TEMPORAL DEPENDENCE IN EARTHQUAKE OCCURRENCE

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SUMMARY

Recent geophysical studies have indicated that an earthquake recurrence interval and the size of the preceding event are positively correlated. This observation is the basis for the deterministic time-predictable recurrence model of Shimazaki and Nakata. Using the basic assumptions of the time-predictable recurrence model, we develop a stochastic model of earthquake occurrence which incorporates temporal dependence. This paper discusses the formulation of the model and the effect of including temporal dependence. Comparisons are made with other models and with observed data. Results indicate that currently used Poisson models may give unconservative estimates of the seismic hazard.

INTRODUCTION

Consideration of the temporal and spatial patterns of earthquake occurrence is an important aspect of seismic hazard analysis and has drawn much attention. Many of the techniques currently used for hazard evaluation rest on Poisson assumptions for earthquake occurrences (Ref. 1). Although a Poisson model is an adequate description for some available occurrence data, it is not consistent with any geophysical description of the earthquake generating process. A model which is based on physical concepts and which leads to more accurate estimates of earthquake occurrence probabilities is desirable in seismic hazard analysis.

Recent developments in seismology and geophysics have lead to two alternative representations for earthquake recurrence patterns. These include the "time-predictable" and "slip-predictable" models (Refs. 2-3). In this paper the time-predictable assumptions are used to develop a stochastic model for earthquake occurrence. The main characteristic of this model is that the time of occurrence of future earthquake events depends on the size and the time of occurrence of the last event. Thus the larger the last earthquake, the longer the time to the next earthquake. The hazard along a section of the San Andreas Fault is computed for illustrative purposes. The results from the proposed model are compared to those from the Poisson model. The comparison shows that the Poisson model yields unconservative results when the last earthquake event occurred a long time before the forecast time. This observation is particularly important when assessing the seismic risk to critical facilities.

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TEMPORAL DEPENDENCE AND THE TIME-PREDICTABLE MODEL

Temporal dependence between seismic events is apparent in some earthquake catalogs. In particular, histories of earthquakes on a single fault tend to indicate a positive correlation between the size of an earthquake and the time to the next event. Patterns in the data indicate that larger earthquakes are followed by longer quiet periods. Thus, a relationship between the time to the next earthquake and the size of the current event enables one to model the time between successive earthquakes. This property, called "time-predictable recurrence", enables the forecasting of the time to the next event given the size of the preceding earthquake.

Time-predictable recurrence can be described by the following model. A time history of the stress accumulation and release on a section of a fault is represented schematically in Figure 1a. Stress build-up occurs at a constant rate until the accumulated stress reaches a threshold. Then an earthquake occurs and some portion of the accumulated stress is released. The size of the earthquake is characterized by the change in stress level on the fault; the greater the change in stress, the larger the event. The occurrence time of the next earthquake is determined by the length of time that is needed to accumulate sufficient stress to reach the threshold and trigger another event. Therefore, the interarrival times are determined by the sizes of the earthquakes and the rate of stress accumulation. Time-predictable recurrence makes it possible to forecast the time of the next earthquake given the size of the preceding event, but gives no information about the size of the following event.

Presently, stress accumulation and release are difficult to measure. Therefore other quantities which are more easily obtained are used to determine parameters for the model. Coseismic slip is proportional to the change in stress level on a ruptured segment of a fault and can be used to estimate the time to the next earthquake. Figure 1b illustrates the relation between coseismic slip and time interval between seismic events. The diagonal line represents the average cumulative slip on a fault over time and its slope is the average slip rate. It is evident that events with larger slip are followed by longer periods of inactivity than smaller slip earthquakes. Therefore, measurements of average slip in an earthquake can be used to forecast the time of the next seismic event; that is, we have time-predictable recurrence.

Evidence of time-predictable recurrence for small strike-slip earthquakes, large thrust events, and large earthquakes along plate boundaries of the convergent and strike-slip type faults is presented in the literature (Ref. 2-5). Investigators have used geological as well as historical data to successfully estimate repeat times for earthquakes. The term "repeat time" refers to the period of time between two earthquakes which cause ruptures to occur along nearly the same portion of plate boundary (Ref. 3). It is significant to note that many types of earthquakes have exhibited time-predictable behavior. However, earthquake catalogs with these characteristics are found mostly along plate boundaries.

MODEL FORMULATION

The basic assumptions of the deterministic time-predictable model are adopted in the formulation of the stochastic model of earthquake occurrence. Stress on a section of the fault accumulates at a constant rate, resulting in a linear stress build-up. Earthquakes occur when the stress reaches a threshold level. Stress is released, and the size of the release determines the size of the associated earthquake. The sizes of successive earthquakes are assumed to be independent and identically distributed random variables. The inter-event times are random variables with a distribution that is conditional on the size of the last event.

A Markov renewal process describes the visits to the magnitude states in time. At present, sufficient information on the stress released in a given size earthquake is not available. Thus, the states of the process are defined by magnitude ranges with associated coseismic slips. Let $E = [1, 2, \ldots, N]$ be the set of mutually exclusive magnitude states, where N is the threshold state. We define Y_n as the state on the fault after the n^{th} transition, and T_n as the time of the n^{th} transition. Thus the set $\{Y_n : n \geq 0\}$ are random variables assuming values in E and the set $\{T_n : n \geq 0\}$ are non-negative random variables such that $0 < T_0 < T_1 < \ldots$ The stochastic process $\{(Y_n, T_n) : n \geq 0\}$ is a Markov renewal process provided that

$$P[Y_{n+1} = j,\ T_{n+1} - T_n \leq t \mid Y_o, \ldots, Y_n;\ T_o, \ldots, T_n] =$$

$$P[Y_{n+1} = j,\ T_{n+1} - T_n \leq t \mid Y_n = i]$$

$$\text{for all } i,\ j \in E,\ n \geq 0 \text{ and } t \geq 0. \tag{1}$$

According to this definition, the joint probability of the next state and the time of the state change depends only on the present state of the process and is independent of past history.

The shape of the release distribution is calculated from the Gutenberg and Richter frequency-magnitude relation

$$\log_{10} N(m) = a - bm \tag{2}$$

where N(m) is the number of earthquakes with magnitude $\geq$ m during a specified time interval in a specified region; a is a constant which describes the level of seismicity; and b is a constant which describes the relative number of large to small earthquakes. It follows that the magnitude M has a truncated exponential distribution, i.e. for $m_0 \leq M \leq m_u$ the cumulative probability distribution is given by

$$F_M(m) = \frac{1 - \exp\,[-B(m - m_0)]}{1 - \exp\,[-B(m_u - m_0)]} \tag{3}$$

where $B = b\ln 10$, m_u is the largest magnitude event which can be expected in the region; and m_0 is the smallest magnitude considered in the model, (Ref. 6). The transition probabilities for the stochastic recurrence model are developed by discretizing the cumulative probability distribution $F_M(m)$.

It is assumed that the time to the next event is Weibull distributed and conditional on the present state i with a cumulative probability distribution given by $F_T(t|i) = 1 - \exp[-\lambda_i t^{\nu_i}]$ for $\lambda_i,\ \nu_i > 0,\ t \geq 0$ (4)

where λ_i and ν_i are constants. The Weibull distribution is desirable because the associated hazard rate $r(t|i)$ increases with time if ν_i is greater than 1. The hazard rate for this distribution is

$$r(t|i) = \lambda_i \nu_i t^{(\nu_i - 1)}$$

In addition, the Weibull distribution has been fitted to some interarrival data for a section of the San Andres fault (Ref. 7). Traditionally the exponential distribution has been used to represent the interarrival time probabilities. The hazard rate for the exponential distribution is a constant, indicating that interarrival times are independent, and the earthquake occurrence process is memoryless.

Probabilities of occurrence and probabilities of exceedence are calculated using recursive relations for a discrete-time semi-Markov process (Ref. 8). Define $G_i(j,k|t)$ as the probability that k visits to state j will occur in time $(0,t)$ given state i is entered at time 0, and define $h_{ir}(m)$ as the probability that a transition from i to r will occur in time m. Then for the present model formulation

$$G_i(j,k|t) = \sum_{\substack{r=1 \\ r\neq j}}^{n} \sum_{m=0}^{t} p_{ir} h_{ir}(m) G_r(j,k|t-m) + \sum_{m=0}^{t} p_{ij} h_{ij}(m) G_j(j,k-1|t) + \delta(k) H_i(t) \qquad (5)$$

Equation 5 is used to formulate the probabilities of exceedence, expected numbers of events, and the probability of an earthquake of size j in time $(t,t+x)$, given no earthquake occurs in time $(0,t)$ and an earthquake of size i occured at time 0. For a more detailed description of the model formulation see Ref. 9.

APPLICATION OF TIME-PREDICTABLE MODEL

As an illustration, the stochastic time-predictable recurrence model is applied to a section of the San Andreas fault that ruptured during the 1906 San Francisco earthquake. Probabilities of at least one event in time $(t,t+x)$ given no events have occurred in time $(0,t)$ are computed. In this example, only events larger than magnitude 5.0 are considered. It is assumed that smaller events could be modeled by a Poisson process, and the resulting probabilities combined with those computed from the time-predictable model. The study zone shown in Figure 2, is the southern part of the 1906 rupture which extends from San Francisco to San Juan Bautista. The San Andreas fault is a strike-slip fault located on the boundary between the Pacific and North American plates. The catalog of earthquakes on this section of the fault indicates that the seismicity is cyclic (Ref. 10). Periods of increased activity are an indication that stress levels along the fault are approaching the threshold. Periods of relative inactivity have occurred after major events, when the stress was at a lower level.

Parameters for the stochastic time-predictable model are determined using slip rates and earthquake histories for this region. A catalog of earthquakes of magnitude greater than 5 between 1855 and 1980 is used to estimate the b-value used in equations 2 and 3 (Ref. 10). A b-value of 0.625 is used as a parameter in the transition probability distribution. Slip is used to represent the accumulation and release mechanism previously described. Data from Ref. 11 for strike-slip faults is used to determine a log-linear relation between Richter magnitude and coseismic displacement. We assume that the holding times are Weibull distributed with $\nu = 2$; it has been shown that interarrival times on other sections of the San Andreas fault can be modeled by this distribution (Ref. 7). The parameter, λ_i, for the holding time distributions was chosen assuming the slip rate is 20 mm/yr (Ref. 10) and that the mean displacement, $\overline{D}$, resulting from a given magnitude earthquake is one half the maximum displacement, D, in meters, estimated from

$$\ln D = 1.26\ M - 8.31 \tag{6}$$

We have chosen parameters which are reasonable and which will illustrate the difference between the time-predictable and Poisson models of earthquake occurrence. The occurrence rate, λ, for the Poisson model is estimated from the catalog of earthquakes greater than magnitude 5 in Ref. 10 and equation 2.

Figures 3 and 4 are plots of the probability of at least one earthquake of magnitude greater than or equal to m in the next 50 years given there has been no major seismic activity since the last large event. In Figure 3, the last event (m_o) is assumed to be a Richter magnitude 7.0 to 7.25 earthquake, while in Figure 4 m_o is in the range 8.0 to 8.25. These probabilities are shown for a set of four different initial conditions; a large earthquake just occurred (t=0) or it occurred 50, 77 or 100 years before the present. Probabilities of exceedence computed from the Poisson model are also shown. All of the probabilities are computed for a 60 km segment. Since, probabilities computed with the Poisson model are insensitive to the time of occurrence of the last event, it is evident that the Poisson model can be unconservative. While the difference in exceedence probabilities is small for larger magnitude earthquakes, this effect is quite pronounced for earthquakes of magnitude less than about 7. As the time since the last earthquake increases, the Poisson model becomes less desirable for computing seismic hazard.

Figures 5 and 6 show the probabilities of having at least one earthquake greater than or equal to magnitude 7.0 in (t, t+x) given no major seismic activity has occurred in (0,t) and the last major earthquake was either a magnitude 7.0 to 7.25 or an 8.0 to 8.25 event, respectively. The probabilities are shown for the same four values of t as in Figures 3 and 4. The Poisson model gives larger probabilities of exceedence than the time-predictable model when the initial event is 8.0 to 8.25. However, when m_o is in the range 7.0 to 7.2 the Poisson model gives unconservative estimates of the seismic hazard. Therefore, the independence assumptions inherent in the Poisson model do not necessarily mean that it will give an upper bound on seismic hazard.

CONCLUSIONS

This paper presents a stochastic model based on the time-predictable hyposthesis of earthquake occurrence. A Markov renewal model was developed as a stochastic representation of the time-predictable hypothesis. It was shown that probabilities of exceedence computed with the Markov renewal model depend on both the time of occurrence and the size of the last earthquake. Comparisons with the Poisson model indicate estimates of exceedence probabilities computed with the Poisson model can be unrealistically low in certain cases. This effect can be extreme if the last event was a moderate earthquake and a long period has passed since its occurrence.

ACKNOWLEDGEMENT

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REFERENCES

1. Cornell, C.A. (1968), Engineering Seismic Risk Analysis, BSSA, V58, 1583-1606.

2. Shimazaki, K. and T. Nakata (1980), Time-Predictable Recurrence for Large Earthquakes, Geoph. Res. Lett., V7, 279-282.

3. Bufe, C.G., Harsh, P.W. and R.O. Burford (1977), Steady-State Seismic Slip - A Precise Recurrence Model, Geoph. Res. Lett., V4, 91-94.

4. Sykes, L.R. and R.C. Quittmeyer (1981), Repeat Times of Great Earthquakes along Simple Plate Boundaries, Earthquake Prediction, Maurice Ewing Series, 4, A.G.U., 217-247.

5. Nishenko, S.P. and W.R. McCann (1981). Quantifying Seismic Potential: A Deterministic Approach, EOS, V62, 963.

6. Der Kiureghian, A. and A.H.-S. Ang (1977), A Fault Rupture Model for Seismic Risk Analysis, Bull. Seism. Soc. Am., V67, 1173-1194.

7. Brillinger, D.R. (1982), Seismic Risk Assessment: Some Statistical Aspects, Earthq. Pred. Res., V1, 183-195.

8. Howard, R.A. (1971), Dynamic Probabilistic Systems, Volume II: Semi-Markov and Decision Processes, Wiley and Sons, New York.

9. Anagnos, T. (1984), A Stochastic Earthquake Recurrence Model with Temporal and Spatial Dependence, Ph.D. dissertation, Dept. of Civil Eng., Stanford University, CA.

10. Ellsworth, W.L., Lindh, A.G., Prescott, W.H. and D.G. Herd (1981). The 1906 San Francisco Earthquake and the Seismic Cycle, Earthquake Prediction, Maurice Ewing Series, 4, A.G.U., 126-140.

11. Slemmons, D.B. (1977). State-of-the-Art for Assessing Earthquake Hazards in the United States, Engineer Waterways Experiment Station Miscellaneous Paper s-73-1, Report 6.

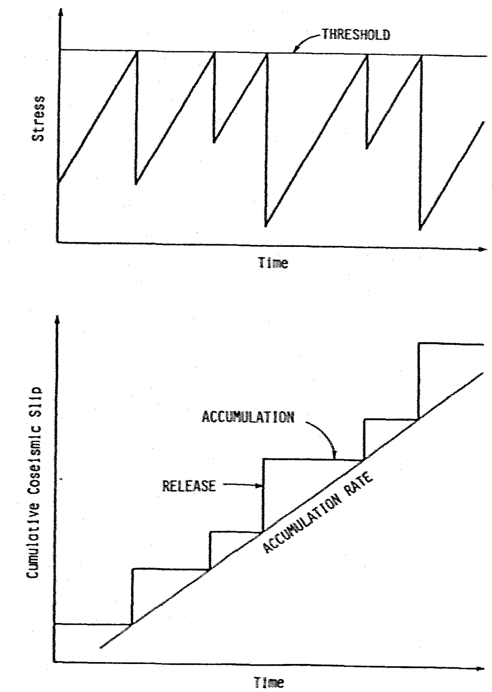

Figure 1. Schematic representation of time predictable behavior.

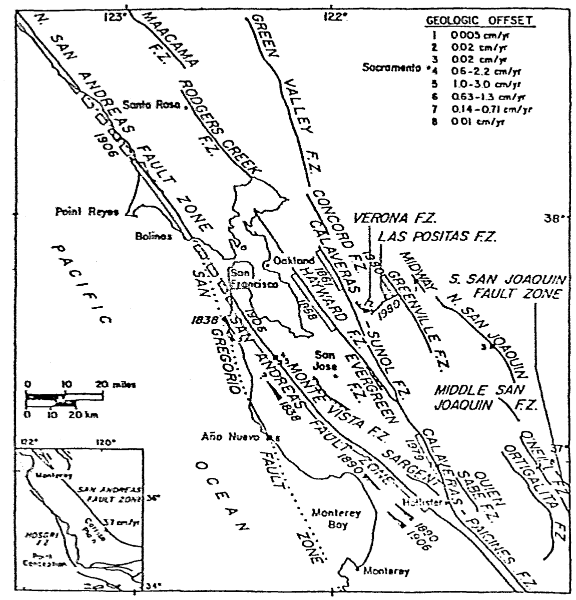

Figure 2. Northern section of the San Andreas fault zone.

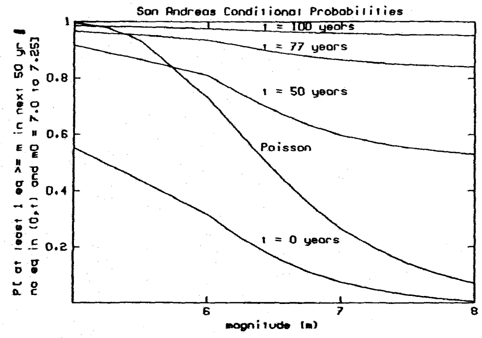

Figure 3. Probabilities of at least 1 earthquake of magnitude ≥ m in the next 50 years given that there were no earthquakes in (0,t) and the event at time 0 had a magnitude of 7.0 to 7.25.

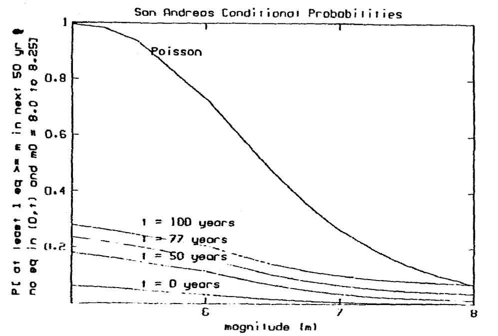

Figure 4. Probabilities of at least 1 earthquake of magnitude ≥ m in the next 50 years given that there were no earthquakes in (0,t) and the event at time 0 had a magnitude of 8.0 to 8.25.

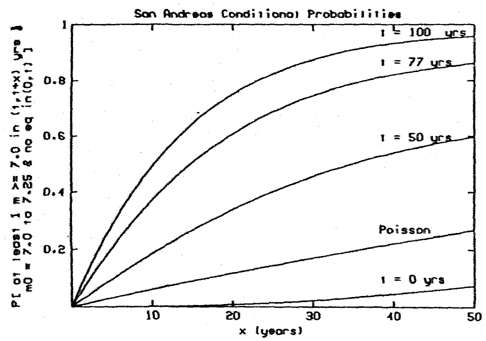

Figure 5. Probability of at least 1 earthquake of magnitude 7.0 or greater in (t,t+x) years given that the initial event is of magnitude 7.0 to 7.25 and there were no earthquakes in (0,t). Comparison between the time-predictable and Poisson models.

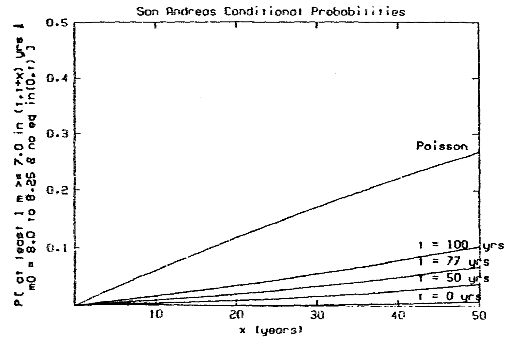

Figure 6. Probability of at least one earthquake of magnitude 7.0 or greater in (t,t+x) years given that the initial event of magnitude 8.0 to 8.25 and there were no earthquakes in (0,t). Comparison between the time-predictable and Poisson models.