A TENTATIVE APPLICATION OF FUZZY SET THEORY TO DAMAGE PREDICTION

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ABSTRACT

Forecasting great earthquake-generated damage to buildings in a city is complicated by vagueness, ambiguity and fuzziness of concepts involved and relations between factors concerned. It is more rational to describe the problem in fuzzy mathematical manner. This paper presents a new model for damage prediction, which is principally on the basis of fuzzy logic and approximate reasoning. Using proposed model we can forecast the general situation of damage to various types of buildings. An approach to damage prediction of single-story buildings is presented here as well. Fuzzy synthetic evaluation is used in this method.

INTRODUCTION

In China, many cities, as ever, face such serious problem as great earthquake-caused severe damage to buildings and urban facilities. In order to avoid and relieve devastation to a city, it is necessary to frame a disaster preparedness plan according to the results of damage prediction, which should be put on the basis of information gained by seismic hazard analysis and seismic microzoning of that city. Since there exist lots of vague and fuzzy concepts involved in the damage-predicting problem, and the relations between concerning factors also have ambiguous nature, it is more rational to deal with the problem of earthquake-related damage forecasting with aid of fuzzy mathematics. The fuzzy set theory was first advanced by L. A. Zadeh in 1964 (Ref. 1). During the past two decades, significant development has been made. A new field as fuzzy mathematics has recieved more and more attention in a wide range of scientific areas, including structure engineering and earthquake engineering (Ref. 2,3). In this paper, a new model, written in fuzzy mathmatical manner, for damage prediction is presented. The model is formulated by multiple component "IF...THEN..." approximate reasoning. Using suggested procedure, we may obtain a general situation of damage in future earthquake to various types of buildings. At first, we will give the representation of damage grade of indivitual building and damage degree of a district in the form of fuzzy set. Then the model, including the establishment of fuzzy

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relation between earthquake excitation and damage to buildings and inference rule used for prediction, will be briefly introduced. At last, a method to predict the earthquake-generated damage to single-story building by means of fuzzy synthetic evaluation is presented as well.

FUZZY REPRESENTATION OF DAMAGE GRADES

Usually, engineers use linguistic description, such as "severely damaged", "basically intact", etc., to depict the damage level to a building in earthquake disaster survey, as well as in earthquake-caused damage prediction. In terms of fuzzy set theory, the damage to a building may be regarded as linguistic variable, and the linguistic descriptions of various damage grades are its values. Generally, engineers use six grades of damage, such as collapse, partial collapse, severely damaged, moderately damaged, slightly damaged, basically intact. Five or four categories of damage are often applied to building. We may use the damage index-ind as the base variable, and to express the six damage grades by fuzzy sets of the base variable $\underset{\sim}{A}_i$ (ind), $i=1, \ldots, 6$. As usual, the damage index 0 denotes exactly intact, 1 completely failure, and the mediate values between 0 and 1 show the intermediate damage levels. So the universe of discourse of ind is [0, 1]. For the purpose of damage prediction, we should explore how engineers use these linguistic description of damage grades, in order that the forecasting procedure may put on the basis of information cited in documents about historical earthquake disaster. In other words, we should establish fuzzy sets $\underset{\sim}{A}_i$ in accordance with what people do really understand about the linguistic terms of damage grades in earthquake disaster field survey. More than 500 samples have been collected, each one is provided with damage grade evaluated at that time and damage state depicted by natural language. Suppose the membership functions $\mu_{\underset{\sim}{A}_i}$ (ind) possesses a normal configuation

$$\mu_{\underset{\sim}{A}_i}(\text{ind}) = e^{-\frac{(ind-\overline{x}_0)^2}{c^2}} \qquad (1)$$

we can give every sample a value of $\overline{x_0}$ and c, according to its description of damage state, and a value $\rho \in [0, 1]$ to reflect the creditability of every piece of information. Then we make fuzzy statistics, which are quite different from probability theory, for samples belonging to each grade of damage, and take the mean value of $\overline{\rho x_0}$, $\overline{\rho c}$ as $\overline{x_0}$ and c in eq. (1) for every category of damage. The suggested fuzzy sets $\underset{\sim}{A}$, $i=1, \ldots, 6$, written in discrete manner and having some modification for easily using, are as following:

$\underset{\sim}{A}_1$ = (basically intact)

$\triangleq 1/0 + 0.91/0.05 + 0.7/0.1 + 0.42/0.15 + 0.2/0.2$

$\underset{\sim}{A}_2$ = (slightly damaged)

$\triangleq 0.3/0 + 0.7/0.1 + 1/0.2 + 0.7/0.3 + 0.2/0.4$

$\underset{\sim}{A}_3$ = (moderately damaged)

$\triangleq 0.2/0.2 + 0.7/0.5 + 1/0.4 + 0.7/0.5 + 0.2/0.6$

$\underset{\sim}{A}_4$ = (severely damaged)

$\triangleq 0.2/0.4 + 0.7/0.5 + 1/0.6 + 0.7/0.7 + 0.2/0.8$

$\underset{\sim}{A}_5$ = (partial collapse)

$$\triangleq 0.2/0.6+0.7/0.7+1/0.8+0.7/0.9+0.2/1.0$$

$\underset{\sim}{A}_6$ = (collapse)

$$\triangleq 0.1/0.8+0.42/0.85+0.7/0.9+0.91/0.95+1/1$$

FUZZY INTENSITY AND FUZZY DAMAGE DEGREE

The earthquake intensity is used to indicate the consequence and effect of an earthquake. Up to date, various intensity scales still give the calibration of every degree of intensity by means of linquistic description of building's damage and people's feeling. In other words, engineers and invastigators rate intensity according to damage state of buildings. So the concept of intensity is intrinsically a fuzzy one, and fuzzy intensity $\underset{\sim}{I}$(ind), which is defined as fuzzy sets of average damage index, is proposed to reflect its fuzzy characteristic (Ref. 4). The average damage index is by definition the average value of all damage indexes within a district, and is used as the base variable in this case. Since the draft of revised China's earthquake intensity scale (1980) utilizes damage to nonengineered masonry and wooden structures to rate intensity, the fuzzy intensity will be also evaluated by the damage state of such structures. We can use the same fuzzy sets as those for fuzzy intensity to indicate the damage level or damage potential, which we now define as damage degree $\underset{\sim}{D}$. For specified type of building the damage degree is written in the form $\underset{\sim}{D}_i$. The samples, which number 81 covering Ⅵ～Ⅺ degree, were drawn from documents on historitical earthquakes and analized through following formula:

$$\underset{\sim}{I} \triangleq \sum_{i=1}^{6} m_i \underset{\sim}{A}_i \tag{2}$$

where $\sum \triangleq$ union and m_i is the percent of buildings classified in ith category of damage. Sometimes the quantity of buildings classified in every category is specified by linguistic terms, such as "many", "a few", "more than half", etc. Under this situation, the quantity terms should be expressed by fuzzy sets $\underset{\sim}{m}_i$ of [0, 1]. Then eq. (2) becomes to

$$\underset{\sim}{I} \triangleq \sum_{i=1}^{6} \underset{\sim}{m}_i \underset{\sim}{A}_i \tag{2'}$$

According to extension principle

$$\underset{\sim}{Z} = f(\underset{\sim}{A}_1, \cdots, \underset{\sim}{A}_t) = \int \alpha f((\underset{\sim}{A}_1)_\alpha, \cdots, (\underset{\sim}{A}_t)_\alpha) \tag{3}$$

where $\int \triangleq$ union, $\alpha \in [0, 1]$, and $(\underset{\sim}{A}_i)_\alpha$ the α-cut, and defining the operation of nterval number as

$$f(L_1, \cdots, L_t) \triangleq \{z = f(l_1, \cdots, l_t) | l_i \in [a_i, b_i], i = 1, \cdots, t\} \tag{4}$$

where $L_i \triangleq [a_i, b_i]$ the interval number, then we can conduct the calculation of eqs. (2), (2'). Our suggestion about fuzzy intensity from degree Ⅵ to Ⅺ is shown in Table 1.

Table 1

intensity \ ind μ_I	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1.0
VI	0.55	0.86	1	0.86	0.55	0.26	0.09														
VII		0.09	0.26	0.55	0.86	1	0.86	0.55	0.26	0.09											
VIII					0.09	0.26	0.55	0.86	1	0.86	0.55	0.26	0.09								
IX									0.09	0.26	0.55	0.86	1	0.86	0.55	0.26	0.09				
X												0.09	0.26	0.55	0.86	1	0.86	0.55	0.26	0.09	
XI															0.09	0.26	0.55	0.86	1	0.86	0.55

FUZZY MATHEMATICAL MODEL FOR DAMAGE PREDICTION

Since the concepts about damage level, earthquake intensity, ..., and the knownledge of the relation between earthquake excitation and resulted damage are not crisp, clear and distinct as mentioned above, the fuzzy reasoning principle is more suitable for us to organize information relating to damage prediction cited in documents. The proposed model for damage prediction contains k-component "IF ... THEN ..." statements and looks as:

$$
\begin{aligned}
&\text{IF}((x \text{ IS } \underset{\sim}{G}^{\text{VI}}),(y \text{ IS } \underset{\sim}{S}^{1})) \text{ THEN } (z \text{ IS } \underset{\sim}{D}^{\text{VI},1}) \\
&\text{ALSO IF}((x \text{ IS } \underset{\sim}{G}^{\text{VI}}),(y \text{ IS } \underset{\sim}{S}^{2})) \text{ THEN } (z \text{ IS } \underset{\sim}{D}^{\text{VI},2}) \\
&\text{ALSO IF}((x \text{ IS } \underset{\sim}{G}^{\text{VI}}),(y \text{ IS } \underset{\sim}{S}^{3})) \text{ THEN } (z \text{ IS } \underset{\sim}{D}^{\text{VI},3}) \\
&\cdots\cdots\cdots \\
&\text{ALSO IF}((x \text{ IS } \underset{\sim}{G}^{\text{X}}),(y \text{ IS } \underset{\sim}{S}^{1})) \text{ THEN } (z \text{ IS } \underset{\sim}{D}^{\text{X},1}) \\
&\text{ALSO IF}((x \text{ IS } \underset{\sim}{G}^{\text{X}}),(y \text{ IS } \underset{\sim}{S}^{2})) \text{ THEN } (z \text{ IS } \underset{\sim}{D}^{\text{X},2}) \\
&\text{ALSO IF}((x \text{ IS } \underset{\sim}{G}^{\text{X}}),(y \text{ IS } \underset{\sim}{S}^{3})) \text{ THEN } (z \text{ IS } \underset{\sim}{D}^{\text{X},3})
\end{aligned}
\qquad (5)
$$

The variables x,y in the antecedants are the intensity of ground motion and the type of site soil, while z in the consquences is the damage level. Since the earthquake disaster survey is always depicted through certain earthquake intensity of soil type in a specified zone, we would rather write every component of the model describing a certain earthquake intensity degree with a certain soil type. Here $\underset{\sim}{G}\text{VI}$, $\underset{\sim}{G}\text{VII}$, ... represent the fuzzy sets of a—the peak value of horizontal component of ground acceleration PGA from VI to X degree of intensity. And $\underset{\sim}{S}^{i} \triangleq 1/s_i + 0/s_j$, $i=1,2,3$, $j \neq i$, s_i denotes the ith soil type. Fuzzy sets $\underset{\sim}{G}\text{VI}, \ldots, \underset{\sim}{G}\text{X}$ have been suggested after analysis of 114 strong motion records, and the nature that variables x and y are basically independent was discussed in detail in Ref. 5. $\underset{\sim}{D}^{\text{VI},i}$ ($\underset{\sim}{D}^{\text{VII},i}$, ...) denotes the damage degrees when the ground motion is $\underset{\sim}{G}\text{VI}$, ($\underset{\sim}{G}\text{VII}$, ...) and the soil type is i. How to get $\underset{\sim}{D}^{\text{VI},i}$, $\underset{\sim}{D}^{\text{VII},i}$, ... will be described later.

Using the implication rule based on Lukasiewicz logic and suggested by Zadeh:

$$\mu_{\underset{\sim}{R}^k}(u,\ z) = (1 - \mu_{\underset{\sim}{M}_k}(u) + \mu_{\underset{\sim}{D}_k}(z)) \wedge 1 \qquad (6)$$

we can get the fuzzy relation $\underset{\sim}{R}^k$ between $u \triangleq (a, s)$ and $z \triangleq$ ind for every component, $\underset{\sim}{M}_k \in F(U)$, $u \in U$. The membership function of $\underset{\sim}{M}$ is defined as bollows

$$\mu_{\underset{\sim}{M}}(u) \triangleq \wedge(\mu_{\underset{\sim}{G}}(a),\ \mu_{\underset{\sim}{S}}(s)) \qquad (7)$$

Then the total relation is

$$\underset{\sim}{R} \triangleq \bigcap_{k=1}^{n} \underset{\sim}{R}^k, \quad n = 3 \times 5 \text{ for VI}, \ldots, \text{X} \qquad (8)$$

The resulted $\underset{\sim}{R}$ may be written in a table, which is too large in size to present here.

As foregoing, $\underset{\sim}{D}$VI$\cdot^i$, ..., $\underset{\sim}{D}$X$\cdot^i$ represent damage degrees when ground excitation is $\underset{\sim}{G}$VI, ..., $\underset{\sim}{G}$X and soil condition is type i. The relation between damage indexes (Fig. 1) under different kinds of soil conditions may be considered as a mapping

$$f:\ \text{ind} \rightarrow \text{ind} \qquad (9)$$

Then we can calculate $\underset{\sim}{D}$VI$\cdot^i$, ..., $\underset{\sim}{D}$X$\cdot^i$, $i = 1,\ 3$ from $\underset{\sim}{D}$VI, ..., $\underset{\sim}{D}$X in Table 1,which represent damage degrees on soil type II, in accordance with extention principle

$$\underset{\sim}{D}^i \triangleq \int \mu_{\underset{\sim}{D}}(\text{ind})/f(\text{ind}) \qquad (10)$$

After the fuzzy sets in consquences of our model eq. (5) are detemined, the fuzzy relation $\underset{\sim}{R}$ may be established through eqs. (6),(7),(8).

Usually seismic hazard analysis and microzoning of a city provide the value of PGA, various kinds of response spectrum, soil and geologic enviroment of every district since the PGA value with a certain exceedance probability estimated for a district is a deteministic one a' rather than a fuzzy set, we may treat it as a fuzzy set $\underset{\sim}{G}'$ having membership function

$$\begin{cases} \mu_{\underset{\sim}{G}'}(a') = 1 \\ \mu_{\underset{\sim}{G}'}(a) = 0 \quad a \neq a' \end{cases} \qquad (11)$$

The fuzzy set $\underset{\sim}{S}'$ may be obtained by means of synthetic evaluation of multiple factors. For the sake of simplicity, only three factors are used, i.e., general description of site soil, the depth of soil layer over bedrock and the area of dynamic amplification factors $\beta(T) = S_A(T)/PGA$ among the periods $T = 0.4 \sim 2.0$ sec., $S_A(T)$ is the acceleration responce spectrum.

According to the nature of discussed problem,we prefer the following formula as inference rule:

$$\underset{\sim}{C}' \triangleq \underset{\sim}{M}' * \underset{\sim}{R}' \qquad (12)$$

$\underset{\sim}{M}'$ is obtained by given values of $\underset{\sim}{G}'$ and $\underset{\sim}{S}'$ of a specified district by eq. (7). $\underset{\sim}{C}'$ is the predicted damage level or damage potential of that district. The rule * is defined as

$$\mu_{\underset{\sim}{C}'}(\text{ind}) = \bigvee_{i=1}^{m} \left(\sum_{j=1}^{3} \mu_{\underset{\sim}{M}'}(a_i, s_j) \times \mu_{\underset{\sim}{R}}(u_{ij}, \text{ind})\right) \qquad (13)$$

m is the number of"a"within the universe of discorse of PGA written in discrete manner.

Assuming $\underset{\sim}{G}$ is $\underset{\sim}{G}$VI, ..., $\underset{\sim}{G}$X respectively, but $\underset{\sim}{S}$ is $\underset{\sim}{S}'$,we can calculate($\underset{\sim}{D}$VI)',..., ($\underset{\sim}{D}$X)' by eq.(12),(13), and trace them together with $\underset{\sim}{C}'$ in Fig. 2(a). If we know the damage degree $(\underset{\sim}{D}_t\,\mathrm{VII}), \ldots, (\underset{\sim}{D}_t\,\mathrm{X})$ of a specified type of building, then it is easy to calculate $(\underset{\sim}{D}_t\,\mathrm{VII})', \ldots, (\underset{\sim}{D}_t\,\mathrm{X})'$ by the same way. Then we may obtain $(\underset{\sim}{D}_t)'$ of this type of building in that district through calculating

$$a = a_1 + (a_2 - a_1)\frac{l_1}{l_1 + l_2}$$
$$b = b_1 + (b_2 - b_1)\frac{l_1}{l_1 + l_2} \tag{14}$$

for a series of values of $\alpha \in [0.1]$, where $[a_1, b_1], [a_2, b_2]$ are α-cuts of adjacent$(\underset{\sim}{D}_t)'$, see Fig. 2(b).

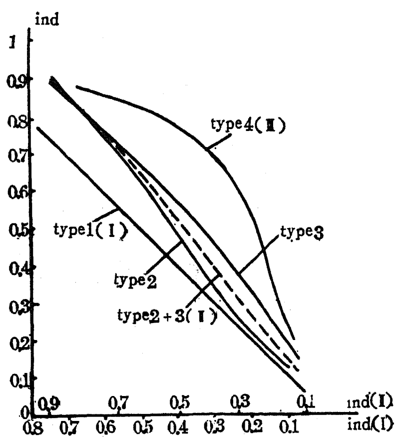

Fig. 1

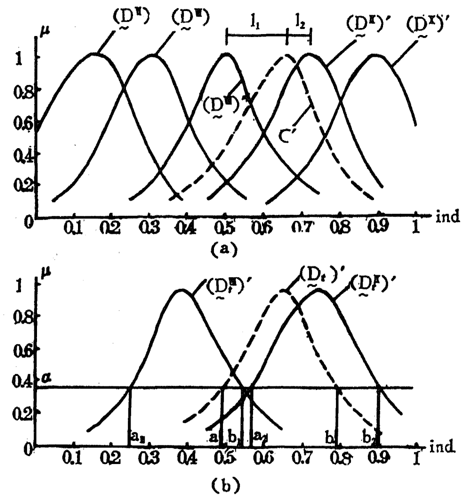

Fig. 2

Fuzzy sets $(\underset{\sim}{D}_t\,\mathrm{VII})', \ldots, (\underset{\sim}{D}_t\,\mathrm{X})'$ for various types of building may be given through two ways: (1) By means of analysis of historical meterials of damage to various types of building, the damage degrees for masonry building, R/c Frame, and so on, have been suggested in Ref. 5. (2) By way of synthetic evaluation, one example will be given in following paragraph.

PREDICTING DAMAGE TO SINGLE-STOREY BUILDING

A method using fuzzy synthetic evaluation has been developed for forecasting the damage to single-storey buildings (Ref. 6). Since the method takes the intensity as earthquake effect, we can only get $(\underset{\sim}{D}_t\,\mathrm{VI})', (\underset{\sim}{D}_t\,\mathrm{VII})', \ldots, (\underset{\sim}{D}_t\,\mathrm{X})'$ by this model. And $(\underset{\sim}{D}_t)'$ should be estimated by eq. (14).

Following five factors are selected as main influence elements. They are

$x_1 = (BLH/I) \times 1000$ to indicate the feature of bearing structure, where B-span, L-bay, H-height to the truss, I-inertia moment of horizontal section of masonry wall.

$x_2 = L/B$ to indicate the feature of horizontal plane, where L-the total length of the building.

x_3 to indicate the feature of roof system, if the bracing is very strong, it takes a value 0.1, the weaker the horizontal stiffness, the greater the value of x_3.

x_4 to indicate the feature of wall, espeacially the gable, $x_4 \in [0, 1]$, the worse quality, the greater the value of x_4.

x_5 to indicate the feature of site soil, $x_5 \in [0, 1]$, the more unfavourable to earthquake resistance, the greater the value of x_5.

The samples are analized and the membership function, which denotes how the factor is compatible to that category of damage, is given. For example, in the case of intensity Ⅷ, and $\underset{\sim}{A}$ the fuzzy set of samples, the damage level of which is above "severely damaged", the membership functions are set up through analysis of such buildings experienced Daofu earthquake on Jan. 24, 1981. They are as follows:

$$\mu_{\underset{\sim}{A}}(x_1) = \begin{cases} 1, & x_1 \geqslant 0.5 \\ x_1/0.6 & 0.2 < x_1 < 0.5 \\ 0 & x_1 \leqslant 0.2 \end{cases}$$

$$\mu_{\underset{\sim}{A}}(x_2) = \begin{cases} 1, & x_2 \geqslant 4.5 \\ 0.5 \times \left(\frac{x_2}{1.5} - 1\right), & 1.5 < x_2 < 4.5 \\ 0, & x_2 \leqslant 1.5 \end{cases}$$

$$\mu_{\underset{\sim}{A}}(x_3) = \begin{cases} 1, & x_3 \geqslant 0.65 \\ 1 + (x_3 - 0.65), & 0.1 < x_3 < 0.65 \\ 0, & x_3 \leqslant 0.1 \end{cases} \quad (15)$$

$$\mu_{\underset{\sim}{A}}(x_4) = \begin{cases} 1, & x_4 \geqslant 0.8 \\ 2 \times (x_4 - 0.3) & 0.3 < x_4 < 0.8 \\ 0, & x_4 \leqslant 0.8 \end{cases}$$

$$\mu_{\underset{\sim}{A}}(x_5) = \begin{cases} 1, & x_5 \geqslant 0.4 \\ 3 \times (x_5 - 0.1) & 0.2 < x_5 < 0.4 \\ 0, & x_5 \leqslant 0.2 \end{cases}$$

Then we can form a fuzzy vector Λ for a sample B_0, the damage to which will be estimated,

$$\Lambda = \begin{pmatrix} \mu_{\underset{\sim}{A}}(x_1) \\ \vdots \\ \mu_{\underset{\sim}{A}}(x_5) \end{pmatrix} \quad (16)$$

Suppose the weight function is $W = \{0.13, 0.22, 0.2, 0.16, 0.19\}$, then we can calculate

$$\mu_{\underset{\sim}{A}}(B_0) = W \cdot \Lambda \quad (17)$$

According to direct classification principle, we may classify B_0 into category "severely damaged or collapse", if $\mu_{\underset{\sim}{A}}(B_0)$ is greater than 0.5. The method that is capable of predicting every damage grade is developed. So we can draw curves$(\underset{\sim}{D}, \text{Ⅶ})'$, ..., $(\underset{\sim}{D}, \text{Ⅹ})'$ in Fig. 2(b) by this method.

CONCLUSION

The approximate reasoning k-component "IF ... THEN ..." model of damage forecast has apparent advantage for processing information about earthquake effect and disaster, which always include lots of fuzzy concepts, such as earthquake intensity, damage grade, etc. So Using fuzzy logic and approximate reasoning principle to develop the damage-predicting model is reasonable and feasible. Since the antecedents of every component contain PGA and acceleration response spectrum, soil depth and soil feature(these factors are used through synthetic evaluation to classify the soil type), the model makes full use of information employed by earthquake hazard analysis and microzoning. The method developed for damage prediction of single-storey building is only an example to express the fact that fuzzy math is an available tool for such a purpose. Since the application of fuzzy set theory to damage prediction is tentative in nature, there still exist lots of problems which need futher effort. To select more suitable implication rule is first above all. Other influence factors, such as soil liquefaction, landslide, and so on, should be considered in the model.

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