STUDY ON THE PREDICTION OF EARTHQUAKE DAMAGE TO BUILDING GROUPS IN URBAN AREAS

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SUMMARY

This paper discusses a method for predicting the earthkuake damage to a group of buildings taking account of the probability distributions of both the building resistance capacity and the earthquake force. Mainly discussed are the probabilistic features of the maximum ground motion during a specified period of time considering the statistical modelling of earthquake occurrence. Seismic risk of cities in Tohoku district, Japan, is studied as an example.

INTRODUCTION

Prediction of the probability of earthquake damage to a group of buildings in a certain urban area within a specified period of time is important in considering the earthquake disaster prevention of urban environment.

Fig. 1 shows an example of the percentage of damaged RC buildings in three different areas with different soil conditions in Sendai city caused by 1978 Miyagi-ken-oki earthquake (Ref. 2). The percentage of damaged buildings to all existing buildings located in a certain area is considered to be influenced by the variety in the resistance capacity of existing buildings and by the variety in the earthquake force exerted on buildings.

The author formerly presented a method for estimating the percentage of damage to a group of buildings due to the maximum ground motion in a specified period of time by use of reliability theory based on the probability distribution of both the resistance capacity and the earthquake force (Ref. 1).

In this paper, mainly considered are the probabilistic features of earthquake force, especially the maximum ground motion in a specified period of time taking account of the statistical model of earthquake occurrence and the attenuation relation.

ESTIMATION OF EARTHQUAKE DAMAGE TO A GROUP OF BUILDINGS

The seismic resistance capacity of a group of buildings and the earthquake force exerted on buildings are assumed to be modelled by the random variables, R and S, having the probability density functions, $P_R(r)$ and $p_S(s)$, respectively as shown in Fig. 2 (Ref. 1).

It is assumed that the failure of buildings will occur if $R< \alpha S$, where α is related to the level of damage considered. The probability of failure p_f is then expressed as follows.

$$P_f = \mathrm{Prob}[R < \alpha S] = \int_0^\infty p_S(s)\int_0^{\alpha s} p_R(r)\,dr\,ds = \int_0^\infty p_R(r)\int_{r/\alpha}^\infty p_S(s)\,ds\,dr \quad \text{--- 1)}$$

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If the lognormal distribution is assumed both for the building resistance R and the earthquake force S, the value of p_f is given as follows.

$$p_f = 1 - \Phi \left[\frac{\ln(\mu_R/\alpha\mu_S) - (1/2)\ln[(1 + v_R^2)/(1 + v_S^2)]}{\sqrt{\ln[(1 + v_R^2)(1 + v_S^2)]}} \right] \qquad \text{----- 2)}$$

where μ_R, μ_S = mean values of R and S, v_R, v_S = coefficients of variation of R and S, and Φ [ ] = standard normal distribution function.

The value of p_f is interpreted as the percentage of damaged buildings to all existing buildings, which expresses the extent of earthquake damage to a group of buildings of a certain structural type in a certain urban area.

The value of α can be related to the degree of inelastic deformation. If we assume the energy conservation rule for the inelastic earthquake response, α is expressed as follows (Fig. 3).

$$\alpha = 1 / \sqrt{2d - 1} \qquad \text{----- 3)}$$

where α = ductility factor corresponding to the level of damage.

PROBABILISTIC MODEL OF RESISTANCE CAPACITY

The probability distribution of the seismic resistance capacity is obtained from the investigation of the structural properties of existing buildings. The probability distribution of a seismic resistance index of RC buildings in terms of seismic coefficient has been studied by Shiga (Ref. 3) and Shibata (Ref. 1) and was shown to be satisfactorily modelled by the lognormal distribution. Onose presented a model for the seismic resistance index of low-rise RC buildings taking account of the number of stories (Ref. 4).

PROBABILISTIC MODEL OF EARTHQUAKE FORCE

We assume that the earthquake force S exerted on buildings is expressed by the idealized acceleration response spectrum, the value of which is random variable having lognormal probability distribution (Fig. 4).

A model of earthquake force S can be expressed as follows.

$$S = \mu_S \, L_S \qquad \text{----- 4)}$$

where μ_S = mean value of response spectrum, and
L_S = random variable having lognormal distribution of which mean value is 1.0 and coefficient of variation is v_S.

The mean value of response spectrum μ_S is given in terms of seismic coefficient in a deterministic manner as follows.

$$\mu_S = \overline{S}(T) \, \gamma \, \eta(h) \, A_m \, / \, G \qquad \text{----- 5)}$$

where $\overline{S}(T)$ = normalized dynamic amplification spectrum, T = natural period, γ = effect of soil condition on maximum ground acceleration,

$\eta(h)$ = effect of damping factor h on spectrum values, and
A_m = maximum ground acceleration expected in a specified period of time at the bedrock of a city (G = gravity acceleration).

Fig. 5 shows a tentative model for the combination of $\bar{S}(T)$ and γ for h = 0.02, considering the effect of local soil condition (Ref. 2). The value of A_m has to be determined on the basis of seismic activity in the surrounding earthquake source areas, which is discussed in the following.

MODELLING OF EARTHQUAKE OCCURRENCE

Fig. 6 shows the plot of epicenters of earthquakes with magnitude greater than 5.0 which occurred in Tohoku district, Japan, during the period from 1926 to 1981. We assume that the activity of interplate earthquakes (ocean earthquakes) can be divided into six earthquake blocks, A to F, as shown in Fig. 6. As for the intraplate earthquakes (inland earthquakes), two earthquake blocks in Miyagi prefecture, G and H, in Fig. 6 are considered as an example.

The activity of each block is modelled by the hypothetical point source having the annual average number N_0 of earthquakes greater than the lower limit magnitude M_0 and the probability density function p(M) of magnitude M.

We assume the modified Gutenberg-Richter formula by Utsu (Ref. 5) for the magnitude-frequency relation in each earthquake block as shown below.

$$n(M) = (M_L - M)\exp(a - \beta M) \quad \text{----- 6)}, \qquad N(M) = \int_M^{M_L} n(M)dM \quad \text{----- 7)}$$

where n(M) = annual average number of earthquakes between magnitude M and M + dM ($n(M) = 0$ for $M < M_0$ and $M > M_L$),
N(M) = annual average number of earthquakes with magnitude greater than M ($N(M) = N_0$ for $M \leq M_0$ and $N(M) = 0$ for $M > M_L$),
M_L = upper limit magnitude, and M_0 = lower limit magnitude.

The probability density function p(M) and the probability distribution function P(M) are derived from Eqs. 6) and 7) as follows.

$$p(M) = n(M) / N_0 = \beta^2(M_L - M)\exp(-\beta(M - M_0)) / D \quad \text{----- 8)}$$

$$P(M) = 1 - N(M) / N_0 = 1 - \{\beta(M_L - M)\exp(-\beta(M - M_0)) + \exp(-\beta(M_L - M_0)) - \exp(-\beta(M - M_0))\} / D \quad \text{----- 9)}$$

where $D = \beta(M_L - M_0) + \exp(-\beta(M_L - M_0)) - 1$.

The values of M_L and β are determined by the method of maximum likelihood (Ref. 5) based on the earthquake data of 1926 - 1981. The estimated values of M_L and β for each earthquake block are shown in Table 1 together with the values of N_0, M_0 and M_{max}, where M_{max} is the maximum magnitude actually occurred. N_0 is taken equal to the value from the data. It is assumed that M_0 is 5.0 for blocks A to F and 4.0 for blocks G and H. Maximum likelihood estimation was done by grouping the data with the magnitude step of 0.5 for blocks A to F and 1.0 for blocks G and H.

Figs. 7 and 8 show the estimated curves of p(M) and P(M) for Block C (

Off Sendai block). Figs. 9 and 10 show the plot of n(M) and N(M) for block C from the earthquake data, together with the model curves from Eqs. 8) and 9) using the estimated values of M_L and β.

The location of each point source is assumed to be the simple average of the latitude and the longitude of earthquake epicenters ($M \geq M_0$) in each block.

PROBABILITY DISTRIBUTION OF MAXIMUM GROUND MOTION

Assuming the properties of hypothetical point sources and the appropriate attenuation relation, the probability distribution function for the maximum ground motion during a specified period of time is derived (Ref. 7).

The probability distribution function $P_t(M)$ of the maximum magnitude during a period of time t is expressed as follows under the assumption of Poisson process for the sequence of earthquake occurrence (Ref. 6).

$$P_t(M) = \sum_{k=1}^{\infty} ((N_0 t)^k \exp(-N_0 t)/k!)(P(M))^k$$
$$= \exp(-N_0 t(1 - P(M))) = \exp(-N(M)t) \qquad \text{----- 10)}$$

where $N(M) = N_0(1 - P(M))$.

As for the attenuation relation, the following Kanai's formula is used.

$$y = a_1 \exp(a_2 M)\ r^{-a_3} \qquad \text{----- 11)}$$

where y = maximum ground acceleration, r = focal distance $= \sqrt{\Delta^2 + e^2}$, Δ = epicentral distance, e = focal depth (assumed to be 40 km for blocks A to F and 10 km for blocks G and H in this analysis), $a_1 = 6.74/\sqrt{T_G}$, $T_G = 0.3$, $a_2 = 1.405$, and $a_3 = 1.73$.

The value of magnitude corresponding to a given value of acceleration is obtained from Eq. 11) as follows.

$$M(y) = (\ln(y) + a_3 \ln(r) - \ln(a_1)) / a_2 \qquad \text{----- 12)}$$

The probability distribution function $P_Y(y)$ of the maximum ground acceleration at a site during t years caused by n independent sources is given as follows (Ref. 7).

$$P_Y(y) = \prod_{i=1}^{n} P_{Yi}(y) = \prod_{i=1}^{n} P_{ti}(M(y))$$
$$= \exp(-\sum_{i=1}^{n} N_i(M(y))t) \qquad \text{----- 13)}$$

The characteristic value of the maximum ground acceleration is determined tentatively by putting the exceedance probability $(1 - P_Y(y))$ equal to 0.632 $(=1 - e^{-1})$. Under this assumption the relation between the characteristic maximum ground acceleration A_m and the period t is obtained as follows.

$$t = 1 / \sum_{i=1}^{n} N_i(M(A_m)) = 1 / \sum_{i=1}^{n} N_{0i}(1 - P_i(M(A_m))) \qquad \text{----- 14)}$$

Fig. 11 shows the period t corresponding to the characteristic maximum acceleration A_m for five cities along the Pacific coast in Tohoku district considering the ocean earthquakes only (blocks A to F). The values of A_m corresponding to specified values of t obtained by interpolation are tabulated in the right-hand side of the figure.

To study the effect of inland earthquakes, the case of Miyagi prefecture is considered. Fig. 12 shows the t - A_m relation for three cities in Miyagi prefecture considering both the ocean earthquakes (blocks A to F) and inland earthquakes (blocks G and H). Values for ocean earthquakes only are also shown for comparison. It is tentatively assumed that in case of inland earthquakes the distance r in Eq. 11) is replaced by $r + r_0$ in order to modify the acceleration values near source point, where r_0 is assumed to be 20 km.

It should be noted that the effect of inland earthquakes near source area is susceptible to the assumption of attenuation relation. Further studies are needed on the nature of attenuation relation near source area considering the spread of source region. It is also to be noted that the effect of various soil conditions on earthquake force has to be accounted for separately from the evaluation of maximum acceleration A_m at bedrock.

CONCLUSIONS

Described is a process for evaluating the probabilistic features of earthquake force to be utilized in the prediction of earthquake damage to a group of buildings. The activity of the earthquake environment in Tohoku district, Japan, is modelled by multiple point sources and the probability distribution of earthquake magnitude for each source is modelled on the assumption of modified Gutenberg-Richter formula by Utsu taking account of the upper limit magnitude. The properties of maximum ground acceleration expected in specified periods of time are studied for seven cities in Tohoku district.

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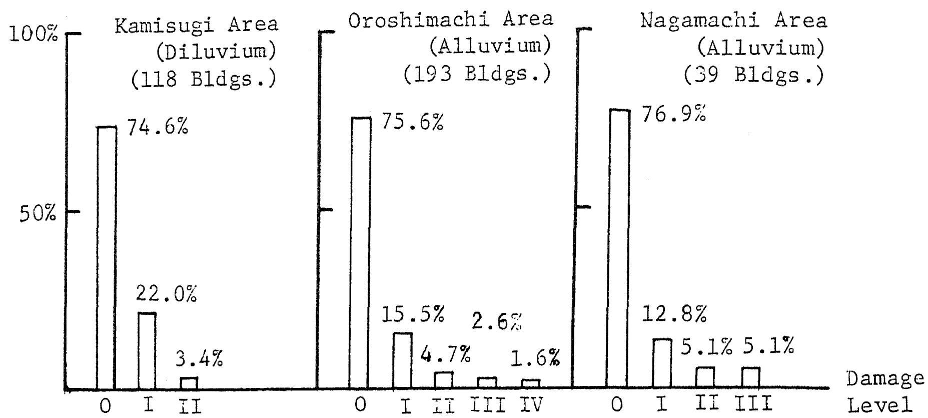


Fig. 1 Percentage of Damaged RC Buildings in Sendai City Caused by Miyagi-ken-oki Earthquake (0=No Damage, I=Small Damage, II=Medium Damage, III=Heavy Damage, IV=Collapse)

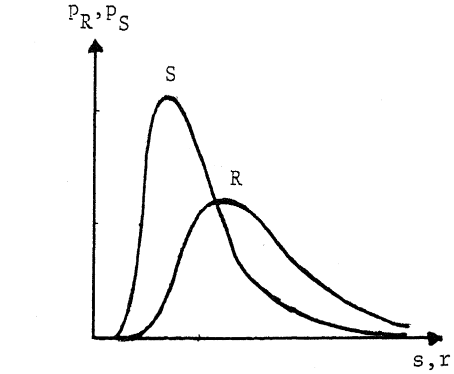


Fig. 2 Probability Distribution of Resistance R and Force S

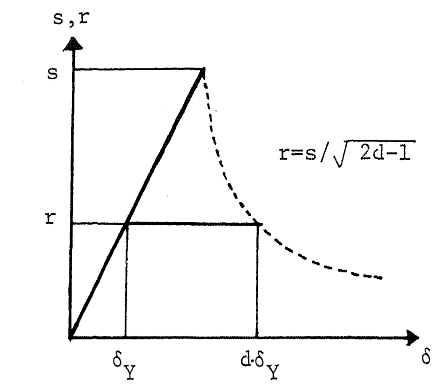


Fig. 3 Criterion of Failure Considering Inelastic Deformation

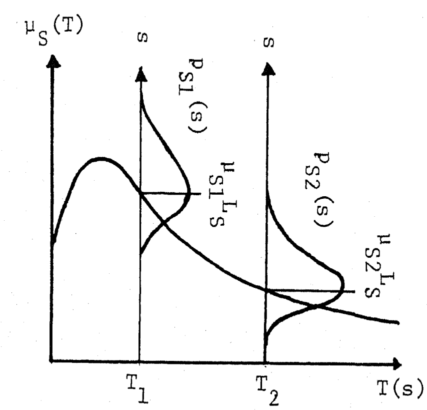


Fig. 4 Probabilistic Model of Response Spectrum

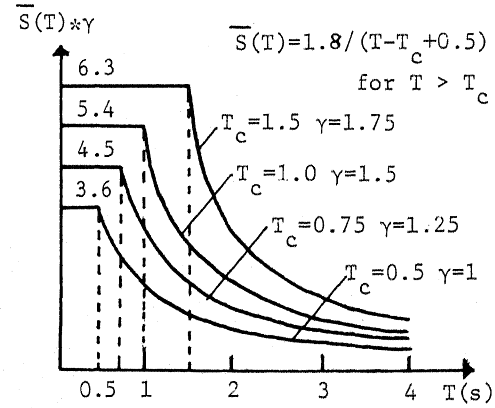


Fig. 5 A Model of Amplification Spectra Considering Soil Condition

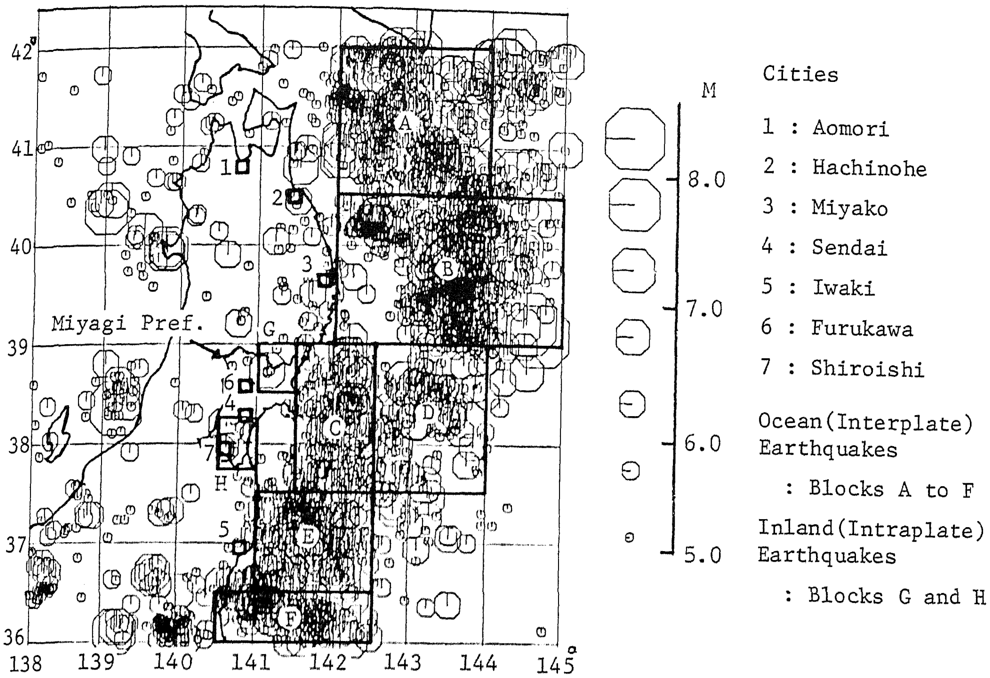


Fig. 6 Epicenters of Earthquakes (M≥5.0) from 1926 to 1981 in Tohoku District

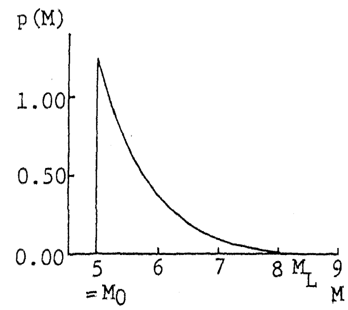


Fig. 7 Probability Density Function of Magnitude

Table 1 Parameters of Earthquake Blocks

Block	N_0	M_0	Mmax	M_L	β
A	7.79	5.00	8.00	8.74	1.40
B	10.54	5.00	8.30	8.70	1.45
C	2.84	5.00	7.70	8.39	0.83
D	2.54	5.00	7.10	7.44	0.47
E	4.57	5.00	7.70	8.43	1.12
F	3.29	5.00	6.90	7.12	0.55
G	0.93	4.00	6.50	6.81	1.09
H	0.43	4.00	6.60	7.08	0.24

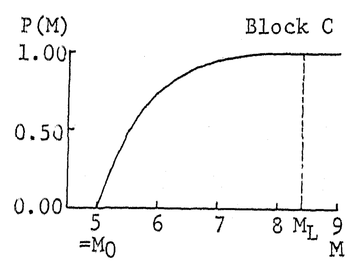


Fig. 8 Probability Distribution Function of Magnitude

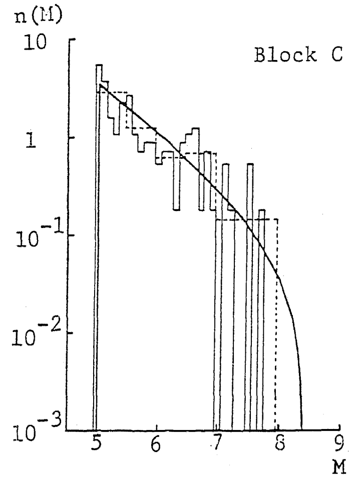


Fig. 9 Annual Average Number of Earthquakes between M and M+dM

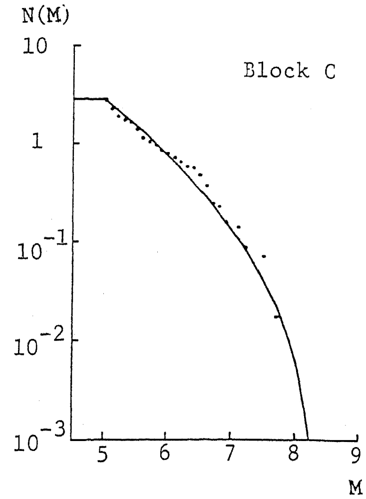
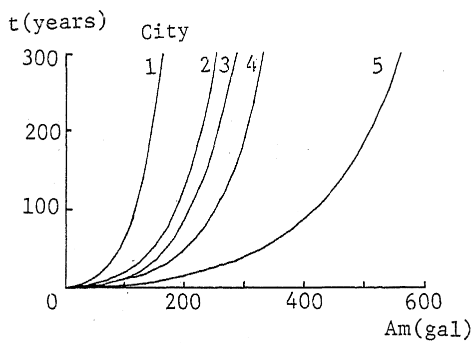


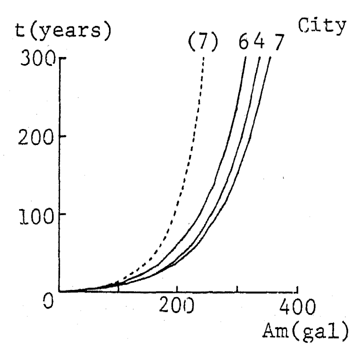
Fig. 10 Annual Average Number of Earthquakes Greater than M



City	t years 20	50	100	200
1	64	93	119	146
2	102	149	188	230
3	123	171	212	258
4	142	205	255	306
5	217	329	421	512

unit gal

Fig. 11 Relation between Characteristic Maximum Acceleration and Period in Tohoku District (Ocean Earthquake Only)



City	t years 20	50	100
4	151 (142)	210 (205)	256 (255)
6	130 (128)	187 (187)	237 (237)
7	155 (115)	219 (162)	269 (197)

r_o=20(km) unit gal

----- and ( ) : ocean eq. only.

Fig. 12 Relation between Characteristic Maximum Acceleration and Period in Miyagi Prefecture (Ocean Earthquakes and Inland Earthquakes)