FREQUENCY DOMAIN CORRECTIONS OF EARTHQUAKE ACCELEROGRAMS WITH EXPERIMENTAL VERIFICATIONS

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SUMMARY

This paper first presents the comparison between the time-domain correction method suggested in the Standard Data Processing of Accelerograms developed at the California Institute of Technology (CalTech) about a decade ago, and its frequency domain equivalents. The Equivalent Method uses the frequency response functions obtained digitally from the CalTech routine. A new and computationally more efficient procedure called the Stanford Accelerogram Correction Procedure (SACP) is presented. Finally, this paper presents an experimental verification of the SACP. Two different motions were simulated on the shake table and recorded simultaneously with a standard accelerograph and a high-precision displacement laser interferometer. The integrated displacements from the two corrected accelerograms were compared with the recorded displacements. Both results showed good agreement.

INTRODUCTION

It is well known that strong-motion data constitute the foundation of earthquake engineering studies and the most complete description of earthquake ground motion is given by the accelerogram which expresses the full time history of the ground acceleration. Therefore, it is essential that the accelerogram be accurately processed and analyzed to remove the unwanted distortions introduced in it.

Although new and sophistiacted digital records are becoming more and more in use, the analog on black and white photographic film will remain the main source of earthquake data for a long time to come. The film on which it is developed, enlarged, and digitized to be ready for computer processing. This digitalized form of the earthquake record is distorted by errors that have been incorporated during all the steps from recording to digitizing. These alterations must be removed from the data in order to retrieve the true ground motion. There is a number of data-processing methods that have been suggested to correct and filter the uncorrected data. The Standard Data Processing of Accelerograms developed at CalTech is by far the most popular in the US and other countries. In the procedure, the data are low-pass filtered using an Ormsby (Ref. 1) filter, instrument-corrected with the standard second differential equation approach, and baseline-corrected by high-pass filtering. The velocity and displacement are then computed from the corrected acceleration using the trapezoidal method. The filtering is achieved in time-domain by means of a direct convolution. The complete CalTech correction procedure is lengthy and intricate.

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In the SACP method, a fast convolution technique is implemented with the use of an Fast Fourier Transform (FFT) algorithm to perform a band-pass digital filtering. The corrected velocity and displacement are obtained through another filter derived directly from the previous one. There are many advantages in using this method. It is simple, fast, theoretically exact, and provides in the course of its processes the Fourier amplitude spectra of the corrected acceleration, velocity, and displacement.

CALTECH ACCELEROGRAM CORRECTION PROCEDURE

The processing necessary to correct accelerograms includes the corrections for instrumental response and true baseline. Preliminary smoothing is carried out first. Then, a low-pass filtering was used. The low-pass filtering using an Ormsby filter can be expressed by the discrete convolution sum:

$$y(n) = x(n) * h(n) = \sum_{k=1}^{N-1} h(k) \cdot x(n-k) \tag{1}$$

where x(n) is the equispaced sequence of the time history to be filtered,
y(n) is the filtered version of x(n), and
h(n) is the sequence of the filter weights of the Ormsby impulse response function.

The instrument correction is performed using the standard second-order differential equation governing oscillator motion. The baseline correction removes from the accelerogram all Fourier components with periods larger than the upper period limit. The standard upper limit for hand-digital records has a cut-off period of 14 sec, and a roll-off termination of 20 sec.

EQUIVALENT CALTECH PROCEDURE

The introduction of the FFT algorithm has rendered the high-speed convolution a much faster operation than the direct convolution. The direct convolution of two discrete-sequence x(n) and h(n) was defined in Eq. 1. This operation is equivalent in the transformed domain to the multiplication of the Discrete Fourier Transforms (DFT) of the two signals

$$x(n) * h(n) \Longleftrightarrow X(f) \cdot H(f) \tag{2}$$

There are actually three steps involved in using the DFT to perform the fast convolution:

1. The DFT of the two signals are computed using an FFT algorithm

$$X(f) = \sum_{n=0}^{N-1} x(n)e^{-2\pi inf/N}, \quad H(f) = \sum_{n=0}^{N-1} h(n)e^{-2\pi inf/N} \tag{3}$$

2. The transform of the signals are multiplied together at all frequency points

$$Y(f) = X(f) \cdot H(f) \tag{4}$$

3. The inverse transform of the product is computed, using again FFT algorithm

$$y(n) = \frac{1}{N} \sum_{f=0}^{N-1} Y(f) e^{2\pi inf/N} \quad (5)$$

The direct discrete convolution of Eq. 1 has been replaced by Eqs. 3, 4, and 5. Because of the computing efficiency of the FFT algorithm, these four equations define in fact a "short-cut by the long way around." Evaluation of the N samples of the direct convolution result y(n) by Eq. 1 requires a computation time proportional to N^2. The computation of FFT is proportional to $N \log_2 N$; computation time of Eqs. 3 and 5 is then proportional to $3N \log_2 N$ and computation time of Eq. 4 is proportional to N.

The frequency response function plays a key role in this Equivalent CalTech procedure. Three different impulse input signals were used to calculate the frequency response functions. The amplitude of these functions are calculated as follows:

$$\text{Amplitude of Freq. Resp. Funct.} = \frac{\text{Amplitude of DFT of Imp. Resp. Funct.}}{\text{Amplitude of DFT of Input Impulse}}$$

The results of these calculations are identical for the three different inputs. The amplitudes of the three functions are plotted in Fig. 1.

COMPARISON OF RESULTS USING CALTECH PROGRAM AND EQUIVALENT CALTECH PROGRAM

A FORTRAN program has been written to compute the corrected acceleration, velocity, and displacement using the frequency response function of the CalTech program. Four accelerogram records from the Imperial Valley earthquake of Oct. 15, 1979, have been chosen to compute their corrected versions from the CalTech program and the Equivalent CalTech program. Only the result of the processing of the first accelerogram is plotted in Fig. 2. Table 1 indicates the peak values and the time of their occurrence for the acceleration, velocity, and displacement, as well as the CPU time required for the computer processing on a DEC-20 system.The last column of this table shows the values of the relative discrepancies between the maximum values from both methods. This relative value is given by

$$\frac{DX}{X} = \frac{|\text{Max. CalTech} - \text{Max. Equiv. CalTech}|}{|\text{Max. CalTech}|} \quad (6)$$

The instrumental characteristics were assumed equal to 14.92 Hz for the natural frequency and 0.58 of critical for the damping.

The results are very close to each other, except at the extremities of the records. The slight discrepancy is due to the end effects that result from the use of the FFT algorithm to transform non-periodic signals. The average saving of CPU time resulting from use of the Equivalent CalTech program is on the average equal to 27%.

THE STANFORD ACCELEROGRAM CORRECTION PROCEDURE (SACP)

In the previous section it was shown that the fast convolution is a more efficient tool for digital filtering of strong-motion data than the direct convolution. The frequency response functions used in the Equivalent CalTech Method were obtained numerically. In this section, the same concept of high-speed convolution will be incorporated in a new and efficient accelerogram correction program that will be referred to as the Stanford Accelerogram Correction Procedure (SACP). The theoretical frequency response function will be used so that the filtering will be more rational.

Acceleration Frequency Response Function

Using the tool of dynamic theory and Discrete Fourier Transform, the true ground acceleration can be derived in terms of $x_r(t)$, time history of relative displacement recorded in an earthquake, i.e.,

$$\ddot{x}_{tg}(s) = \sum_{k=0}^{N-1} \left[-DMF_k e^{i\phi_{1k}}\left(\frac{1}{N}\sum_{q=0}^{N-1} x_r(q) e^{-2\pi ikq/N}\right)\right] e^{2\pi iks/N} \tag{7}$$

with

$$DMF_k = p^2\left[\left(1-\frac{w_k^2}{p^2}\right)^2 + \left(\frac{2\gamma w_k^2}{p^2}\right)^2\right]^{1/2}, \quad \phi_{1k} = \tan^{-1}\frac{2\gamma w_k/p}{1-w_k^2/p^2} \tag{8}$$

The operation to get $\ddot{x}_{tg}(s)$ via Eq. 7 is equivalent to a digital filtering via a fast convolution. Its filter has an amplitude $-DMF_k$ and a phase ϕ_{1k}. The amplitude DMF_k should be band-limited because of the various sources of noise. The value of the roll-off and cut-off frequencies suggested in the CalTech data-processing program are maintained. The transition bands of the filter are tappered with a half cosine wavelength on each side. Figure 3 shows the construction of the amplitude of this band-pass filter.

So far, the input data to be corrected were regarded as relative displacement. However, in practice, it is calibrated as an acceleration. In other words, the digital value corresponds to $\ddot{x}_g$ instead of x_r. This means that all the values of the relative displacement have been multiplied by $-p^2$. Hence, in order for Eq. 7 to apply for the calibrated acceleration, DMF_k should be divided by $-p^2$. Finally, the amplitude of the band-pass filter that converts the acceleration raw data into the true ground acceleration is:

$$DMF_{1k} = \frac{-DMF_k}{-p^2} = \frac{DMF_k}{p^2} \tag{9}$$

The phase remains the same, i.e., in complex notation this filter can be written as

$$H_{1k} = DMF_{1k} e^{i\phi_{1k}} \qquad k = 0,1,\ldots,N-1 \tag{10}$$

The substitution of Eq. 10 into Eq. 7 yields

$$\ddot{x}_{tg}(s) = \sum_{k=0}^{N-1} \left[H_{1k}\left(\frac{1}{N}\sum_{q=0}^{N-1} \ddot{x}_g(q) e^{-2\pi ikq/N}\right)\right] e^{2\pi iks/N} \tag{11}$$

For an instrument having a natural frequency of 14.92 Hz and 58% of critical damping, the amplitude and phase of the corresponding band-pass filtered are plotted in Fig. 4a.

Velocity Frequency Response Function

If $F(w)$ is the Fourier transform of $f(t)$, then $F(w)/iw$ is the Fourier transform of the integral of $f(t)$ from 0 to t. This implies that the Fourier transform of the true ground velocity is:

$$H_{1k}\left(\frac{1}{N}\sum_{q=0}^{N-1} \ddot{x}_g(q)e^{-2\pi ikq/N}\right)/iw_k \qquad (12)$$

The corresponding filter that generates the true ground velocity from the raw data is

$$H_{2k} = H_{1k}/iw_k = DMF_{2k}e^{i\phi_{2k}} \qquad (13)$$

With

$$DMF_{2k} = DMF_{1k}/w_k\ , \quad \phi_{2k} = \phi_{1k} - \pi/2 \qquad (14)$$

The true ground velocity is then

$$\dot{x}_{tg}(s) = \sum_{k=0}^{N-1} \left[H_{2k}\left(\frac{1}{N}\sum_{q=0}^{n-1} \ddot{x}_g(q)e^{-2\pi ikq/N}\right)\right]e^{2\pi iks/N} \qquad (15)$$

The amplitude and phase of the corresponding band-pass filtered are plotted in Fig.4b.

<u>Displacement Frequency Response Function</u>

The band-pass filter H_{3k} that retrieves the true ground displacement from the raw data can be obtained in a similar manner as H_{2k}, i.e.

$$H_{3k} = H_{2k}/iwk = DMF_{3k}e^{i\phi_{3k}} \qquad (16)$$

with

$$DMF_{3k} = -DMF_{1k}/w_k^2\ , \quad \phi_{3k} = \phi_{1k} \qquad (17)$$

The amplitude and phase of the corresponding band-pass filtered are plotted in Fig. 4c.

<u>Comparison of Results Using the CalTech Program and the Stanford Program</u>

The four earthquakes studied in the previous section are again processed using the Stanford Method. The results are compared with those already obtained with the CalTech Method. Only the plot of the result of example 1 is shown in Fig. 5. Table 2 indicates the peak values and the CPU time taken to process each file on a DEC-20 system. The results show that for examples 1, 2, and 4, the two methods agree with less than 4% relative discrepancy. Example 3 shows a difference of about 10% in the PGA. Also, the time of occurrence of the peaks do not coincide. The reason for such a large difference is explained by the large frequency response function in the Stanford Method (see Fig. 6), which corresponds exactly to the theoretical curve.

EXPERIMENTAL VERIFICATION

The objective of this experiment was to evaluate the accuracy with which the integrated displacement is obtained from the uncorrected accelerogram by comparing it with the measured displacement of the table. Unlike the test done by Trifunac and Lee (Ref. 2), where the displacement was measured with a laser beam interferometer with an accuracy of 10^{-6} in.

Two signals were used as inputs:

- The lowest frequency sine wave which could be stimulated without drift and other mechanical problems such as sticking, friction, etc.

The frequency generated was 0.5 Hz. The duration of the test was 48 sec.

- A random noise with a frequency band of 0.3-15 Hz lasting 18 sec.

The accelerograms of the two tests were digitized and processed with the CalTech and the Stanford procedures. The results (see Figs. 7, 8) show that the accuracy of the integrated displacement was excellent for the first test. The second test shows a reasonable discrepancy in the amplitude and a good concordance in the wave form.

CONCLUSIONS

The goal of this paper was to develop a computer program, SACP, that uses the fast convolution technique as a means to correct and filter the uncorrected digital data obtained from an accelerogram.

The currently used data-processing procedure developed at CalTech, was transformed into an equivalent form that implemented an FFT algorithm. From the four accelerograms that were processed. The Equivalent CalTech Procedure gives 27% saving in CPU time. The new procedure called the Stanford Accelerogram Correction Procedure (SACP) proves to be even more advantageous than the Equivalent CalTech Method, since it resulted in a savings of 37% of CPU time over the CalTech Method. Finally, two different inputs were simulated on the shake table. In these tests, the displacement of the table was measured with a laser beam interferometer, while the acceleration was recorded on a standard accelerogaph. The synthetic displacement obtained by double integration of the corrected accelerogams were compared with the measured displacements. Both results showed good agreement. In the meantime, it was discovered that the accuracy of the corrected displacement is very sensitive to the lower cut-off frequency. Further work on the lower corner frequencies should be studied to improve the current data-processing method as well as the SCAP.

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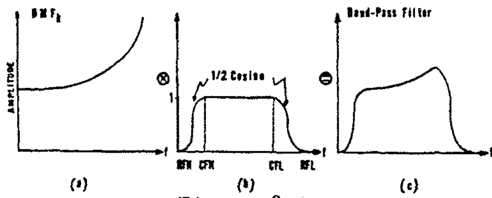


Figure 3.

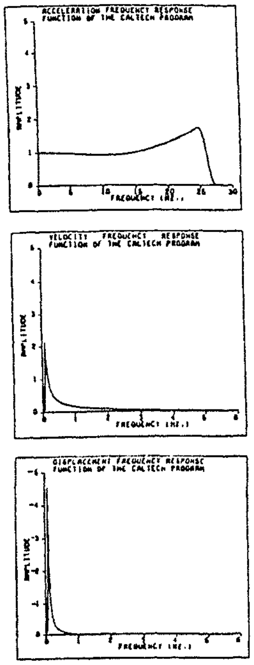


Figure 1.

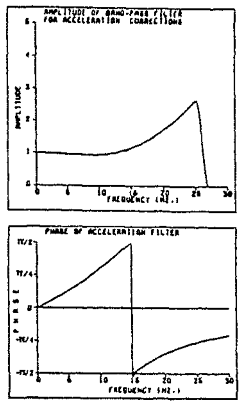
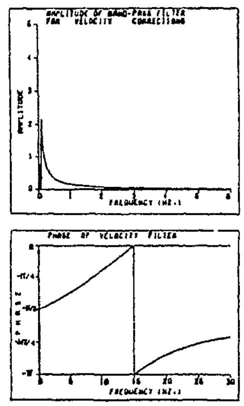


Figure 4a.



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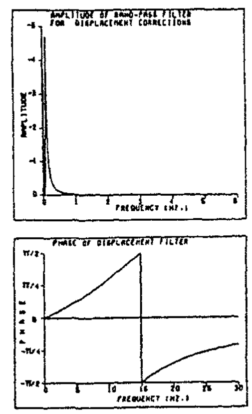

Figure 4c.

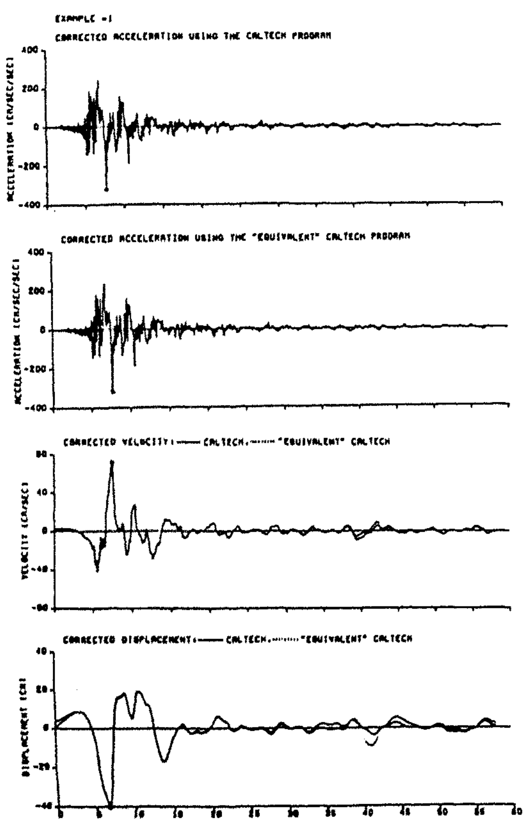

Figure 2. Example #1.

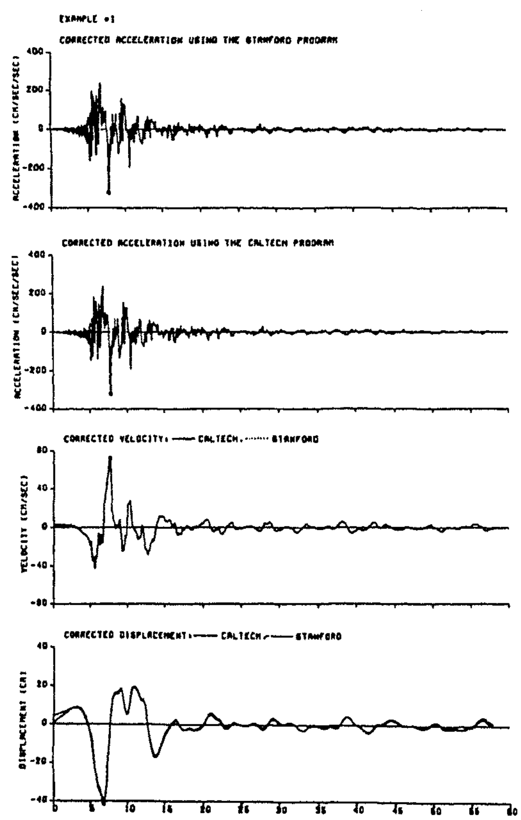

Figure 5. Example #1.

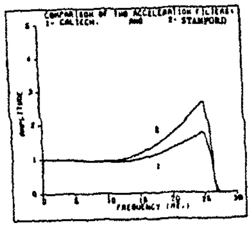

Figure 6.

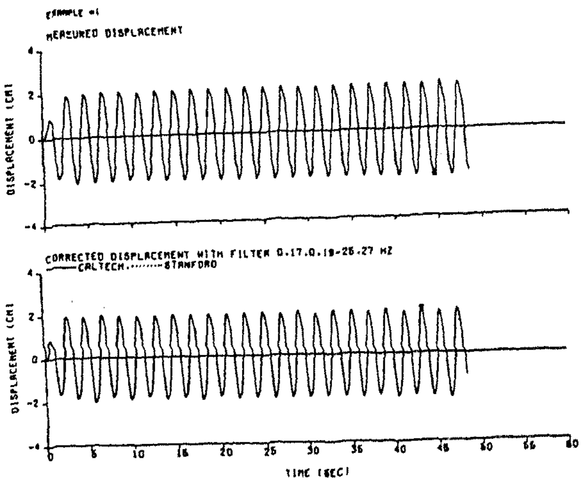

Figure 7.

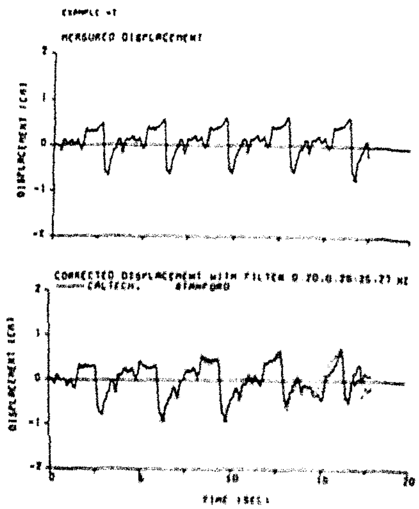

Figure 8.

In this table, the acceleration is in cm/sec², the velocity is in cm/sec, and the displacement is in cm.

Avrg. CPU	4	3	2	1	Example #		
	-204.33	-243.99	-284.67	319.80	Max.	Accelar.	Uncorrected Data
	8.96	5.11	6.44	7.72	Time	Accelar.	Uncorrected Data
	208.60	246.72	284.88	-318.66	Max.	Accelar.	Corrected Data with Caltech Program
	8.96	5.09	6.43	7.71	Time	Accelar.	Corrected Data with Caltech Program
	38.34	-16.76	-45.47	72.07	Max.	Velocity	Corrected Data with Caltech Program
	6.70	6.00	9.90	7.53	Time	Velocity	Corrected Data with Caltech Program
	-16.75	-10.42	-15.98	-40.08	Max.	Displac.	Corrected Data with Caltech Program
	6.32	6.90	12.96	6.68	Time	Displac.	Corrected Data with Caltech Program
4:15.84	4:11.74	4:16.86	4:16.06	4:18.68	CPU		Corrected Data with Caltech Program
	206.07	263.33	283.62	-318.14	Max.	Accelar.	Corrected Data with Equivalent Caltech Program
	8.96	5.11	6.44	7.72	Time	Accelar.	Corrected Data with Equivalent Caltech Program
	38.56	-18.57	-45.70	71.84	Max.	Velocity	Corrected Data with Equivalent Caltech Program
	6.68	5.99	9.89	7.52	Time	Velocity	Corrected Data with Equivalent Caltech Program
	-16.66	-10.18	-16.08	-40.89	Max.	Displac.	Corrected Data with Equivalent Caltech Program
	6.33	6.91	12.98	6.69	Time	Displac.	Corrected Data with Equivalent Caltech Program
3:07.11	3:06.23	3:05.46	3:07.80	3:08.96	CPU		Corrected Data with Equivalent Caltech Program
	.012	.067	.004	.002	Accelar.		δX/X
	.006	.011	.005	.003	Velocity		δX/X
	.005	.023	.006	.020	Displac.		δX/X

Table 1.

In this table, the acceleration is in cm/sec², the velocity is in cm/sec, and the displacement is in cm.

Avrg. CPU	4	3	2	1	Example #		
	-204.33	-243.99	-284.67	319.80	Max.	Accelar.	Uncorrected Data
	8.96	5.11	6.44	7.72	Time	Accelar.	Uncorrected Data
	208.60	246.72	284.88	-318.66	Max.	Accelar.	Corrected Data With Caltech Program
	8.96	5.09	6.43	7.71	Time	Accelar.	Corrected Data With Caltech Program
	38.34	-16.76	-45.47	72.07	Max.	Velocity	Corrected Data With Caltech Program
	6.70	6.00	9.90	7.53	Time	Velocity	Corrected Data With Caltech Program
	-16.75	-10.42	-15.98	-40.08	Max.	Displac.	Corrected Data With Caltech Program
	6.32	6.90	12.96	6.68	Time	Displac.	Corrected Data With Caltech Program
4:15.84	4:11.74	4:16.84	4:16.06	4:18.68	CPU		Corrected Data With Caltech Program
	310.81	269.72	292.50	-321.66	Max.	Accelar.	Corrected Data With Stanford Program
	8.92	8.87	6.47	7.75	Time	Accelar.	Corrected Data With Stanford Program
	38.58	-18.22	-45.77	73.16	Max.	Velocity	Corrected Data With Stanford Program
	6.68	6.10	9.90	7.53	Time	Velocity	Corrected Data With Stanford Program
	-16.50	-9.96	-16.35	-41.58	Max.	Displac.	Corrected Data With Stanford Program
	6.32	6.91	12.92	6.68	Time	Displac.	Corrected Data With Stanford Program
2:41.60	2:41.50	2:41.50	2:41.70	2:41.70	CPU		Corrected Data With Stanford Program
	.011	.093	.027	.008	Accelar.		δX/X
	.006	.030	.007	.005	Velocity		δX/X
	.010	.042	.017	.037	Displac.		δX/X

Table 2.