STRESS ANALYSIS AND STRENGTHENING TECHNIQUES OF MASONRY BUILDINGS

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SUMMARY

The design procedure here explained has been extensively applied to restore and to strengthen masonry buildings in Southern Italy areas after the seismic event of november 1980.

The basic concept is to premise a F.E.M. analysis of the walls composing the multi-cellular structure of the building. Each wall is considered as a plane plate with one or more rectangular openings under the action of shearing forces. The analysis is performed in the inelastic range,with incremental procedure,using imposed dislocations to account for inelastic strains. In fact the F.E.M. inelastic stress analysis has proved itself as an efficient and in some cases an indispensable tool to define the extent and the localization of strengthening operations,on condition that the fracturing is considered.

SOME THEORETICAL REMARKS

The analysis of post-elastic behaviour in plane-stress state of structures whose constitutive material is characterized by different values of yield stress in tension and in compression as is the case of materials having stone-type nature is performed in the context of the limiting envelope proposed by Drucker and Prager (Ref.1). The well-known analytical expression of this surface in the principal stress space is:

$$F = \alpha\, T_1 + \sqrt{T_{2d}} - K = 0 \qquad (1)$$

with :

$$T_1 = \sigma_1 + \sigma_2 + \sigma_3 \qquad (2)$$

$$T_{2d} = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1 \cdot \sigma_2 - \sigma_2 \cdot \sigma_3 - \sigma_3 \cdot \sigma_1)/3 \qquad (3)$$

and α ,K are material constants to be determined from experimental data. For soils these constants depend on friction angle and cohesion. For masonry, instead,the usual experimental tests lead to evaluate the uniaxial yield stresses σ_{o1}, σ_{o2} in tension and in compression respectively. We have therefore expressed the above parameters α ,K in terms of σ_{o1}, σ_{o2} obtaining (Ref.2):

$$\alpha = \frac{1}{\sqrt{3}} \frac{\sigma_{o1} + \sigma_{o2}}{\sigma_{o2} - \sigma_{o2}} \qquad K = \frac{2}{\sqrt{3}} \frac{\sigma_{o1} \cdot \sigma_{o2}}{\sigma_{o2} - \sigma_{o1}} \qquad (4)$$

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Cutting now the surface represented by (1) with one of the planes of equation $\sigma_i=0$, $(i=1,2,3)$, we obtain the limiting conical curve in plane stress state. [1] This curve is always hyperbole for values of the parameter $\rho=|\sigma_{o1}/\sigma_{o2}|>3$ (Ref.3). This result don't agree with the experimental data obtained for materials like those mentioned above (Ref.4). We proposed as yielding surface that one defined by the D.-P. cone for values of $T_1 > \sigma_{o2}$ and by the von-Mises cylinder in the other cases.The intersecting curve between these two surfaces is a circumference with radius $R=|\sigma_{o2}|\cdot\sqrt{2/3}$ laying in the octaedral plane. The resulting surface,defined by only two parameters,fits with sufficient approximation the experimental results obtained,for ex.,by Kupfer et al.(Ref.4) testing on concrete flat plates in biaxial state of stress. This surface we have used now to design the numerical procedure explained in the following (Ref.3).

The material behaviour in post-elastic range has been supposed to obey to the plastic flow-rule,assuming the yield surface as plastic potential.

The essential features of the numerical incremental procedure are strictly based on the concept of considering inelastic strains as "dislocations" enforced on the elemental sub-region deriving from the discretization of the actual bi-dimensional continuum medium whose constitutive low is assumed to be linearly elastic. Under this hypotesis and with the assumption of stress-constant finite element,it is possible to account for the softening occurring when the stress point P overcrosses the yielding curve in tension-tension or in tension compression quadrant of the σ_1, σ_2 plane (Fig.I). The central point of the procedure is the evaluation of dislocation to enforce elastically on an element for which the plastic compatibility condition is violated because of an increment $\Delta\underline{R}$ of applied external loads $\underline{R}$. For the sake of simplicity and without loss of generality we shall refer to an element for which the previously achieved equilibrium configuration under loads $\underline{R}$ may be represented by stress point P_o inside the yield curve; let its new stress state corresponding to the increment $\Delta\underline{R}$ be represented by point $P'(\underline{\sigma})$ outside the yield curve.

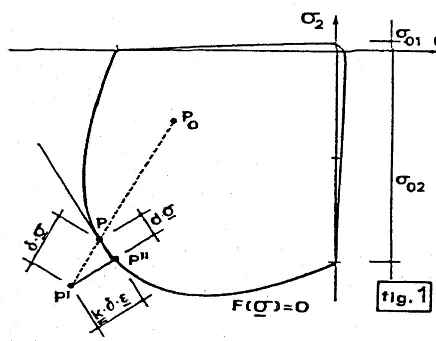


fig.1

The value of dislocations to enforce on such an element is determined according to the following steps (Colonnetti's method):

i) The neutral position of point P laying on the yield curve is defined directly solving the system formed by the equation of the curve itself and the equation of the straight-line P_o-P'. The length of the segment PP' measures the excess of stress $\delta\underline{\sigma}$ violating the plastic compatibility condition.

ii) The gradient of the function $F(\underline{\sigma})$ is evaluated at P; its components $a_1=\partial F/\partial\sigma_1$, $a_2=\partial F/\partial\sigma_2$ are proportional to the plastic strain increments $\delta\varepsilon_1$, $\delta\varepsilon_2$ through a positive scalar parameter λ ,according to the relation:

$$\delta\underline{\varepsilon}=\lambda\cdot\underline{a} \qquad (5)$$

where is $\underline{a}^T=[a_1\ a_2]$

iii) The value of dislocation $\delta\underline{\varepsilon}$ to enforce on the element is defined by the condition that the associated

$$\delta'\underline{\sigma}=\underline{\underline{K}}\ \delta\underline{\varepsilon}=\lambda\underline{\underline{K}}\underline{a} \qquad (6)$$

(where $\underline{\underline{K}}$ is the material elastic stiffness matrix), is such that the plastic compatibility is satisfied

$$F (\underline{\sigma} + \lambda \underline{\underline{K}} \ \underline{a}) = 0 \qquad (7)$$

Obviously, greater is the loading step, so the error.

When during the loading step the stress-point P' overcrosses the yield curve in tension-tension or in tension-compression quadrant, the yield curve (7) corresponding to the D.P. cone, degenerates to the $\sigma_1 - \sigma_2$ axes (σ_{01}=0), so accounting for the fracturing behaviour.

A detailed description of the different ways by which it is possible to return on the new yield curve can be found in (Ref.5). We assumed only the tensile ultimate stress of the material diminishes to 0, while the other is not affected by fracture and therefore remains constant. This is equivalent to suppose the plastic flow-rule to be still valid for the new yield curve.

The whole procedure can be summarized as follows: the equilibrium and plastic compatibility of the structural system under the action of a set of loading forces $\underline{R}$ (loading step) is obtained by means of a sequel of phases each of one characterized by modification applied to the non-compatible current stress state. A set of dislocation defined according to (i), (ii), (iii), is enforced on a fixed nodes element in order to satisfy the plastic compatibility condition. The set of elemental stresses corresponding to the dislocation, transformed into nodal elemental forces $\delta \underline{R}$ is subtracted to the external nodal applied loads $\Delta \underline{R}$ and under this new loading set, it is achieved again an equilibrium configuration, for which the plastic compatibility must be satisfied. The iterative process stops when the norm of the elemental nodal forces corresponding to the dislocations becomes vanishingly small.

Starting from the consideration previously discussed, we used a standard general purpose F.E. system running on a HP 9845B, 16-bit desk-top computer. The modular characteristics of the numerical code, called NINFEA, allow the implementation of the plastic rutines in the body of the program itself without any relevant changement of its logical structure. In fact the procedure explained, using the same stiffness matrix of the assembled structural model for the whole loading history, makes possible to solve the successive linearly elastic problems by an easy backward and forward substitution of the loading column-matrices on the Cholesky's factorized form of initial stiffness matrix.

The Table I contains a brief listing of the rélevant statements of the BASIC routine concerning the determination of the value of dislocation to apply on a plastic element. In the listing some REM explain what is necessary.

THE NUMERICAL CHECK OF STRENGTHENING OPERATION

The first set of applications is concerned with the two-story wall represented in Fig.2, where there are also indicated the values of all the mechanical and geometrical parameters. This case can well simulate the wall of a two storeys masonry building, under a vertical constant loading set, and horizontal forces uniformly distributed acting at level of floors, whose value is increased monotonically by the β factor. The result of the stress-analisys for β=1.75, is in Fig.3, where it is possible to evaluete the contribution to the strength due to the presence of crosswalls well-connected. Fig.4 shows the results of the strengthening of the spandrel, obtined increasing the Young modolus E, and ultimate tensile stress σ_{01}, by a factor 3. It is possible to do a comparison

TABLE I

```
10  REM  S01 = Yield stress in tension
20  REM  S02 =  "      "    " compression
30  REM   S1 = Principal stress
40  REM   S2 =   "        "
50  REM   P'(S1,S2) = Stress-point outside the yield curve (current step)
60  REM   Po(X2,Y2) =   "      "   inside   "    "    "   (previous step)
70  REM    P(X,Y)   = Neutral stress-point                 (current step)
80  REM    K = Material stiffness matrix
90  REM    A = Direction cosines of Grad F at P
100 REM   Normsig = Direction cosines of associated stress
110 REM   To satisfy (7) we intersect yield surface with the straight line
         having Normsig as direction cosines and passing through P'
120       DIM K(2,2),Normsig(2,1),A(2,1)
130       T1=S1+S2                                 !  (2)
140       T2d=(S1^2+S2^2-S1*S2)/3                  !  (3)
150       Alpha=(S01+S02)/(SQR(3)*(S02-S01))       !  (4)
160       Key=2*S01*S02/(SQR(3)*(S02-S01))         !  (4)
170 REM   Expression of yield surface
180 Mises:  DEF FNMises(T2d,S02)=3*T2d-S02^2
190 Drucker:DEF FNDrucker(T1,T2d,Alpha,Key)=Alpha*T1+SQR(T2d)-Key   ! (1)
200       Di=SQR((X2-S1)^2+(Y2-S2)^2)
210       L1=(X2-S1)/Di
220       M1=(Y2-S2)/Di
230 REM   The following label is used twice:the first time to determine
    REM   the intersection between the yield surface and the straight line
240 REM   Po-P'. The second time to determine the return stress point P''
250 Intersez: IF T1<=S02 THEN Druckerb
260       Aa=L1^2+M1^2-L1*M1
270       Bb=2*S1*L1+2*S2*M1-S1*M1-S2*L1
280       Cc=S1^2+S2^2-S1*S2
290       GOTO Coord
300 Druckerb: Beta=1/3-Alpha^2
310       Gamma=1/3+2*Alpha^2
320       Aa=Beta-Gamma*(L1*M1)
330       Bb=2*Beta*(S1*L1+S2*M1)-Gamma*(S1*M1+S2*L1)+2*Key*Alpha*(L1+M1)
340       Cc=Beta*(S1^2+S2^2)-Gamma*(S1*S2)+2*Key*Alpha*(S1+S2)-Key^2
350 REM   X01,X02 = Distances of moving neutral stress-point P from P'
360 Coord:Disc=SQR(Bb^2-4*Aa*Cc)
370       X01=(-Bb+Disc)/(2*Aa)
380       X02=(-Bb-Disc)/(2*Aa)
390       IF ABS(X1)>ABS(X02) THEN Puntoq
400 Puntop: X=S1+L1*X01
410       Y=S2+M1*X01
420       GOTO Subend
430 Puntoq: X=S1+L1*X02
440       Y=S2+M1*X02
450 Subend:IF Flag1=1 THEN RETURN
460       T11=X+Y
470       T22d=(X^2+Y^2-X*Y)/3
480       IF T1<S02 THEN Normmises
490 Normdrucker:  A(1,1)=Alpha+1/2/SQR(T22d)*(X-T11/3)
500       A(2,1)=Alpha+1/2/SQR(T22d)*(Y-T11/3)
510       GOTO Num
520 Normmises:    A(1,1)=2*X-Y
530       A(2,1)=2*Y-X
540 Num:  Mod=SQR(A(1,1)^2+A(2,1)^2)
550       FOR I=1 TO 2
560       A(I,1)=A(I,1)/Mod
570       NEXT I
580       MAT Normsig=K*A
590       Mod=SQR(Normsig(1,1)^2+Normsig(2,1)^2)
600       MAT Normsig=(-1)*Normsig
610       L1=Normsig(1,1)/Mod
620       M1=Normsig(2,1)/Mod
630       Flag1=1
640       GOSUB Intersez
650       SUBEND
```

between the strengthening of the whole spandrel or of its only part above the opening. Finally, in the Fig.5, the plot of horizontal displacement of the corner of wall versus β makes possible to quantify the different response of the wall assuming perfecty-plastic or fracturing behaviour of masonry.

The second set of applications analyses the wall considered as reference example in the Italian Standard Code (Ref.6). In Fig.6 it is possible to read all the mechanical and geometrical parameters, and to see the mesh of the discretization of wall. The horizontal ultimate force according to the mentioned code and computed assuming a constant value of ultimate shear strength has value H=32330 Kg. The vertical constant loading set distributed at the top of the wall values $-2.75\ Kg/cm^2$. For this wall we have studied the effect of the following strengthening techniques :

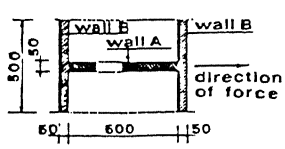
(a)= Reinforced Concrete beam collar
(b)= Openings boxing in r.c. alround, or with steel frame.
(c)= Binding of the wall along the edges to the crosswalls
(d)= Vertical reinforcement of wall by steel bars.

The results are shown in Figg.6,7,and 9. From Fig.9 we observe that it is possible to attain the ultimate shearing force H only in the presence of both the r.c.collar beam and the bound crosswalls. In the presence of frames boxing the openings the structural element response tends to that of the solid shear wall, i.e. without openings. If we set for the beams of the frames

$$EJ=GtpA/48$$

with t, thickness of wall; A, area of opening; p, perimeter; G, shear modulus of masonry; E, Young modulus of frame, the wall's behaviour is that of the solid wall. In the case studied we used $J_1=68566\ cm^4$ for the door, and $J_2=31510$ for the window, with $E=300000\ Kg/cm^2$.

Finally in Fig.6 and 7 the spread of plastic zones are indicated for different values of β .



$E=6600\ kg \cdot cm^{-2}$
$\nu=0{,}2$
$\gamma=0{,}0016\ kg \cdot cm^{-2}$
$\sigma_{01}=1\ kg \cdot cm^{-2}$
$\sigma_{02}=-30\ kg \cdot cm^{-2}$

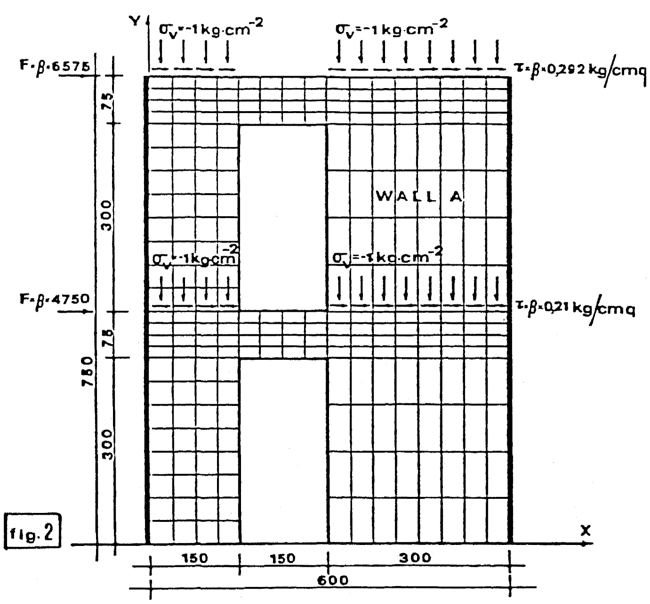


fig.2

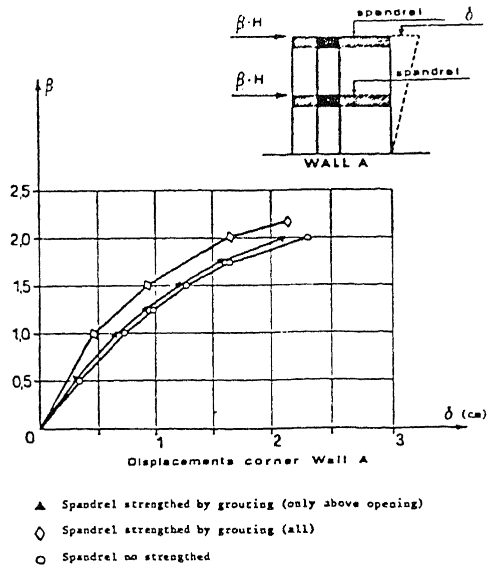


fig. 4

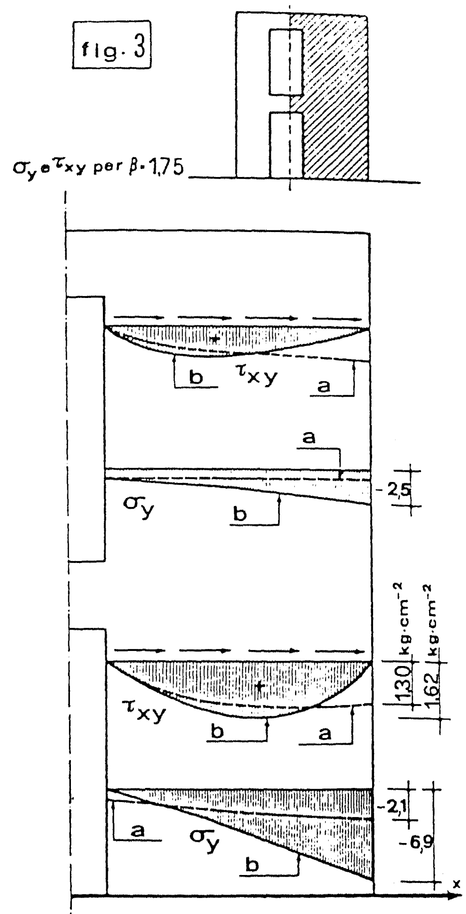


fig. 3

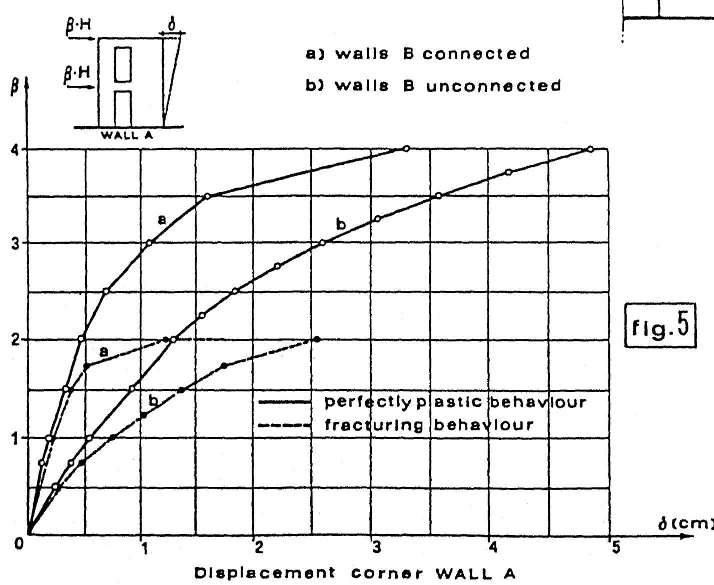


fig. 5

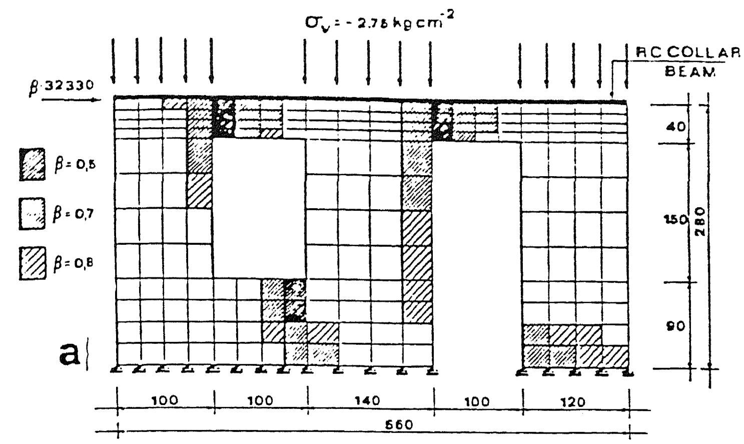


fig.6

$E = 2904\ kg\,cm^{-2}$
$\upsilon = 0.20$
$G = 1210\ kg\,cm^{-2}$
$\sigma_{01} = 0.984$
$\sigma_{02} = -30$
$\gamma = 0{,}0014\ kg\,cm^{-3}$

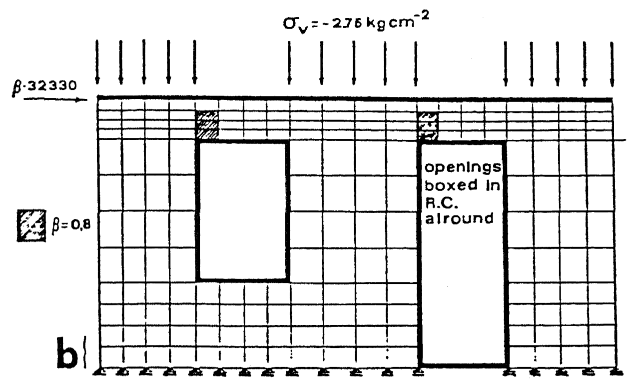


fig. 7

In Fig.8 it is shown the deformed shape of the wall under the horizontal force H. It is worth noting that the stiffness of the adopted boxing frames has been determined by equating the diagonal's elongations of the masonry panels virtually occluding the openings,to that of the frames. The firsts are computed assuming a pure shearing stress state of masonry,the seconds neglecting the axial flexibility of the beams.

We furthermore suggest that the curves I,II,and IV of Fig.9 can also represent the behaviour of the wall without bound crosswalls,but with steel vertical bars at the edges. The area of each bar is,obviously,

$$A = 2904\text{x}50\text{x}150/2100000 = 10.37\ cm^2.$$

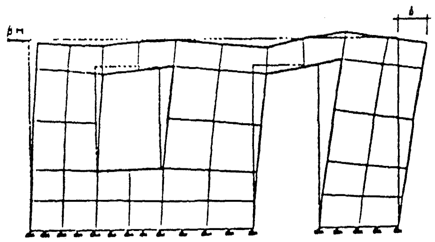


fig.8

I Without any Strengthening, with walls B connected

II With Reinforced Concrete collar beam, without walls B

III With R.C. Collar beam (a), with walls B connected

IV Openings boxed, (b) with R.C. Collar beam aud walls B connected

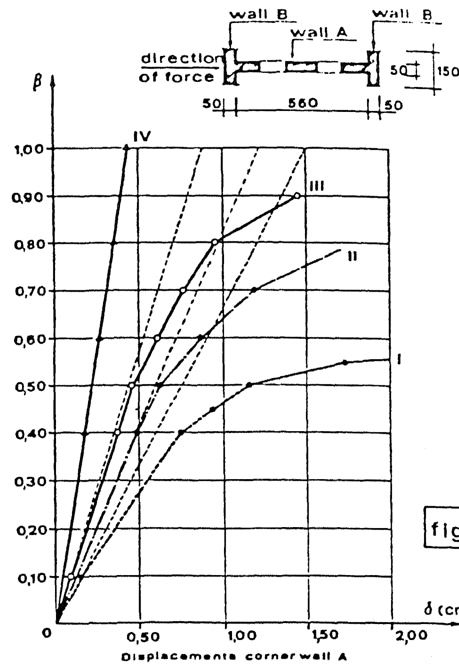


fig.9

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