ON THE USE OF RENEWAL PROCESSES IN SEISMIC HAZARD ANALYSIS

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SUMMARY

Renewal processes are used in this paper for the interpretation of earthquake occurrences. The probability density function of interoccurrence times is a mixture distribution. The first component accounts essentially for aftershocks, the second component accounts essentially for longer recurrence times and is particularly useful when the occurrence process is not a memory-less process. Some consequences of the use of the model in earthquake prediction and in damage evaluation are discussed in the paper.

INTRODUCTION

Many authors agree on the opinion that Poisson process is not always suitable for the interpretation of earthquake occurrences. It is true that a more complicated model could lead to serious misleadings due to the paucity of data. On the other hand, there are problems in which the non-stationarity and the memory of the process, if they exist, are of the utmost importance. This is the case, for instance, of special structures with finite life and of earthquake prediction.

It is impossible to mention here all the models that have been proposed as improvements of the Poisson model. Main references are Ref. 1 to 8.

The model that is presented in this paper derives primarily from the remark that renewal processes (r.p.) lead to the simplest model that includes the memory of past events. A r.p. is a semi-Markov process in that it has memory only of the last event. A r.p. is defined by initial conditions (time t_o measured backward to the last event) and by the probability density function (pdf) $f_\tau(t)$ of interoccurrence time (i.t.) τ.

Let us now remember some properties of r.p. that will be utilized in the next sections.

The r.p. of larger events is defined when their conditional probability π (given an earthquake) is known. Precisely, the i.t. τ_y of larger events has a pdf f_y such that

$$f_y^*(\gamma) = \frac{f_\tau^*(\gamma)}{1-(1-\pi)\, f_\tau^*(\gamma)} \, , \qquad (1)$$

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where * indicates Laplace transform (L.t.) (Ref. 9, 10). Taylor expansion of (1) furnishes the moments of f_y. In particular the mean value μ_y is simply:

$$\mu_y = \frac{\mu}{\pi} \ . \tag{2}$$

where μ is the mean value of i.t. τ.

The renewal density $h(t, t_o)$ is defined by:

$$h(t, t_o)dt = P\{\text{(at least) one event in } [t,t+dt)\} \ .$$

$h(t, t_o)$ is related to i.t. as follows (Ref. 10, 11):

$$h^*(\gamma, t_o) = \frac{f^*_{w_1}(\gamma, t_o)}{1-f^*_\tau(\gamma)} \ , \tag{3}$$

where w_1 is the waiting time to the first event (or recurrence time) when t_o elapsed from the past one.

The dependence on t_o vanishes in equilibrium processes. This is the case when the time origin is a sampling point chosen at random over a very long time interval (Ref. 11, 15). In this case

$$f_{w_1}(t) = \frac{1-F_\tau(t)}{\mu} \tag{4}$$

and the expected value μ of i.t. τ coincides with the return period as usually defined.

Assume now that, after an earthquake, the next one can occur

- either, with probability p, rather soon because energy is not yet completely released (aftershock)
- or, with probability 1-p, rather late because energy is already completely released and it has to be accumulated again ("long term").

Call ξ the i.t. of aftershocks and T the i.t. of long term shocks. The above-mentioned assumption can be written(1)

$$f_\tau(t) = p\, f_\xi(t)+(1-p)\, f_T(t) \ . \tag{5}$$

The choice of f_ξ and f_T can be suggested by the wanted characteristics of their hazard rate (h.r.). The h.r. of f_ξ should be suddenly rather large and then it should decrease to zero. If long-term process is with memory, the h.r. of f_T should be increasing, possibly with a finite asymptotic value.

Two pdf with the wanted characteristics, and simple enough for handling, are represented by a log-normal distribution (LN) and a Γ distribution with shape parameter a>1. If only integer values of a are considered for simplicity (Erlang distribution), a = 3 is the minimum value for which the renewal process defined by f_T has a renewal density $h(t, t_o)$ oscillating around $1/\mu_\Gamma$

(1) Kameda and Ozaki already proposed to distinguish different behaviours in a seismic r.p.. They used a double Poisson process (Ref. 8).

(μ_Γ is the mean value of the Γ distribution). With a = 3 the oscillations will die out more rapidly than with a>3.

In the next sections a few applications of the model to Italian sites are presented. Moreover, the consequences of the application of the model to earthquake prediction and to damage evaluation are discussed.

CASE STUDIES

We refer first to seismic hazard at a site: the city of Messina. Intensities MCS from 1670 to 1970 are used with the following attenuation law of epicentral intensity I_o (d = epicentral distance in km):

$$I = I_o \quad \text{if} \quad d \leq 23 \; ; \quad I = I_o + 7.32 - 2.33 \ln d \quad \text{if} \quad d > 23 \; ,$$

which gives a rather weak attenuation.

Only events with local intensity $I \geq 6$ are considered, with no more than one event per day.

The whole set of i.t. has been fitted with maximum likelihood method using an exponential distribution, a Γ_3 distribution and a mixture distribution lognormal-Γ_3. The same has been done after the exclusion of events with i.t.<Δt (Δt = 0.3, 0.5, 1, 2 years). The whole set corresponds to Δt = 0.003 years (one day).

Table 1 shows the index S_n^*, based on the differences between theoretical and observed cumulative distributions (Ref. 12). In each column the values of S_n^* are divided by the one corresponding to the mixture distribution. More statistical details are contained in Ref. 13.

Table 1 - Messina - $I \geq 6$ - Values of S_n^*

$f_\tau(t)$ \ Δt	.003	.3	.5	1	2
exp	8.4	2.2	1.4	1.0	1.4
Γ_3	10.2	4.6	3.6	2.0	1.4
LN-Γ_3	1	1	1	1	1

The mixture distribution shows generally better fits than the others. When Δt = 2, exponential and Γ_3 distributions are equivalent.

The results concerning the LN-Γ_3 distribution are shown in Table 2. The mean value of i.t. is plotted in fig. 1 versus Δt (continuous line).

The values of T are conventional (they depend on the definition of aftershocks). If aftershocks are defined by means of a fixed time window Δt, the mean value $\overline{T}$ coincides with μ and largely depends on the value assumed for Δt. If aftershocks are interpreted by the lognormal component of the mixture distribution, then the mean μ_Γ of the Γ_3 component is an estimate of $\overline{T}$. Table 2 shows that μ_Γ is fairly independent of Δt.

Obviously a value of Δt for which $\mu = \mu_\Gamma$ (Δt = 0.003) does exist. However it would be greater than 2 years and shows little meaning.

It is interesting that using the mixture distribution on the whole set of data we get reliable results for T distribution without introducing an (arbitrary) time window Δt.

Table 2 - Messina - $I \gtrsim 6$ - LN-Γ_3 distribution

	Δt	0.003	0.3	0.5	1	2
number of i.t.	n	164	66	58	49	37
mean i.t.	μ	1.64	4.06	4.62	5.47	7.25
	p	0.83	0.51	0.44	0.32	0.17
	μ_{LN}	0.34	0.77	0.95	1.31	2.81
	μ_Γ	7.71	7.47	7.48	7.44	8.18
scale parameter of Γ_3	ρ_Γ	0.389	0.402	0.401	0.403	0.367

The results obtained with events $I \gtrsim 8$ are shown in Table 3.

Table 3 - Messina - $I \gtrsim 8$ - LN-Γ_3 distribution

Δt		0.003	0.3	0.5	1	2
n		15	10	10	9	8
μ		16.0	23.9	23.9	26.5	30.0
p		0.76	0.59	0.59	0.55	0.37
μ_Γ		54.6	54.3	54.3	54.1	45.8
s_n^*	exp	3.4	2.1	2.1	1.8	1.7
	Γ_3	4.3	3.1	3.1	2.7	2.4
	LN-Γ_3	1	1	1	1	1

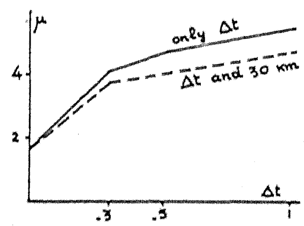

Fig. 1

The general trend is similar to the one shown in Table 2, even if the paucity of data makes the results less reliable. In particular, to get $\mu = \mu_\Gamma$ (Δt = 0.003) it would be necessary to use a value of Δt which is out of proportion with any aftershock definition. With regard to the estimation of $\overline{T}$, note that: a) with usual values of Δt we obtain $\mu = 24 \div 30$; b) a classic application of Gumbel theory gives $\mu_G = 70$; c) from the LN-Γ_3 distribution we obtain $\mu_\Gamma = 55$. Eeven if conventional, μ_Γ has the advantage of being independent of Δt.

However μ_Γ and μ take account only of events $I \gtrsim 8$, which are few. It seems better to rely on μ_Γ ($I \gtrsim 6$) and to derive from it the value of μ_Γ ($I \gtrsim 8$), using eq. (2).

An estimate of π can be obtained from the ratio between the number of events $N(I \gtrsim 8)$ and the number $N(I \gtrsim 6)$, which is plotted in fig. 2 (continuous line) versus Δt. In order to read π on the diagram we can use the value Δt_ξ for which $F_\xi(t) = 0.9$. We obtain $\Delta t_\xi = 0.45$, $\pi = 0.185$, and hence $\overline{T}(I \gtrsim 8) = 7.71/0.185 = 42$ years, which looks more reliable than $\mu_\Gamma = 55$. In any case the definition of $\overline{T}$ remains a conventional one. However it does not depend strongly on the particular distribution (LN) assumed for the

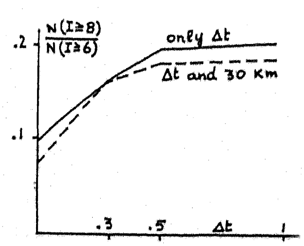

fig. 2

interpretation of aftershocks. In Table 4 a comparison is shown between LN-Γ_3 and two other mixtures, one (w_1-Γ_3) with the first component represented by a Weibull distribution with shape parameter a = 1/2, the other (w_2-Γ_3) with the shape parameter free, under the condition a<1 in order to make h.r. decreas ing.

Table 4 - Messina - Δt = 0.003

	$I \geq 6$			$I \geq 8$		
	LN-Γ_3	w_1-Γ_3	w_2-Γ_3	LN-Γ_3	w_1-Γ_3	w_2-Γ_3
p	0.83	0.83	0.80	0.76	0.73	0.74
μ_Γ	7.71	8.02	7.34	54.6	54.4	54.6

The results discussed till now do not change substantially if a classic space-time window is used for aftershocks, instead of a mere time window Δt. For example, in fig. 1 and 2 the dashed lines derive from a space (30 km)-time window. The values of μ_Γ for Δt = 0.003 are $\mu_\Gamma(I\geq6)$ = 7.72, $\mu_\Gamma(I\geq8)$=54.9.

The attenuation law that has been used is fairly "weak", so that the results could be referred to a seismogenetic zone. If a steep attenuation law is used, the numerical results obviously change substantially. However the general trend remains the same.

Analogous tests have been carried out for different Italian sites. The general trend can be summarized as follows: a) a mixture distribution shows always the advantages that we saw for Messina; b) if the site is well inside a seismogenetic zone (Irpinia, Friuli) the best fitting is obtained when the second component of the mixture has a memory (Γ_3); c) if the site is influ enced by various seismogenetic zones (Firenze), the best fitting is obtained when the second component is memory-less (exponential).

Mixtures better than LN-Γ_3 could obviously be found for particular cases with memory. However it is already justified to look at the influence of the interoccurrence model on the results regarding particular problems, by comparing exponential and Γ_3 interpretation of i.t. T.

INFLUENCE OF INTEROCCURRENCE MODEL ON PREDICTION

Consider a seismogenetic zone with $\overline{T}(I\geq6)$ = 7.5 years, π = 0.2 and hence $\overline{T}(I\geq8)$ = 37.5 years. Assume π constant for $T>\Delta t_\xi$. If t is the time elapsed from the last event $I\geq6$, and $t>\Delta t_\xi$, the "immediate risk" of an event $I\geq6$ is defined as $P(E) = P\{t<T\leq t+dt/T>t\}$ and is given by:

$$P(E) = \phi(t)\, d\, t$$

where $\phi(t)$ is the h.r. of an exponential distribution or of a Γ_3 distribution, depending on the mixture used. The immediate risk of an event $I\geq8$, i.e. $P\{(t<T\leq t+dt)\cap I\geq8/T>t\}$, is given by:

$$P(E) = \pi\, \phi(t)\, d\, t \qquad (6)$$

Suppose now that: a short-term precursor F is monitored; F is correlated only to events $I_o \geq 8$; its useful warning time is δt = 1 week (if the earthquake does not occur within 1 week, F is a false alarm). The average probability of false alarm $\overline{p}$ is derived from the number f of false alarms and the number s of successful alarms over a long period of observation: $\overline{p} = f/(f+s)$. Suppose $\overline{p} = 0.5$.

The background risk (disregarding the precursor) at t, of an event $I \geq 8$ in the next week, is approximately given by eq. (6) with dt = 1 week:

$$P(E) = \frac{0.2}{54} \phi(t) = 0.0037 \phi(t) .$$

Suppose that false alarms are due only to the fact that the precursor is sensitive also to another phenomenon, independent of the seismogenetic process and with constant frequency. In this case (Ref. 14) the probability of false alarm at t is $p = 1/|1+\alpha P(E)|$ with α = constant, and hence p depends on $\phi(t)$.

If an exponential distribution is assumed for T, $\phi(t)$ is constant, p = constant = $\overline{p}$, and the conditional probability $P(E/F) = 1-p = 0.5$, while $P(E) = 0.000494$.

If a Γ_3 distribution is assumed for T, the constant α can be determined as indicated in Ref. 14 and the conditional probability P(E/F) may be quite different from the previous one as shown in Table 5, where t is the instant at which F occurs (t is always measured from the last $I \geq 6$).

Table 5

t(years)	2	5	10	20	30	40
P(E/F)	0.157	0.330	0.431	0.489	0.510	0.522
P(E)	0.000223	0.000592	0.000910	0.00115	0.00125	0.00131

DAMAGE EVALUATION

Consider the process of all events $I \geq 6$ at a site. Given an earthquake, the mean value $\overline{C}$ of damage cost is defined when the pdf of local intensity and the vulnerability of building sample are known (Ref. 16). The expected present cost of all future damage depends on t_o and on the r.p. defined by eq. (5). Δt_ξ being small, f_ξ influences the total cost only in that it amplifies $\overline{C}$. So a weight p_{cl} can be defined according to the importance of clustering. For example:

$$p_{cl} = \exp\left(p + \frac{\Delta t_\xi}{\mu}\right) .$$

Call C the amplified cost $C = p_{cl} \overline{C}$. Then the expected present cost E(D) of all future damage (only governed by f_T) is:

$$E(D) = C \sum_1^\infty{}_i \int_o^\infty \exp(-\gamma t) \, f_{w_i}(t) \, dt = C \, \Sigma f^*_{w_i}(\gamma) = C \frac{f^*_{w_1}(\gamma,t_o)}{1-f^*_T(\gamma)} \qquad (7)$$

where $f_{w_i}(t)$ is the waiting time to i-th event and γ is the discount factor. Due to eq. (3), from eq. (7) we obtain:

$$E(D) = C\, h^{*}(\gamma,\, t_o) \tag{8}$$

If T has an exponential distribution, eq. (8) becomes:

$$E(D) = \frac{C}{\gamma\,\overline{T}}\ ; \tag{9}$$

so E(D) does not depend on t_o and corresponds to an annual cost $C/\overline{T}$.

If T has a Γ_3 distribution, eq. (8) becomes:

$$E(D) = C\,\frac{27}{(3+\gamma\overline{T})^3-27}\;\frac{t_o^2\,(3+\gamma\overline{T})^2+2t_o\overline{T}\,(3+\gamma\overline{T})+2\overline{T}^2}{9t_o^2+6t_o\overline{T}+2\overline{T}^2}\ ; \tag{10}$$

so E(D) depends on t_o. In the case of equilibrium process eq. (7), due to eq. (4), does not depend on t_o and E(D) has the same value in the two cases.

The influence of t_o on E(D) given by eq. (10) may be illustrated by means of the ratio ν

$$\nu = \frac{E(D)_{t_o}}{E(D)_{t_o=0}} = \frac{2\overline{T}^2+2t_o\overline{T}\,(3+\gamma\overline{T})+t_o^2\,(3+\gamma\overline{T})^2}{2\overline{T}^2+6t_o\overline{T}+9t_o^2}\ . \tag{11}$$

Obviously ν increases with t_o. For example if $t_o = \overline{T} = 50$ years and $\gamma = 0.07$, we obtain $\nu = 3.37$. If the process had stronger memory, for instance $a = 10$, with the same data we would obtain $\nu = 9.8$, and with $a \to \infty$ $\nu \to 33.1$ (Ref. 17).

The ratio ν does not change if only the damage due to the first event is considered.

It is possible to show that, for $t_o = 0$, the value of E(D) derived from an exponential distribution is greater than the value derived from a Γ_3 distribution. However, for t_o sufficiently large, the second value is greater than the first one. For example, with previous $\overline{T}$ and γ, this happens when $t_o > 0.48\,\overline{T}$.

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