RELIABILITY OF TALL COLUMNS IN INDUSTRIAL FACILITIES

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SUMMARY

This paper presents a simplified method for formulating the limit state equation for the failure of tall columns subjected to seismic ground motion. Probabilities of failure conditional on input ground motion are obtained for a hypothetical distillation column found at oil refineries based on the first order reliability method. The two predominant failure modes of such columns are first yield in the anchor bolts at the base of the column and buckling of the supporting skirt. The limit state equation based on the simplified approach is compared to a limit state equation obtained from random vibrations theory for the input motion and the response of the column. The latter limit state equation is not explicit and first order reliability methods become very difficult to implement. The simplified approach has the potential of being very practical and easily implementable especially in a large scale study.

INTRODUCTION

The need for assessing the risk from failure of major industrial facilities has been long recognized, however attention has been focused on these facilities only recently. The failure of such facilities can result in major economic loss or in direct damage to the environment or a community. Evaluation of the risk from an industrial facility involves a systems representation and analysis of that facility. A fundamental problem which arises in the systems analysis is the evaluation of the component failure probabilities. Many of the key components at such facilities are structures (e.g. distillation columns, pipe supports, furnaces, etc) and require separate reliability estimations.

In this paper existing methods for reliability analysis of strucutres are reviewed and a simplified approach is presented. The proposed method is based on the first order reliability FOR method (Refs. 1-3). A key to this method is the construction of the limit state equation for every failure mode. The proposed procedure considers simplifications for the formulation of the limit state equation for structures subjected to seismic ground motion. For illustrative purposes, the reliability of a tall distillation column found at oil refineries (see Figure 1) is evaluated for the two most important limit states of the structure.

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METHODS FOR COMPONENT RELIABILITY ANALYSIS

In general, structural failures occur when the load on a structure is greater than the structure's resistance. The interaction between the load and the resistance is defined in terms of the margin of safety, $M = R - S$, where R is the resistance and S is the load. The load and the resistance are functions of many random variables. The margin of safety is generalized to a function $g(X)$, where $X = [x_1, x_2, \ldots x_n]$ is the vector of basic random variables. The region defined as $\omega_s = \{X: g(X) > 0\}$ is the safe domain, $\omega_f = \{X: g(X) < 0\}$ is the failure domain and $\partial\omega = \{X: g(X) = 0\}$ is the failure surface. The probability of failure is found as follows

$$P_f = \int_{X\varepsilon\omega_f} f_X(X)\, dX \qquad (1)$$

where $f_X(X)$ is the joint probability density of the basic random variables. In most practical cases, however, the integrations involved are unwieldy even with numerical techniques. For this reason, some approximate methods have evolved for estimating the total probability of failure.

The most widely used approximate technique is the first order second moment method (Ref. 4,5). In this approach the resistance and load are assumed independent and normally distributed and the failure probability is approximated by

$$P_f = 1 - \Phi(\beta) = \Phi(-\beta) \qquad (2)$$

where $\Phi(\cdot)$ is the cumulative normal distribution and β is the safety index defined by

$$\beta = (\mu_R - \mu_S)/\sqrt{\sigma_R{}^2 + \sigma_S{}^2} \qquad (3)$$

In equation 3, μ_R and μ_S are the mean values, and $\sigma_R{}^2$ and $\sigma_S{}^2$ are the variances of the resistance and the load respectively. The means and variances are approximated by the first order term of the Taylor series expansion of safety margin evaluated at the mean values of all random variables. It should be noted that the safety index, β, as defined by equation 3, will vary depending on the form of the failure function. The accuracy of the probability approximation depends on the degree of non-linearity of the failure surface, the deviation of the distributions for R and S from a normal distribution, and the deviation of the safety index defined by equation 3 from the "minimal" safety index as is defined later in this section.

A better approximation of the failure probability defined by equation 2 is obtained by the first order reliability approach. As described earlier, the limit state function g(X) is expressed in terms of the basic random variables X which are not necessarily normally distributed. These variables describing the failure surface are transformed to standard normal variables. The transformation will be exact at all points for a normally distributed basic random variable. For a non-normal basic variable, the probability density and distribution of the transformed variable will equal the density and distribution of the original variable only at the transformation point. This transformation explicitly accounts for dependent random variables.

An invariant measure of the safety index, is provided by the shortest distance from the origin to the transformed failure surface denoted by β_{HL} (Ref. 1):

$$\beta_{HL} = \min_{U\varepsilon\partial\omega} \left(\sum_{i=1}^{n} U_i^2 \right)^{1/2} \qquad (4)$$

where $\partial\omega$ is the failure surface in the transformed space. The point on the failure surface closest to the origin is known as the design point or the most likely failure point. β_{HL} and the design point are usually calculated iteratively. The failure surface is approximated by a first-order Taylor series evaluated at the design point. Then the integral of equation 1 is given by equation 2 where β is replaced by β_{HL}. Ditlevsen (Ref. 2) has shown that this approximation is almost always close to the correct value of the failure probability. A number of different solutions are possible, as this is a classical problem in non-linear equations. An iterative procedure with rapid convergence (Ref. 3) is adopted in this study.

FORMULATION OF LIMIT STATE

A key to the implementation of the first order reliability approach is the formulation of the limit state function, g(X). For purposes of reliability of structures subjected to seismic ground motion the following simplified form for the limit state function is suggested:

$$g(X) = R - A \cdot G_1(X_1) \cdot G_2(X_2) \qquad (5)$$

where R is a resistance parameter and A is a ground motion parameter such as peak ground acceleration, velocity or displacement although not restricted to these. $G_1(X_1)$ is a transfer function from ground motion parameter to input load on the structure (e.g. base shear) and reflects variations due to local soil conditions and soil-structure interaction. $G_2(X_2)$ is a transfer function from structure input load to failure mode measure parameter such as stress at potential failure point or displacement at a critical point on the structure. The vector of random variables X is divided conveniently into two vectors.

Such generalized formulations are developed below for a tall column found at oil refineries. Such columns are designed so that the anchor bolts at the base of the column would yield before any other failure can occur. Thus, the limit state equation for failure by yielding in the anchor bolts can be simply stated

$$g(X) = \sigma_y - \sigma_{b,max} \qquad (6)$$

where σ_y is the yield stress of the bolts and $\sigma_{b,max}$ is the maximum stress on the bolts due to vibrations of the column from earthquake ground accelerations. The objective is to describe the maximum stress in the bolt as a function of the input ground motion. Two approaches are considered for that purpose - a random vibrations analysis and equivalent static analysis. Numerical results from the equivalent static analysis will be presented.

Random Vibrations Approach

The theory of random vibrations is employed to develop the limit state equation for the predominant failure mode of a tall column. Let earthquake ground acceleration be a stationary wide-band Gaussian process, $\ddot{X}(t)$, with a power spectral density $S_{\ddot{X}}(\omega)$. This assumption is often made when considering only the high amplitude portion of a signal (Ref. 6) and when the structure has very low damping in the elastic range (Ref. 7). Tall columns, such as the ones considered in this paper, have been shown to have very low damping (Ref. 8). Linear time-invariant behavior conditions for the column are also met for the specific limit state of first yield in the bolts.

In order to evaluate the maximum stress in the anchor bolts of the column, it is first necessary to determine the distribution of the peak acceleration amplitudes at the top of the column (see Figure 1). Assume the input power spectrum to be essentially constant, S_o, across the width of the column transfer function. The column is modeled as a cantilever with a constant mass and cross section along its height. The fundamental period is assumed to dominate. With these assumptions, the displacement at the top of the column will be a narrow band process with a power spectrum given by (Ref. 9)

$$S_y(\omega) = \begin{cases} \left(\frac{\pi K}{\Delta\omega pkL}\right)^2 \cdot S_o & \text{for } p - \frac{\Delta\omega}{2} \leq |\omega| \leq p + \frac{\Delta w}{2} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where K is a normalizing constant, $\Delta\omega = \pi\xi p$ is the width of the transfer function, ξ is the damping ratio, p is the fundamental period of the column, L is the height of the column, and $k = 1.875/L$ is a shape constant (see Ref. 11). For this response process the distribution of the peak accelerations, a, is Rayleigh (Refs. 9,11)

$$f_A(a) = \frac{a}{\sigma_{\ddot{y}}^2} \exp\left[-a^2/2\sigma_{\ddot{y}}^2\right] \tag{8}$$

where $\sigma_{\ddot{y}}^2$ is the variance of the response acceleration. The moment at the base of the column is found to be a function of the peak accelerations, a, the mode shape, and the mass distribution along the height of the column. The moment is given by

$$M_{base} = (aWLB)/(g(kL)^2) \tag{9}$$

where W is the weight of the column, g is the acceleration of gravity and B = 1.566 is a normalizing constant related to the mode shape (Ref. 12). The maximum stress in the bolts at the base of the column, $\sigma_{b,max}$, is expressed as an implicit function of the weight of the column and the moment at the base using equilibrium conditions (see Figure 2):

$$W = B_1\,[A_1 - R'A_2] - A_3 \tag{10}$$

$$M_{base} = B_1\,[R_s^2 A_2 - R'A_1]/2 + A_4 \tag{11}$$

$$\text{where } A_1 = \sqrt{x'(2R_s - x')}\ ;\ R' = R_s - x'$$

$$A_2 = \cos^{-1}(R'/R_s);\ B_1 = 2\,R_s t E_c \varepsilon_{cm}/x'$$

$$A_3 = E_s \varepsilon_{cm} A_b \Big[\sum_{x_i < R'} (R' - x_i)/x'\Big]$$

and

$$A_4 = E_s \varepsilon_{cm} A_b \Big[\sum_{x_i < R'} (R' - x_i)^2/x'\Big];$$

where E_s and E_c are Young's moduli for steel and concrete respectively, A_b is the cross sectional area of a bolt, R_s, t and x' are as shown on Figure 2. Equations 9, 10 and 11 have to be solved iteratively in order to find ε_{cm} and the neutral axis x'. The maximum bolt stress is then given by:

$$\sigma_{b,max} = E_s \varepsilon_{cm} (R_s + R_b)/x' \tag{12}$$

where R_b is the bolt radius. Equations 9 through 12 are needed to describe the limit state equation 6. In order to apply the FOR method beyond this point, it is necessary to evaluate the function g(X) numerically. This formulation of the limit state equation proves to be rather lengthy and difficult to apply, paticularly when repeated applications of the model are necessary. In the following section we present a simpler method for formulating the limit state function which lends itself to easy implementation.

Equivalent Static Method

In order to simplify the formulation of the limit state equation the following assumptions are made. The ground motion parameter is a peak ground acceleration (PGA) value. The transfer function is the dynamic amplification factor taken from a response spectrum. An exact mode shape is replaced by an assumed shape when calculating the moment at the base and the iterative solution for equilibrium at the base is replaced by the assumed stress distribution used in the design equation. These simplifications provide a closed-form expression for the limit state equation. The procedure is illustrated in the following examples. The two limit states considered are first yield in the bolts and buckling of the supporting skirt.

For the analysis of first yield in the bolts, it is assumed that the ground motion is sufficiently well characterized by peak ground acceleration (PGA). The shear at the base of the column is computed following ATC-3 design recommendations:

$$V = A \cdot G_1(X) = PGA \cdot BDQW = C_1 \cdot PGA \tag{13}$$

where $B = 0.7$ is the ductility factor; $D = 2\sqrt{0.3/T}$ is the mean dynamic amplification factor with T = natural period of the structure; Q is quality factor taken as unity to remain mean centered; and W is the weight of the column. The shear is assumed to have a triangular distribution with height with an additional force at the top of the column to account for higher modes of vibration. The moment at the base is given by

$$M_{base} = L(0.07T + 2/3(1 - 0.07T)) \cdot V = C_2 \cdot V \tag{14}$$

Following design recommendations for design of tall distillation columns (Ref. 14) the stress in the bolts is expressed as:

$$\sigma_{b,max} = \frac{1}{A_b}\left(\frac{4\,M_{base}}{ND_b} - \frac{W}{N}\right) \tag{15}$$

where N is the total number of bolts, A_b is the cross sectional area of a bolt and D_b is the bolt circle diameter. The limit state equation 5 is then given by

$$g(X) = \sigma_y - PGA \cdot G_1 + G_2$$

$$\text{where } G_1 = 4W \cdot C_1 \cdot C_2/(A_b N D_b) \quad \text{and} \quad G_2 = W/(NA_b). \tag{16}$$

The random variables in equation 16 are listed in Table 1. The corresponding probability distributions are also identified in that table. Implementation of the FOR method with the limit state given by equation 16 is relatively straight-forward. Probabilities of failure of a hypothetical column 100 ft high and with a diameter of 6 ft where obtained as a function of the peak ground acceleration level. Figure 3 shows the failure probability conditional on peak ground accelerations for first yield in the bolts of that column.

The second failure mode important for tall columns is buckling of the supporting skirt. A rigorous random vibrations analysis for this limit state is very complicated because of the non-linear behavior of the structure after the first yield in the bolts. The simplified approach, however, can be modified as follows.

The failure state of the column is defined as the stress in the supporting skirt shell reaching the critical buckling load given by

$$\sigma_{cr} = \frac{2Et}{D\sqrt{3(1-\nu^2)}} \cdot \psi\,(D/2t) \qquad (17)$$

where E is Young's modulus, t is the thickness of the skirt, D is the outer diameter of the skirt, ν is Poisson's ratio, and ψ is a reduction factor based on D/2t. Using the well known equation for maximum stress $\sigma_{max} = M_{base}/S + W/A$, where S is the section modulus and A is the cross sectional area of the column, the limit state equation is obtained as follows

$$\begin{aligned} g(X) &= \sigma_{cr} - PGA \cdot G_3 + G_4 \\ G_3 &= L(0.67 + 0.0233T)(1.4\sqrt{0.3/T})D' \\ G_4 &= D'(D^2 + D_i^{\,2})/(8D) \qquad (18) \\ D' &= 32\ WD/[\pi \cdot (D^2 + D_i^{\,2})^2] \end{aligned}$$

where D_i is the inner diameter of the column. Table 1 lists the variables in equation 18 which were considered random and their corresponding probability distributions. Probabilities of failure for the hypothetical column described above were also computed using equation 18. Figure 3 shows the probability of failure conditional on the peak ground acceleration. As expected, the failure probabilities for first yield of the bolts are consistently greater than these for buckling of the skirt. For overall systems analysis, however, both failure modes may be of importance.

CONCLUSIONS

Several methods are available for the evaluation of failure probabilities of structures at large industrial facilities. These methods were reviewed with the objective of identifying their advantages and disadvantages when implemented in a large systems analysis procedure. A simplified method for reliablity analysis of structures is presented based on the first order reliability approach. Simplifications of the limit state equations are especially useful for the application of the first order reliablity method. The simplified form of the limit state equation is used for the specific example of tall distillation columns. A theoretical formulation of the limit state based on random vibrations is also presented to illustrate the degree of difficulties encountered in implementing the FOR method. The simplified equation appears to provide sufficiently good description of the failure state and corresponding failure probabilities are easily estimated.

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Table 1 Variables used in Limit State Equation

First Yield in Bolts		Buckling of Skirt	
Variable	Distribution & Parameters	Variable	Distribution & Parameter
C_1	Normal $(\overline{C}_1, 0.4\ \overline{C}_1)$ $\overline{C}_1 = 48.55$	C_1	Normal $(\overline{C}_1, 0.4\ \overline{C}_1)$ $\overline{C}_1 = 62.6\sqrt{0.3/T}$
C_2	Normal $(\overline{C}_2, 0.15\ \overline{C}_2)$ $\overline{C}_2 = 825.1$	C_2	Normal $(\overline{C}_2, 0.15\ \overline{C}_2)$ $\overline{C}_2 = 69T$
σ_y	Normal $(\sigma_y, 0.05\ \sigma_y)$ $\sigma = 33$ ksi	T	Normal $(\overline{T}, 0.10\ \overline{T})$ $\overline{T} = 0.898$-s
N	Deterministic; $N = 16$ bolts	Ψ	Lognormal $(\ln\overline{\Psi}, \sigma_\Psi)$ $\overline{\Psi} = 0.349$; $\sigma_\Psi = 0.0744$
W	Deterministic; $W = 60$ kips	W	Deterministic; $W = 60$ kips
D_b	Deterministic; $D_b = 78$ in	E	Deterministic; $E = 29\times10^6$ psi
A_b	Deterministic; $A_b = 2.3$ in^2	ν	Deterministic; $\nu = 0.25$
		t	Deterministic; $t = 0.25$ in
		D	Deterministic; $D = 6.0$ ft
		D_i	Deterministic; $D_i = 5.979$ ft.

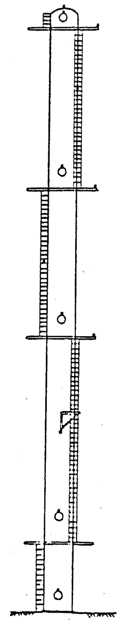

Figure 1. Diagram of a distillation column

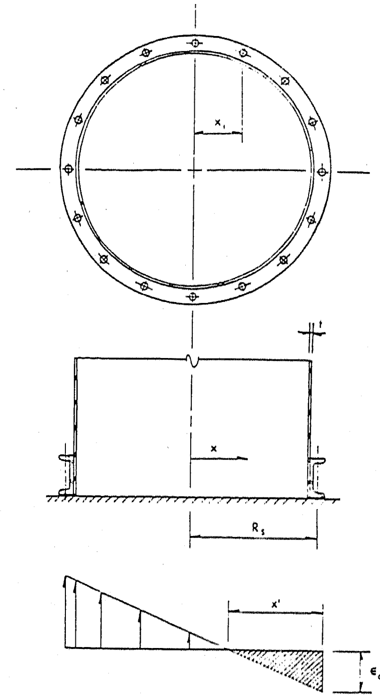

Figure 2. Diagram of cross section of the base of the column and corresponding stress distribution.

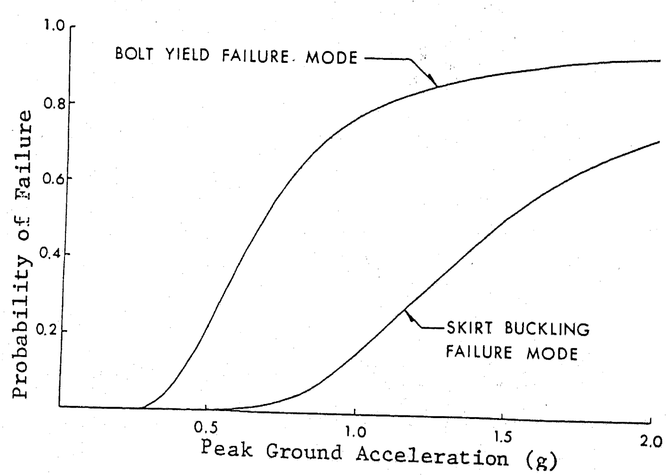

Figure 3. Probabilities of failure of column conditional on input ground acceleration.