STOCHASTIC SEISMIC PERFORMANCE EVALUATION OF STRUCTURES

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SUMMARY

A method is presented for determining the probabilities of a structure sustaining various levels of damage due to seismic activity during its lifetime. Uncertainties in the loading, the resistance, and the structural response analysis are considered. The method is based on a nonlinear random vibration analysis and an analytical technique for evaluating the sensitivity of the response to various structural and load parameters. The method is illustrated by an analysis of a seven-story reinforced concrete building.

INTRODUCTION

The assessment of the performance of a structure under seismic excitations is a complex problem involving numerous uncertainties. For this reason, safety factors are traditionally used to ensure a degree of conservatism in the design. However, the actual risk of failure implied in such a design remains unknown. Described herein is a methodology in which the variability in the loads and resistances, and the potential inaccuracies of the underlying assumptions are considered and quantified. Structural safety or damage are then expressed in probabilistic terms. Decisions concerning structural improvements to increase safety (or reduce lifetime expected damage costs) may, therefore, be made rationally based on these probabilities.

The method includes the following studies: (1) estimation of the probabilities of exceeding specified ground motion intensities at the specified site (seismic hazard); (2) evaluation of the structural response statistics for a loading of given intensity, in which the randomness of earthquake time histories and uncertainties in structural response prediction are considered; (3) a probabilistic evaluation of damage based on the structural response statistics; and (4) convolution of the seismic hazard with the structural damage and response exceedance probabilities (conditional on loading intensity) to obtain the lifetime damage probabilities and structural safety evaluation.

SEISMIC HAZARD AND GROUND MOTION MODEL

The seismic hazard is established using the "fault-rupture" model of Der Kiureghian and Ang (Ref. 1), which allows an evaluation of the probabilities of exceeding all significant ground shaking intensities at a particular site over a specific time duration. The model is based on the assumption that an earthquake originates as an intermittent series of fault ruptures in the

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earth's crust, and that the intensity of motion at a site is mainly contributed by the segment of the rupture closest to the site (in contrast to "point-source" models in which the total energy is assumed to radiate from the focus). All uncertainties in the hazard analysis, i.e., uncertainty in the physical relations used (e.g., the intensity attenuation equation) and the parameters of the model (e.g., the slope of the magnitude-recurrence relation) are considered in the model.

The seismic hazard model may be used to evaluate the probability associated with a given maximum ground motion intensity; however, the detailed characteristics of the ground motion can vary widely from one occurrence to another. For this reason, the actual ground motion is modeled as a filtered Gaussian shot noise to account for the random nature of earthquake time histories. The well-known Kanai-Tajimi (K-T) filter is used to define the frequency content of the process and a semi-deterministic temporal multiplier (Ref. 2) is used to model the nonstationarity of the load intensity.

Values of the filter natural frequency, ω_g, and damping ratio, β_g, were obtained by fitting the K-T spectrum to empirical power spectra recently obtained by Moayyad and Mohraz (Ref. 3) using a least squares procedure. Figure 1 compares the empirical power spectra (obtained from Fourier analysis of a large number of accelerograms in Ref. 3) and the K-T spectra obtained herein. Based on the work of Lai (Ref. 4), the coefficients of variation of ω_g and β_g are both taken as 0.43 for soft and intermediate ground and 0.40 and 0.39, respectively, for hard ground.

Since the K-T spectra approach zero more slowly in the high frequency range than do the empirical power spectra it is necessary to apply scale factors when evaluating the mean square of the process (the area under the power spectral density function). By matching the areas under the empirical and corresponding K-T spectra, the mean square accelerations of the processes represented by the K-T spectra with the proposed parameter values were determined to be 67.7 s_o, 95.7 s_o, and 101.2 s_o for soft, intermediate, and hard ground, respectively, where s_o is the spectral value at $\omega = 0$.

The functional form of the temporal multiplier is given in Ref. 2. The duration of the strong stationary phase is treated as a random variable with mean values of 10.0 sec., 7.0 sec., and 5.5 sec. for soft, intermediate and hard ground, respectively, and corresponding coefficients of variation of 0.9, 0.9 and 1.0. These statistics are based on the results reported in Refs. 3 and 5.

Since the seismic hazard is obtained in terms of the probabilities of exceeding given maximum accelerations and the intensity of the random process is measured by the root-mean-square acceleration, it is necessary to relate the two quantities. The "peak factor" (ratio of maximum to root-mean-square acceleration) is taken as 3.0 (Ref. 6).

STRUCTURAL MODELING

Although the method used for the random structural response analysis is general and not restricted to any particular class of structural models, a lumped mass single-degree-of-freedom per story model is found to be adequate.

In this method the linear elastic stiffness of each story is obtained from the requirement that the resulting fundamental frequency and mode shape of the simplified model exactly matches that obtained from a detailed frame analysis. It has been shown (Ref. 6), that with this approach the higher modes of the simplified model do not significantly differ from those of the detailed model. An equivalent story yield strength is used for each story following the method by Anagnostopoulos (Ref. 7). The hysteretic characteristic of the structural restoring force is based on the models developed by Wen (Ref. 8) and Baber and Wen (Ref. 9) which also allows deteriorations. Some details of this model are described as follows.

The hysteretic restoring force of a particular story is given by

$$q = c\dot{u} + \alpha k u + (1 - \alpha)\, kz \tag{1}$$

where the parameters c and k represent damping and elastic stiffness, respectively. $(1 - \alpha)\, kz$ is the hysteretic part of the restoring force, and z is governed by a differential equation,

$$\dot{z} = [A\dot{u} - \nu(\beta|\dot{u}||z|^{n-1} z + \gamma\dot{u}|z|^{n})]/\eta \tag{2}$$

The parameters α, A, β, γ and n control the shape and yield level of the hysteresis loop, and the parameters ν and η control the rates of strength and stiffness degradation, respectively. The hysteresis deterioration may be governed by dissipated hysteretic energy (Ref. 9) or by maximum displacements (Ref. 6), depending on how the parameters ν and η are defined. Figure 2 shows a comparison of experimental load deflection curves and those obtained using the model with displacement-based stiffness degradation. System identification techniques have also been developed to evaluate the values of the shape parameters based on experimental test results (Ref. 10). From these studies, simple rules have been developed for establishing the values of the parameters for structural modeling purposes. The results illustrated in Fig. 2 are based on such rules.

RESPONSE UNCERTAINTY

In evaluating the structural response for a loading of given intensity, it is necessary to consider the uncertainty associated with the structural and ground motion parameters (e.g., stiffness, filter frequency, etc.), the uncertainty inherent in the modeling (mathematical idealization) and the uncertainty associated with the randomness of the earthquake time history. The uncertainty of the structural response may be obtained by first-order analysis as

$$\mathrm{Var}[X] \simeq (E[\hat{X}])^2 \cdot \mathrm{Var}[N] + (E[N])^2 \cdot \mathrm{Var}[\hat{X}] \tag{3}$$

where $E[\hat{X}]$ is the mean response obtained from the model using mean parameter values, E[N] and Var[N] represent the expected bias and variance of the error in the response underlying the mathematical idealization of the structure, and $\mathrm{Var}[\hat{X}]$ is the sum of the variances in the response associated with the parameter variabilities and the randomness of the loading history.

The variance of the response associated with the parameter variabilities is also obtained by first-order approximation as

$$\mathrm{Var}_{\underset{\sim}{p}}[\hat{X}] \simeq \sum_{i}\sum_{j} \left(\frac{\partial \hat{X}}{\partial p_i}\right)_{\bar{\underset{\sim}{p}}} \left(\frac{\partial \hat{X}}{\partial p_j}\right)_{\bar{\underset{\sim}{p}}} \rho_{ij}\sigma_{p_i}\sigma_{p_j} \tag{4}$$

where $\bar{P}$ is the set of mean model parameters, ρ_{ij} is the correlation coefficient of the i^{th} and j^{th} parameter, and σ_{p_i} is the standard deviation of the i^{th} parameter. The variance of the response due to the random nature of the loading history is obtained through a random vibration analysis as described in the following section.

Based on comparisons of observed and predicted maximum structural responses reported in Ref. 11, the coefficient of variation of N was determined to be approximately 0.20. Values for the coefficients of variation of the structural model parameters were also established from a literature review and are summarized in Table 1.

Table 1 Parameter Coefficients of Variation

Parameter	Coefficient of Variation	
	Reinforced Concrete Structures	Steel Structures
Lateral Story Stiffness	0.30	0.10
Damping Ratio	0.65	0.65
Story Mass	0.12	0.11
Story Strength	0.25	0.23

RANDOM RESPONSE STATISTICS AND SENSITIVITY COEFFICIENTS

Exact solution for the random vibration response statistics of hysteretic, degrading systems, as considered herein, is generally not possible. Using an equivalent linearization technique, however, approximate results (shown to be accurate as compared with Monte-Carlo simulation results) may be obtained (Ref. 9). The response statistics are evaluated by solving an equivalent linear system, with coefficients that are response-dependent. The zero time lag solution of the equivalent linear system satisfies the matrix equation

$$\dot{S} + GS + SG^T = B \tag{5}$$

where, S is the unknown zero time lag response covariance matrix, G is a matrix of the system parameters and the response-dependent equivalent linear coefficients, and B is a matrix describing the random process input. In the stationary case, the matrix S is constant in time implying $\dot{S}=0$ and Eq. 5 is reduced to a set of algebraic equations; an algorithm for the solution of this sytem is given in Ref. 12. In the nonstationary case, for which $\dot{S}\neq 0$, numerical integration is necessary. A similar set of equations is solved for the two-time response covariance matrix (Ref. 6).

Based on the solution for the response covariance matrices (zero time lag and two-time), the first and second moment statistics for the maximum response and dissipated hysteretic energy (necessary for estimating structural damage and safety) may be obtained as described in Refs. 6 and 13, respectively.

From Eq. 4 it is seen that the derivatives of the response statistics with respect to the various model parameters are required for evaluating the response uncertainty. In order to obtain these "sensitivity coefficients," Eq. 5 is differentiated with respect to the parameter of interest (e.g., story stiffness, load duration, K-T filter frequency, etc.), giving

$$\frac{\partial}{\partial t}\left(\frac{\partial S}{\partial p}\right) + G\frac{\partial S}{\partial p} + \frac{\partial G}{\partial p}S + \frac{\partial S}{\partial p}G^T + S\frac{\partial G^T}{\partial p} = \frac{\partial B}{\partial p} \tag{6}$$

where p represents the parameter of interest, and $\frac{\partial S}{\partial p}$ is the unknown derivative of the response covariance matrix. Although Eq. 5 is nonlinear (because the coefficients in the G matrix are response-dependent), Eq. 6 is linear in terms of the unknown derivative matrix. Hence, the sensitivity coefficients are obtained by solving a linear system of equations of the order of the original system. Efficient solution schemes for Eq. 6 are given in Ref. 6.

ILLUSTRATION

The procedure described above is illustrated with an analysis of the Holiday Inn Building in Van Nuys, California. The building is a seven-story reinforced concrete frame structure that sustained extensive nonstructural damage and minor structural damage during the 1971 San Fernando Earthquake. The actual total repair cost was approximately 11% of the initial construction cost.

A nonstationary stochastic seismic analysis was performed with the loading, structure and uncertainties modeled as previously outlined. The seismic hazard curves were developed using the data for the Los Angeles area and the ground condition was taken as intermediate soil. Figure 3 shows the mean and standard deviation of the maximum story drift for the first and second stories. It may be pointed out that the mean drifts at .25g acceleration (the maximum acceleration to which the structure was subjected during the 1971 earthquake) compare well with those estimated during the actual shaking (Refs. 6, 11).

Depending on the load level, approximately 50-80% of the total response coefficient of variation is contributed by the parameter uncertainties, 10-30% from the randomness of the loading history, and 10-20% from the structural modeling uncertainty. The total response coefficient of variation ranged between 60% and 80%.

A type I extreme value distribution is assumed for the maximum drift at a given acceleration. With the seismic hazard curves evaluated for the region, the annual and 50-year exceedance probabilities for given drift values were calculated. The results are plotted in Fig. 4. The dashed lines show the probabilities corresponding to the mean maximum response; i.e., assuming no uncertainty in the calculated response. Observe that in the low probability range, the response uncertainties become more significant; this range is of prime importance for critical structures such as hospitals or nuclear power plants. For example, the return period for the first-story drift exceeding 2.5% (a level at which structural failures may occur) changes from approximately 330 years to 200 years when the response uncertainties are included.

Based on these drift exceedance probabilities and damage functions of the form suggested in Ref. 14, both nonstructural and structural damage probabilities were evaluated. Figure 5 shows these results, in which the damage intensity scale may be interpreted as follows: (0.0) = no damage, (0.1-0.3) = minor damage, (0.4-0.5) = moderate damage, (0.6-0.7) = substantial damage, and (0.8-0.9) = major damage. The 50-year probabilities of exceeding the damage levels actually experienced during the 1971 San Fernando earthquake (i.e., nonstructural damage intensity $\geq$ 0.95 and structural damage intensity around 0.6) are of particular interest. From Fig. 5, the pertinent probabilities are approximately 35% for both stories, implying that the structure had a 35% chance of experiencing damage equal to or greater than that sustained in the earthquake during its lifetime of 50 years. Assuming that safety may be evaluated in terms of the probability of the structural damage intensity exceeding 0.9, the chance of a full time occupant of the structure being exposed to a potentially hazardous seismic event in 50 years is approximately 25%.

ACKNOWLEDGMENTS

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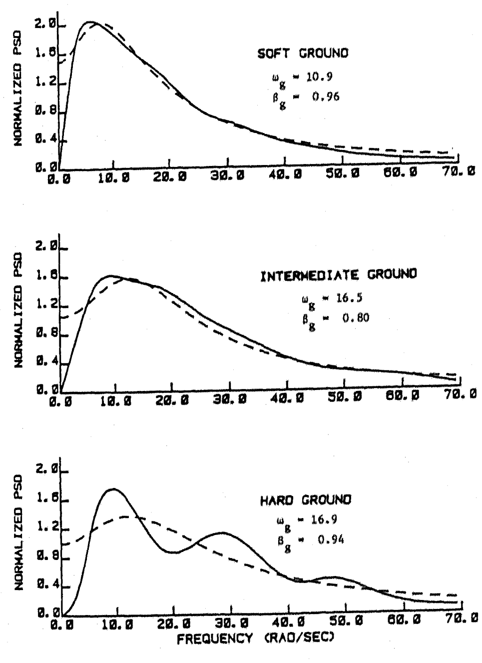

Fig. 1 Best Fit Kanai-Tajimi Spectra (----) and Moayyad and Mohraz (Ref. 3) Empirical Spectra (——)

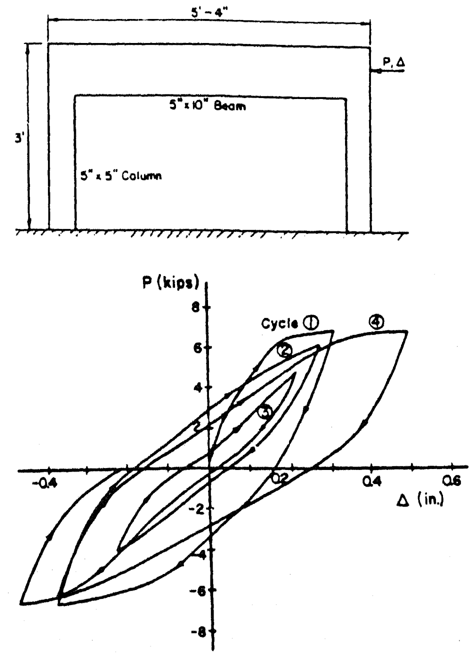

Fig. 2a Load-Deflection Curves for Reinforced Concrete Frame Model (after Gulkan and Sozen, 1971)

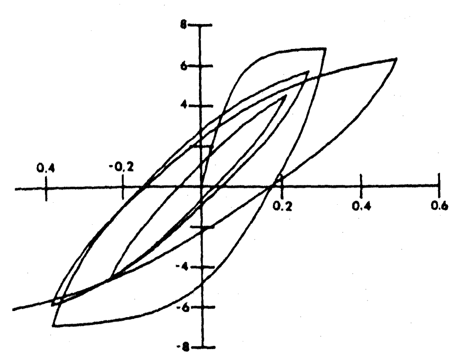

Fig. 2b Displacement Based Stiffness Degrading Model

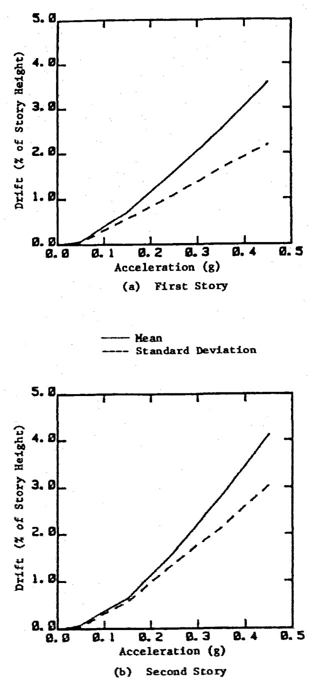

Fig. 3 Maximum Drift Statistics

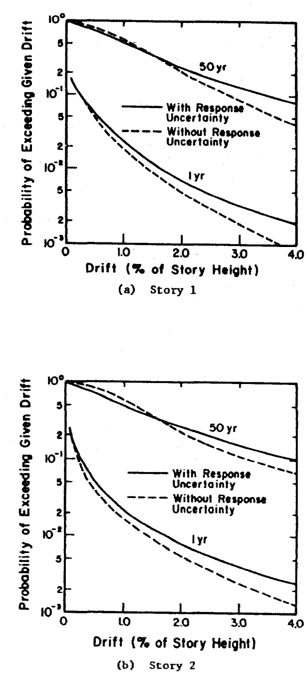

Fig. 4 Drift Exceedance Probability Curves

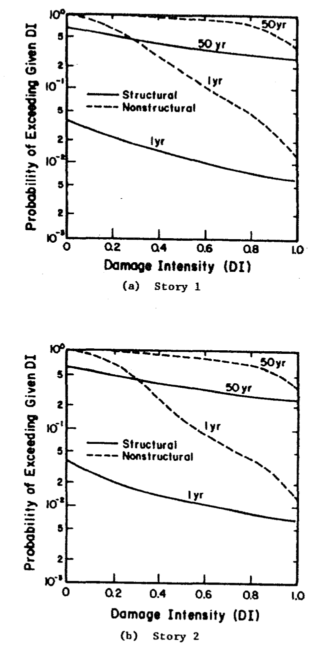

Fig. 5 Damage Intensity Probability Curves