, the order of the filter, $C_k = C_k(\lambda)$, $k = o, N$, the coefficients of the filter, will all depend on λ. The coefficients will be symmetric, with $C_k = C_{N-k}$, this will ensure the phase of the filter to be $-\pi/2$, as in the case of the ideal integrator, so that no phase errors occur.

(2) that the coefficients $C_k = C_k(\lambda)$ be chosen to satisfy Chebyshev minimax criterion within the frequency band $o \; \omega \; \omega_L$: to minimize the maximum error in approximating the ideal integration

$$H(\omega) = \begin{cases} \dfrac{1}{i \; \pi\omega/\omega_N} & |\omega| < \omega_L \\ 0 & |\omega| > \omega_L \end{cases} \; , \tag{18}$$

which is cut-off at the same cut-off frequencies used in band-pass filtering, and finally,

(3) that the ratio $|\text{design filter/idealfilter}| \to 1$ as $\omega \to o$, so that they have the same tangency at $\omega = o$.

An example of such a design of a 3 point formula by Tick was given in the book "Digital Filters" by Hamming (1977)(Ref. 1).

CONCLUSIONS

With modern developments in digital signal processing and the increased speed and efficiency of digital computers, we are approaching the time when the limitations and constraints on data processing and accuracy of recorded strong earthquake ground motion will be mainly influenced by the recording instrumentation. In this paper a summary has been presented on several recent refinements in routine data processing of accelerograms and on the selected physical principles which should govern the choice of the appropriate data processing scheme.

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