A STOCHASTIC MODEL OF EARTHQUAKE OCCURRENCE

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SUMMARY

The gamma distribution is proposed to describe the distribution of the seismic moments of earthquakes and earthquake sequences. Our results indicate that the major part of small earthquakes are dependent shocks, i.e. aftershocks. Even among earthquakes of intermediate magnitudes the number of dependent events may exceed that of independent events. The stochastic multidimensional model is used to extrapolate a catalog of earthquakes in time. This extrapolation makes unnecessary many unjustified assumptions that are usually made in the estimation of seismic risk.

INTRODUCTION

Reliable earthquake catalogs are too short to permit construction of estimates of return times of large earthquakes without the addition of one or more, usually unjustified assumptions. Typical among these assumptions are those of Poissonian independent processes plus the extrapolation of frequency-magnitude formulas. We present an alternative to the above which makes use of all documented statistical data about earthquakes, including their aftershocks and foreshocks. We find that a remarkably different set of conclusions can be drawn regarding predicted risk, even though the same data base may be used.

Several earthquake catalogs have been analyzed as a multidimensional point process to study statistical interrelations among earthquakes (Ref. 1). Under a suitable magnitude-time-distance scaling, the patterns of interrelationships among earthquakes of different magnitude ranges are found to be almost identical. The rate of occurrence of dependent shocks increases approximately as 1/t as the origin time of the main shock is approached, allowing us to model every earthquake as a multi-shock event. After the non-uniformity of the depth distribution is taken into account, the spatial moment functions indicate the absence of any preferred scale of distance or configuration size. While the results for two- and three-point distributions are not inconsistent with the usual model that an earthquake fault is an isolated plane, the results for the four-point moments rule out the likelihood that faulting is plane (Refs 2,3).

From the above results, we have developed a kinematic stochastic model of the earthquake source (Refs 4,5). An earthquake fault is assumed to consist of a system of infinitesimal shear dislocations. After an initial microevent, subsequent ruptures occur according to a critical branching process, with time and space coordinates and focal mechanism controlled by simple scale-invariant distributions. With only a small number of independent parameters in a Monte Carlo procedure, we have produced sequences that simulate all of the known

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statistical properties of shallow seismicity. Thus, given an initial configuration of earthquakes and using the above model, we can extrapolate the seismic process in time and therefore estimate quantitatively the future seismic risk in the area.

In this paper we use the above model to extrapolate in time an earthquake history considered as a multidimensional stochastic point process. As an example we take a catalog of fault-plane solutions from the of East Bay Region (east of San Francisco), which we supplement by earthquake information taken from the catalog for Central California (called the USGS catalog here). Because our model has been described extensively in the above-mentioned publications, here we concentrate our discussion on topics which are of a more immediate interest to engineering seismology.

SEISMIC MOMENT DISTRIBUTION

In this paper $\underline{M}$ denotes seismic moment, and $\underline{m}$ local magnitude. We refrain from using the differential version of the well-known Gutenberg-Richter relation for magnitudes:

$$\phi(M) = \beta M_c^{\beta} \cdot M^{-1-\beta} \qquad \text{for } M_c \leq M < \infty. \tag{1}$$

Here M_c is a lower detection threshold seismic moment. We write $\beta = b/c$, where $\underline{b}$ is the standard coefficient in the magnitude-frequency relation and $\underline{c}$ is an empirical coefficient relating magnitude and seismic moment: $\lg(M) = cm + d$. We use $\lg(.)$ to mean a base 10 logarithm and $\ln(.)$ a natural logarithm. In this paper we use $c=1.43$ and $d=16.2$ (Ref. 6), which were derived for Southern California, a region which adjoins Central California.

The b-value cannot be uniform over the entire range of seismic moments or magnitudes if the amount of seismic energy released is finite. Therefore we introduce an upper magnitude cutoff. Lomnitz-Adler and Lomnitz (Ref. 7) argue persuasively that the upper cutoff of a distribution such as (1) cannot be abrupt since it discriminates between a finite number of events just below the cutoff magnitude and zero events just above it.

We use a distribution which has the property of a finite moment flux, and at the same time reflects our ignorance of the size distribution of large earthquakes. We use the gamma distribution (Ref. 8):

$$\phi(M) = C^{-1} \cdot M^{-1-\beta} \cdot \exp(-aM) \qquad \text{for } M_c \leq M < \infty, \tag{2}$$

where $\underline{a}$ is a second parameter of the distribution that controls the distribution in the upper ranges of $\underline{M}$, $\underline{C}$ is a normalizing coefficient, $C = a^{\beta} \cdot \Gamma(-\beta, aM_c)$, and Γ is the incomplete gamma function (Ref. 9). The distribution (2) is consistent with the principle of maximum entropy (Refs 10,11) for a distribution of seismic moments which is scale-invariant, i.e. obeys a power-law type distribution. To avoid the 'infrared catastrophe' of an infinite number of events, we truncate the distribution at M_c, just as in equation (1). As we see below, the parameter $\underline{a}$ can be equated to M_x^{-1} with dimensions of energy^{-1}, where M_x is a measure of the maximum moment.

To estimate the parameters from experimental data we apply a maximum likelihood procedure. We introduce the logarithm of the likelihood function, ℓ. Numerical exploration of the $\ell(\beta, M_x)$ space gives an extremum at $\beta = 0.58 \pm 0.01$. Although the value of β is well defined by the data, the method does not provide any useful information concerning the value of $\underline{a}$. At the above value of β, the the likelihood function is flat above a presumed maximum magnitude of about 6.5. Thus any value of m_x greater than 6.5 would fit our data. The largest earthquake in the USGS catalog has a magnitude $m = 5.2$, so we could not have expected a different result. We suspect however, that even if we were to use more extensive catalogs, the value of $\underline{a}$ would still be difficult to obtain by this method.

We use geological information to determine the upper limits of the seismic moment rate, M_x, in the region (Ref. 12). We write

$$\dot{M} = \mu L H s, \tag{3}$$

where μ is the shear modulus, $\underline{L}$ the length of the fault, $\underline{s}$ is the slip rate, $\underline{H}$ the effective depth of the fault. For that part of Central California which is covered by the USGS network, we take $\mu = 3\times10^{11}$ dyne/cm², $L = 378$ km, $s = 5.5$ cm/y (Ref. 13) and $H = 8$ km (Ref. 2). The calculation gives $\dot{M} = 5\times10^{25}$ dyne-cm/y. We integrate (2) to obtain an average seismic moment $\overline{M}_c$ for lower cutoff moment M_c. The moment rate is then $\dot{M}_c = \alpha_c \cdot \overline{M}_c$, where α_c is the rate of occurrence of earthquakes with seismic moment greater than M_c. For the distribution (2) we have

$$\dot{M} = (\alpha_0 \beta / a) \cdot \exp(a M_0) \Gamma(1-\beta,\ a M_0), \tag{4}$$

where M_0 and α_0 are a reference moment and its associated rate of occurrence, and $1/a \gg M_0$. We compare (4) with a similar quantity computed for the distribution (1) (cf. Ref. 12)

$$\dot{M}_0 = \alpha_0\ M_x^{1-\beta}\ M_0^{\beta}\ \frac{\beta}{1-\beta}\ . \tag{5}$$

We can now define M_x^{eff}, the value of the upper bound cutoff M_x, which yields the same value of the seismic rate for the Gutenberg-Richter distribution (1) as for the gamma distribution (2):

$$M_x^{eff} = a^{-1}\left[\overline{\Gamma}(2-\beta)\right]^{1/(1-\beta)}, \tag{6}$$

where $\overline{\Gamma}$ is the ordinary gamma function (Ref. 9).

BRANCHING MODEL OF EARTHQUAKE OCCURRENCE

We have shown (Ref. 4,5) that the standard methods of assigning a given earthquake into an aftershock or foreshock subsequence, which is the basis of any study of earthquake size distribution, lacks an appropriate rational basis. Indeed, this assignment depends strongly on properties of the seismographic network such as amplitude detection threshold and frequency response of the instruments. We consider as an alternative the entire earthquake sequence, including foreshocks, main shock(s) and numerous aftershocks. In this case, the usual definition of the seismic moment, requires consideration of all causally connected shocks in a sequence; otherwise one has to decide where to draw a boundary where one earthquake ends

and another begins. Some authors try to make this separation: if aftershocks are removed from earthquake catalogs, presumably only main shocks remain. This removal is usually done in an ad-hoc or purely intuitive manner. We formalize the rule that identifies dependent shocks.

We derive the distribution of total seismic moment of an earthquake sequence, instead of the more usual distribution of individual events. This distribution was first considered by Vere-Jones (Ref. 14) who showed that a critical branching process yields a value of $\beta = 1/2$ for the distribution (1). He also demonstrated that a slightly subcritical process for certain choice of distribution of sizes of branching cracks (geometrical distribution) produces a distribution of the total size of a crack system which is equivalent to the gamma distribution, discussed above. We have shown (Ref. 15) that the geometrical distribution is a natural consequence of some rather general assumptions regarding the development of the seismic process.

We use the gamma distribution (2) for the total seismic moment of an earthquake sequence and set the value of the parameter β equal to 1/2. Below we indicate experimental evidence to justify this assumption. If $\beta = 1/2$, the incomplete gamma function becomes the error-function (Ref. 9) and several of the formulas above simplify. The moment rate function of (5) becomes

$$\dot{M}_0 = \frac{\sqrt{\pi}}{2} \lambda_0 \, (M_0/a)^{1/2}, \tag{7}$$

where λ_0 is the number of earthquake <u>sequences</u> with $M \geq M_0$. The 'maximum seismic moment' is

$$M_x^{eff} = \pi/4a. \tag{8}$$

If we compare (3) and (7) we can estimate λ_0. We choose three possible values of M_x: 1) $\lg M_x = 28.0$ for the San Francisco earthquake of 1906 (Ref. 16), 2) $\lg M_x = 28.4$, obtained by equating the value of the seismic moment rate from (5) with that of equation (3), 3) $\lg M_x = 29.25$ which is close to the upper limit of possible earthquakes in the area. Values of <u>a</u> can be computed using (6) or (8).

Table 1.
Values of the rate of earthquake sequences, λ.

Ref. mag. m_0	Meas. rate α	Calc. rate α	lg(a)		
			-28.1	-28.5	-29.35
1.5	1051.4	1051.4	335.3 (32%)	214.7 (20%)	79.5 (7.6%)
5.0	0.285	1.28	1.06 (83%)	0.68 (53%)	0.25 (20%)
7.0	-	0.028	0.039(141%)	0.0247(89%)	0.009(34%)

In Table 1 we list the results of calculations of λ for the above choices of <u>a</u> and $\beta = 0.5$. The column 'measured rate' shows the yearly numbers of earthquakes as measured from the magnitude distribution, the 'calculated rate' values were computed using formula (1). In addition we indicate the ratio in

percent of λ to α_{calc}, this ratio is the percentage of earthquakes that are independent.

We see that most of the earthquakes with magnitudes $m \geq 1.5$ are not independent, i.e. they are mostly aftershocks. The earthquakes of intermediate and large magnitudes are, on the other hand, largely independent. The relatively high value of λ in the first column for $m = 7.0$ indicates that the value of $\lg(M_x^{eff}) = 28.0$ is probably too small. It should be kept in mind, a) that not all deformations which are calculated by (3) are concentrated on the San Andreas fault and b) some of these deformations could be released by aseismic creep. Thus the estimates of λ shown in Table 1 should be considered as upper bounds.

The above calculations illustrate why it is so difficult to use the results of studies of small earthquakes for the estimation of large-magnitude seismicity. Most studies simply avoid this problem by using earthquakes with magnitudes greater than 5 or 6, where only a few aftershocks can be removed by ad-hoc methods.

EXTRAPOLATION OF EARTHQUAKE CATALOGS TO FUTURE TIME

We have analyzed the space-time-magnitude interrelations between earthquakes in the USGS catalog 1971-1977. One goal was to establish a lower limit for the number of dependent shocks in this catalog. A second was to find the values of the parameters which can be used as input for extrapolation of present-day seismicity as a function of time.

In a catalog of only seven years duration, more than half of the earthquakes are dependent shocks. These results are in agreement with our previous observations (Ref. 17) that most of the earthquakes in the available catalogs are aftershocks or, in other words, dependent events. The coefficient β in (2) is $\beta = 0.54$; this value is is consistently lower than the β-value we found for the Poissonian model (see above). This result supports our hypothesis that $\beta = 0.5$ for earthquake sequences.

We now have all of the ingredients that are needed to extrapolate the available catalog of earthquakes to future times. For this purpose we need the full history of earthquake deformations, that is, the time-space-size parameters of the earthquake catalog plus the seismic moment tensor data. The latter data are available in a form of an incomplete fault-plane solution catalog (Ref. 18). For those events for which fault plane solutions are not determined, we have arbitrarily assigned the focal mechanism of the spatially closest earthquake which has a published solution. To conform with our model, we converted the standard fault-plane parameters to normalized quaternion quantities which determine the three-dimensional rotations of the fault focus geometry (Ref. 5).

The Eastbay catalog has 1742 earthquakes in the time interval 1970-1980; the magnitude cutoff was taken to be 1.5. To ensure uniformity of coverage, we removed all earthquake epicenters from the catalog which lie outside the area covered by the seismographic network in 1971 (Ref. 2). The positions of epicenters in the catalog are shown in Fig. 1a.

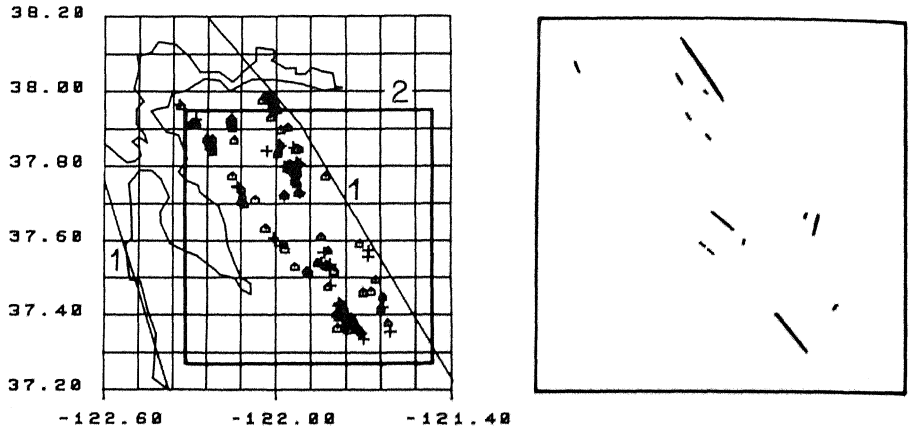

Fig. 1a, b. a) Epicenter map of Eastbay catalog.
Magnitude limits: $\Delta m = 2.5\text{-}2.9$; $+ m = 3.0\text{-}3.9$.
1 -- boundaries of 1971 network coverage. 2 -- see b).
b) Fault-plane traces for one extrapolation simulation. The area inside the box 2 in Fig. 1a is displayed.

We have taken the value of parameter ϕ_0, which controls the degree of spatial branching of earthquake faults (Ref. 5), to be 5×10^{-6}. This value corresponds to the four-point distribution of earthquake foci in the USGS catalog (Ref. 3). The value of the criticality coefficient κ, which controls the maximum size of simulated faults, can be estimated from the value of $\underline{a}$ in (2): $\lg(\kappa) = 0.5 \lg(aM_c) + \lg 2.0$. The formula can be easily deduced from Vere-Jones (Ref. 14, p. 721). In order to reduce the volume of computation we have taken M_c to have the relatively large value corresponding to magnitude $m = 5.0$.

As a first step we calculate how many dependent earthquakes, mostly aftershocks, that will be produced in the next 50 years by earthquakes in the Eastbay catalog. The interval 1970-1980 was relatively quiet; the largest earthquake only had a magnitude of 3.9. Thus it is not surprising that, even if we take the most 'favorable' values of the parameters, the number of aftershocks with $m \geq 5.0$ is only about 0.5. We simulate the origin time of these future dependent shocks and then calculate the number of independent sequences in the same way we derive Table 1. The length $\underline{L}$ of the fault in equation (3) was taken to be 89 km for the Eastbay catalog. Independent shocks were distributed uniformly randomly in the 50 year interval; their position and focal mechanism was also taken by chance from the population of shocks in the Eastbay catalog. Both sets of initial shocks were then used as elementary events in the simulation procedure described previously (Refs 4,5).

In Fig. 1b one of the realizations of this process is shown. The lines inside the box show the intersections of the fault planes of the simulated earthquakes with a horizontal plane at the depth of 6 km. The origin time as well as the slip vector of these events were also simulated. Thus, in

principle, we have all of the necessary input data to calculate ground motion of a structure of engineering interest assuming, of course, that we also know the effects of seismic wave propagation.

The estimates of seismic risk that were obtained differ markedly from standard estimates that employ Poissonian models of earthquake occurrence. The mean values of the predicted seismicity may be similar in both cases, in consideration of the fact that in the latter method one has to remove aftershocks by some subjective procedure. The estimates of the uncertainties, on the other hand, are much larger in our model. In the Poissonian model one should expect the ratio of the variance to the mean to be about unity. In our case this ratio, calculated for five year intervals, is as large as 300 and even higher. These uncertainties seem to increase as the prediction time increases. This is a direct consequence of the modeling of the seismicity by a critical branching process. Although the first moment of the process should be constant, the variance increases with time.

We believe that this feature of our model better reflects the real errors of seismic risk estimates. Our model not only reproduces the long-term behavior of the seismicity, but we can also use the model to develop a stochastic model of an individual earthquake source-time-moment function, thus making it suitable for use as a 'design earthquake' in engineering seismology investigations. An elaboration of the ideas in this paper will be presented elsewhere.

CONCLUSIONS

1. The gamma distribution is proposed to describe the distribution of the seismic moments of earthquakes and earthquake sequences. The distribution is both a result of the maximum entropy reasoning and the natural outcome of our model. The model predicts the value of 0.5 for the β parameter of the distribution. There is evidence that the distribution of the total seismic moment of earthquake sequences satisfies this prediction.

2. Our results indicate that the majority of small earthquakes are dependent shocks, i.e. aftershocks. Even among earthquakes of intermediate magnitudes the number of dependent events may exceed that of independent events. We have developed a maximum likelihood procedure to calculate the probability that an earthquake is a dependent event.

3. The stochastic multidimensional model we have described can be used to extrapolate a catalog of earthquakes in time. This extrapolation makes unnecessary many unjustified assumptions that are usually made in the estimation of seismic risk.

4. The simulation of future seismicity provides a more direct input to the tasks of engineering seismology. Estimates of possible errors can be made more reliably and systematically.

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