SEISMIC RISK ANALYSIS BASED ON WEIBULL DISTRIBUTION

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SUMMARY

Based on the Weibull distribution, a quantitative procedure is developed for the evaluation of seismic risk in a particular region. Utilizing the maximum-likelihood criterion, an efficient algorithm for estimation of the parameters of Weibull distribution is developed. This procedure is applied to the earthquake data of Iran recorded during 1902-1975. Typical results are presented in terms of the probabilities of exceedance of the peak ground accelerations for two major cities of Iran. The results of this investigation are compared with the previously obtained results of seismic risk analysis in Iran based on the stationary Poisson model.

INTRODUCTION

A realistic and rational approach to the earthquake resistant design of major structures must undoubtedly take into account the probabilistic nature of earthquake occurrence. Probabilistic assessment of seismic hazard at a site has been the focus of considerable research and several analytical methods have been developed in recent years. In conventional method of seismic risk analysis, the Poisson probabilistic model is usually employed as the stochastic model of earthquake occurrence in time (Refs. 1-7). Seismic events in this model are assumed to be independent in time and space.

The expected number of earthquakes in a given period in this model is constant and often given by the following log-linear relationship:

$$\log N(m) = \alpha + \beta m$$

Where $N(m)$ is the number of earthquakes of magnitude greater than m, and α and β are regional parameters to be found by a regression analysis of existing and pertinent data. However, the hazard function, $h(t)$, which represents the instantaneous probability of failure for the Poisson process is a constant. This function is defined by

$$h(t) = \frac{f_T(t)}{1-F_T(t)} \tag{1}$$

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Where, T is the arrival time of an earthquake of certain magnitude, and $f_T(t)$ and $F_T(t)$ are the probability density function and cumulative distribution function for the earthquake arrival time, respectively. For the Poisson model $h(t) = \lambda$ = mean arrival rate of the earthquake. This simply means that the occurrence or non-occurrence of an earthquake, e.g., in this year does not change the probability of occurrence or non-occurrence of another earthquake in the future.

However, according to the well-known elastic rebound theory of earthquake occurrence, when an earthquake occurs, it releases the accumulated strain energy, which may in turn, decrease the possibility of observing another earthquake until the strain energy has again accumulated. The use of stationary Poisson model may be appropriate for regions with relatively uniform seismic activity. However, application of this stochastic model for regions where infrequent but large earthquakes occur after long periods of quiescence is inconsistent with the very phenomenon of strain build-up in parts of the earth.

To overcome this shortcoming, other stochastic models for seismic risk analysis have been proposed including Markov and Semi-Markov processes (Ref. 8). However, most of these models are handicapped by the need to a large quantity of data for evaluation of their parameters. Use of the Weibull distribution which is a somewhat modified form of an asymptotic extreme-value model for seismic risk evaluation is proposed in Ref. 9. The Weibull distribution has been used extensively in failure and reliability analyses (Refs. 10-11). In Ref. 12, using the motion data obtained with the geophysical model, Weibull distribution has been utilized to calculate the consistent probability power spectral density of the ground acceleration at a site. In this paper, based on the Weibull distribution, a quantitative procedure is developed for the evaluation of seismic risk in a particular region in which the time dependency of earthquake occurrence is taken into account. In the following section, utilizing the maximum-likelihood criterion, an efficient algorithm is presented for the estimation of the parameters of Weibull distribution.

ALGORITHM FOR ESTIMATION OF THE PARAMETERS OF THE WEIBULL DISTRIBUTION

If we denote the arrival time of an earthquake of magnitude greater than a given value m by T, the probability density function, $f_T(t)$, and the cumulative distribution function, $F_T(t)$, of the Weibull distribution can be written in the following form

$$f_T(t) = \frac{b_m}{a_m}\left(\frac{t}{a_m}\right)^{b_m - 1} \exp\left[-\left(\frac{t}{a_m}\right)^{b_m}\right] \qquad t \geq 0 \tag{2}$$

$$F_T(t) = 1 - \exp\left[-\left(\frac{t}{a_m}\right)^{b_m}\right] \qquad t \geq 0 \tag{3}$$

The parameters a_m and b_m are called the scale and shape parameters of the Weibull distribution. In the present application, the subscript m refers to the class of earthquakes with magnitude greater than m. The hazard function for the Weibull distribution is given by

$$h(t) = \frac{b_m}{a_m}\left(\frac{t}{a_m}\right)^{b_m - 1} \tag{4}$$

If the value of the parameter b_m is greater than one, the hazard function increases with increasing time which is consistent with the elastic rebound theory of earthquake generation.

In order to estimate the shape and scale parameters of the Weibull distribution, we employ the method of maximum likelihood (Ref. 13). Denoting the arrival time of the ith earthquake occurrence by t_i and the number of occurrences by n, the likelihood function for the Weibull distribution and its natural logarithm can be written as

$$L(a_m, b_m \mid t_1, t_2, \ldots, t_n) = \prod_{i=1}^{n} f_T(t_i \mid a,b) = \left(\frac{b_m}{a_m}\right)^n \left(\frac{t_1, t_2, \ldots t_n}{a_m^n}\right)^{b_m - 1} \exp\left[- \sum_{i=1}^{n} (t_i/a_m)^{b_m}\right] \tag{5}$$

$$\log(L) = n(\log b_m - b_m \log a_m) + (b_m - 1) \sum_{i=1}^{n} \log t_i - \sum_{i=1}^{n} (t_i/a_m)^{b_m} \tag{6}$$

Now, we set the partial derivatives of the function log(L) with respect to a_m and b_m equal to zero.

$$\frac{\partial \log L}{\partial a_m} = 0 \; ; \quad \sum_{i=1}^{n} (t_i/a_m)^{b_m} = n \tag{7}$$

$$\frac{\partial \log L}{\partial b_m} = 0 \; ; \quad \frac{n}{b_m} - n\log a_m + \sum_{i=1}^{n} \log t_i - \sum_{i=1}^{n} (t_i/a_m)^{b_m} \log (t_i/a_m) = 0 \tag{8}$$

The above two equations, after some manipulation, can be written as

$$a_m = g(b_m) = \left[\sum_{i=1}^{n} (t_i)^{b_m}/n\right]^{1/b_m} \tag{9}$$

$$f(b_m) = \frac{n}{b_m} + \sum_{i=1}^{n} \log t_i \left[1 - (t_i/a_m)^{b_m}\right] \tag{10}$$

One may substitute for a_m in Eq. (10) from Eq. (9), the result will be a nonlinear equation for the shape parameter b_m. In order to solve the nonlinear equation (10) for b_m we use the Newton-Raphson iteration procefure (Ref. 14). According to this procedure, the value of the shape parameter at the end of the iteration j+1 is given by

$$(b_m)_{j+1} = (b_m)_j - f\left[(b_m)_j\right] / f'\left[(b_m)_j\right] \tag{11}$$

The denominator in the last expression which is the derivative of function $f(b_m)$ (Eq. 10) can be shown to be

$$f'(b_m) = -\frac{n}{b_m^2} - \sum_{i=1}^{n} \left\{ (t_i/a_m)^{b_m} \log t_i \left[-(b_m/a_m)\ g'(b_m) + \log(t_i/a_m) \right] \right\} \quad (12)$$

in which $g'(b_m)$ is the derivative of function $g(b_m)$ and given by

$$g'(b_m) = \frac{1}{nb_m}\left[\sum_{i=1}^{n} (t_i)^{b_m}/n\right]^{1/b_m - 1}\left[\sum_{i=1}^{n} (t_i)^{b_m} \log t_i\right] - \frac{1}{b_m^2}\left[\sum_{i=1}^{n} (t_i)^{b_m}/n\right]^{1/b_m} \log\left[\sum_{i=1}^{n} (t_i)^{b_m}/n\right] \quad (13)$$

Now, Eqs. (9) - (13) can be used iteratively to find the parameters of the Weibull distribution.

To start the solution, an initial estimate of the shape parameter b_m must be given. We may start the solution with $b_m=1$. At each iteration, values of functions $f(b_m)$, $g'(b_m)$, and $f'(b_m)$ will be evaluated from Eqs. (10), (13), and (12), respectively. Then, a new value for b_m will be estimated from Eq. (11). The convergence of this procedure is excellent as will be shown in the following section.

SEISMIC RISK IN A REGION

To evaluate the seismic risk in a region, we define the seismic risk, $R(m,t)$, as the probability of observing at least one earthquake of a magnitude greater than m in a time period t.

$$R(m,t) = 1 - \exp\left[-(t/a_m)^{b_m}\right] \quad (14)$$

In order to present numerical results, we apply this concept to the earthquake data of Iran during the period 1902-1975. This data consists of 1787 earthquakes which is collected and analyzed in Ref. 15.

Seismic risk as defined by Eq. (14) is evaluated for four time periods of 5, 10, 20, and 50 years. Results are plotted in Fig. 1. It is seen that, e.g., the probabilities that at least one earthquake of a magnitude greater than 7 would occur within this area during the time periods of 5, 10, 20, and 50 years, respectively, are 0.62, 0.88, 0.98, and 1. The corresponding values for an event of magnitude greater than 7.5 are 0.24, 0.45, 0.69, and 0.92.

In a previous work (Refs. 1 and 6), the stationary Poisson process with a long-nonlinear recurrence relationship was used for the forecasting of the earthquake risk in Iran. According to this model the risk equation is defined by

$$R(m,t) = 1 - \exp\left[-t \exp\left(\alpha' + \beta m + \gamma/m\right)\right] \tag{15}$$

where α', β, and γ are the coefficients of the recurrence relation and are found by a regression analysis of the pertinent data. Using the data of Iran, $\alpha' = 21.300$, $\beta = -2.718$, and $\gamma = -30.318$. Results obtained from Eq. (15) are also plotted in Fig. 1 with dashed lines. It is observed that, the probabilities that at least one earthquake of a magnitude greater than 7 would occur within this area during the time periods of 5, 10, 20, and 50 years, respectively, are 0.47, 0.72, 0.92, and 1.00. The corresponding values for an event of magnitude greater than 7.5 are 0.20, 0.35, 0.58, and 0.89. Figure 1 clearly reveals that the stationary Poisson process consistently underestimates the seismic risk.

It should be noted that having chosen a tolerance limit of 0.001 the number of iterations for finding the parameters a_m and b_m of the Weibull distribution in most cases is from 2 to 5 and sometimes 6 or 7. Therefore, it may be concluded that the proposed algorithm is an efficient method for evaluation of the parameters of the Weibull distribution. Also, it is of interest to note that the number of iterations for a given tolerance limit generally increases as the number of data for the class of seismic event considered decreases.

SEISMIC RISK AT A SITE

In order to evaluate the seismic risk at a site, the spatial distribution of earthquakes should be accounted for using different source models. In order to compare the application of Weibull distribution with the previous investigation of the seismic risk in Iran, the earthquake data of Iran was divided among 19 line source models and 2 area source models, as presented in Ref. 6. In this modeling, only one focal depth equal to the average of the focal depths of all the pertinent earthquakes is assigned to each line or area source. Also, a peak ground acceleration (PGA) attenuation relation in the form

$$a = C_1 \, e^{C_2 m} \, (R_h + C_3)^{C_4} \tag{16}$$

is used where a is the PGA, R_h is the hypocentral distance and C_1, C_2, C_3, and C_4, are the regional constants.

Assuming the spatial independance of earthquake occurrence on different sources, one obtains the following equation for the risk due to the multi-source model occurrences (Ref. 12):

$$R(m,t) = 1 - \exp\left[-\sum_{i=1}^{N} p_{im} \, (t/a_{im})^{b_{im}}\right] \tag{17}$$

In this equation, N is the number of the source models and p_{im} is the probability of occurrence of an earthquake of greater than m, given that an event has occurred at source i.

Because, according to Eq. (16), there is a one-to-one relation between the PGA and the magnitude, using Eq. (17) one can find the probability of observing at least one earthquake with PGA greater than some specific value a at a given site. Details of this analysis will be published elsewhere. However, preliminary results of the seismic risk analysis in terms of the probabilities of exceedance of different PGA levels for two cities are given here. These cities are Tabriz in the northwest of Iran and Bandar Abbas in the south of Iran. The results are presented with solid curves in Figs. 2 and 3. For ths sake of comparison, the results of a previous investigation based on a stationary Poisson process (Ref. 6) are also shown with dashed lines on the same figures. These lines are based on the 1902-1975 data period. However, the available data base in Iran is nonhomogeneous. For instance about 97 percent of the 1787 earthquakes have been recorded during the 50-year period from 1925 to 1975. To take the effect of this nonhomogeneity into account, the recurrence relation parameters for use in the Poisson model are also calculated based on a data period of 50 years (Ref. 6). The results are shown with the dash-dotted curves in Figs. 2 and 3.

An examination of Figs. 2 and 3 clearly reveals that application of Weibull distribution for evaluation of the seismic risk at a site produces consistently much higher probabilities of exceedance than the nonstationary Poisson model.

In conclusion, the results of this investigation show that the use of the Weibull distribution is a more realistic approach to the problem of seismic risk at a site. Also, the nonstationary Poisson process apparently underestimates the seismic risk at a site.

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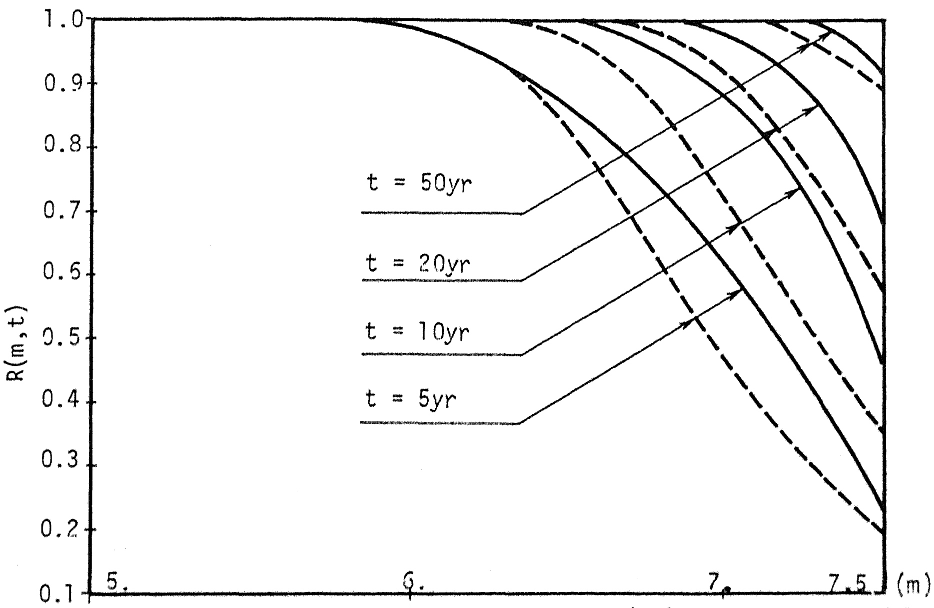

Figure 1 Risk Evaluation by Weibull Distribution (—) and Poisson Model (--)

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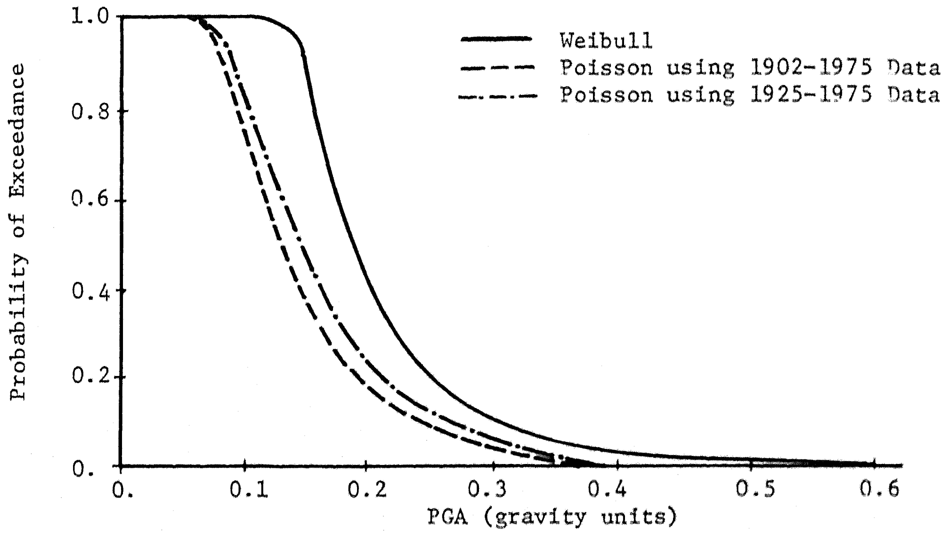

Figure 2 Seismic Risk at Tabriz for a Period of 50 Years

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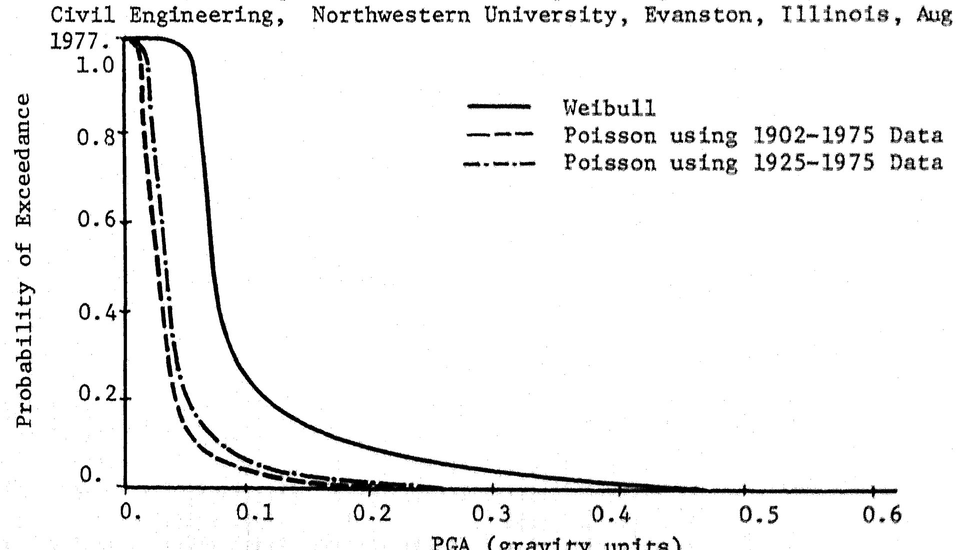

Figure 3 Seismic Risk at Bandar Abbas for a Period of 50 Years