ON THE THREE-DIMENSIONAL SEISMIC RESPONSE OF EARTH STRUCTURES

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SYNOPSIS

An analysis is made of the lateral response of three-dimensional models of earth and rock-fill dams subjected to seismic motion. A two-dimensional finite element discretization of a cross-section together with a Fourier expansion of the solution in the longitudinal direction are used to solve the equations of motion, and simplified approximate formulas are obtained for the natural frequencies and mode shapes of prismatic dams. Numerical results indicate that for dams located in narrow canyons the stresses in the linear range of behavior may differ significantly from those that would occur if the dams were infinitely long. Whether an increase or decrease will result depends upon the geometry and mechanical properties of the dam, and the characteristics of the excitation.

INTRODUCTION

Investigations of the response of earth and rock-fill dams to earthquake excitation have been conducted by many authors, mostly on the assumption of plane strain behavior of the dam. Computations are a serious task in two dimensions. In three they are a major undertaking. Yet consideration of the third dimension is often important. Hatanaka [1] has shown that if attention is confined to shearing strains in a dam of cohesionless material and triangular cross-section restricted at the ends by transverse vertical abutments, a ratio of length to height that does not exceed about 5:1 is sufficient to introduce significant increases in capacity to resist transverse ground acceleration relative to the two-dimensional solution. Many practical problems involve more severe restrictions. It is desirable, therefore, to have an expedient procedure that will allow translating two-into the three-dimensional solutions without excessive loss of accuracy. The main objective of this study is to develop such a method, as well as simplified rules for incorporating three-dimensional effects in the preliminary design of earthfill dams. The analysis is confined to the linear range of behavior, and the influence of the stored water is ignored.

ANALYSIS OF THE SYSTEM

In order to formulate the problem of the elastic response of an earth dam to seismic excitation as an initial value problem in classical elastodynamics let R —the open region occupied by the dam— be filled with an isotropic, elastic material with shear modulus μ, Poisson's ratio ν, and density ρ, and let S be its boundary. The problem consists in determining the displacement, strain, and stress fields in R at any time in the interval $[0, t_1]$ for prescribed body forces and boundary conditions, when the material is initially at rest. The boundary conditions are specified on two complementary parts, S_1 and S_2, of S. Displacements are specified on S_1, whereas S_2 is traction-free. As a concrete example let us consider the prismatic dam with vertical abutments shown on

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Fig. 1. Dams with inclined abutments will be examined subsequently. For the dam shown on Fig. 1, S_1 consists of the base and abutments, while the lateral inclined faces define S_2. The coordinates of a point $\underset{\sim}{x}$ in the cartesian coordinate system shown will be denoted by x_i $(i = 1, 2, 3)$.

We will assume that μ, ν and ρ vary over the cross-section of the dam but remain constant along its length. In addition, the seismic excitation $\hat{\underset{\sim}{u}}$ is assumed to be uniform on S_1, i.e., $\hat{\underset{\sim}{u}}(\underset{\sim}{x}, t) = u_g(t)\underset{\sim}{b}$ on S_1, where $u_g(t)$ is a prescribed function of time, and the unit vector $\underset{\sim}{b}$ defines the direction of motion. The relative displacement of the dam, $\underset{\sim}{u}$, with respect to that of the boundary S_1 will be used as the primary dependent variable. The corresponding problem for $\underset{\sim}{u}$ is then one in which S_1 is held fixed, S_2 is traction-free, and a body force $\underset{\sim}{F}$ equal to $-\rho\ddot{u}_g(t)\underset{\sim}{b}$ acts on R. Total displacements in the dam are given by $\underset{\sim}{u} + \hat{\underset{\sim}{u}}$, but the corresponding stresses are only functions of the relative displacements, since $\hat{\underset{\sim}{u}}$ is independent of the spatial coordinates (Notice the similarity between this formulation and that for a building subjected to uniform excitation of its basement). Rather than expressing the preceding problem directly in terms of the equations of motion for $\underset{\sim}{u}$ an alternative formulation based on a variational principle will be used herein. For any continuous function $\underset{\sim}{u}$ with piecewise continuous first derivatives define the functional

$$\Omega(u) = \frac{1}{2}\int_R \left\{ \rho\dot{u}_i * \dot{u}_i + \mu\left[u_{i,j} * u_{j,i}) + \frac{2\nu}{1-2\nu} u_{i,i} * u_{j,j}\right] - 2u_i * F_i\right\} d\underset{\sim}{x} \qquad (1)$$

Then $\underset{\sim}{u}$ is a solution of the problem under consideration if and only if the first variation $\delta\Omega(u)$ of Ω vanishes at $\underset{\sim}{u}$ [2, 3].

In (1) u_i, F_i are the components of the displacement field $\underset{\sim}{u}$ and body force $\underset{\sim}{F}$, respectively, and the asterisk denotes a modified convolution, with t_1 replacing t as the upper limit of the integral over time. Indicial notation and summation convention will be understood unless otherwise specified. Latin subscripts will have the range 1 to 3, while greek subscripts will take the values 1 and 2. Since this paper is limited to the evaluation of the response of the dam to lateral excitation, and the relative displacement at the ends is not allowed by the abutments, it is reasonable to neglect the longitudinal component, u_3, of the displacement field $\underset{\sim}{u}$ altogether. In solving for the two nonvanishing components of displacement $u_\alpha(\alpha = 1, 2)$ it is convenient to assume a solution of the form

$$u_\alpha(x_1, x_2, x_3, t) = \sum_{j=1,2,\ldots} u_{\alpha j}(x_1, x_2, t) \sin[\pi j(x_3 + L/2)/L] \qquad (2)$$

which automatically satisfies the condition of fixity at the end walls; $u_{\alpha j}$ are unknown functions of the cross-sectional coordinates x_1 and x_2, time t, and the parameter j; the $u_{\alpha j}$ will be determined from the variational principle. L is the length of the dam.

Expanding the prescribed components F_α of the effective body force $\underset{\sim}{F}$ into a Fourier series in terms of the longitudinal coordinate x_3, and substituting the resulting expression together with (2) into (1) leads to

$$\Omega(u_{\alpha j}) = \frac{L}{4}\sum_{j=1,2,\ldots}\int_A \left\{\rho\dot{u}_{\alpha j}*\dot{u}_{\alpha j} + \mu\left[u_{\alpha j,\beta}*(u_{\alpha j,\beta}+u_{\beta j,\alpha}) + (\pi j/L)^2 u_{\alpha j}*u_{\alpha j}\right.\right.$$
$$\left.\left. + 2\nu/(1-2\nu)u_{\alpha j,\alpha} + u_{\beta j,\beta}\right] - 2u_{\alpha j}*a_{\alpha j}\right\} d\underset{\sim}{x} \qquad (3)$$

where $a_{\alpha j}(t)$ are the coefficients of the Fourier expansion for F_α, and A is the cross-sectional area of the dam. This equation shows that the problem of finding the displacement field $\underset{\sim}{u}$ in three spatial dimensions is equivalent to solving an infinite number of uncoupled problems in two dimensions; in fact, except for the term $(\pi j/L)^2 u_{\alpha j} * u_{\alpha j}$ the expression for each j is identical to that for a plane strain problem. This result can be useful in practical applications, especially if only a few harmonic terms as those indicated in (2) are required to attain desired accuracy. The same procedure has been used by others for analyzing problems in elastostatics [4], and in the solution of dynamic problems of axisymmetric bodies [5]. Clearly, each two-dimensional problem must be solved numerically as exact analytical solutions are difficult to obtain even for simple cases. The finite element method is ideally suited for treating arbitrary geometry and material inhomogeneities. Thus, let us divide the cross-section of the dam into a finite element mesh and let the unknown functions $u_{\alpha j}$ be approximated within each element e by

$$u^e_{\alpha j}(x_1,\ x_2,\ t) = \underset{\sim}{N}^T_{\alpha}(x_1,\ x_2)\ \underset{\sim}{v}_j(t);\ \alpha = 1,2,;\ j = 1,2, \ldots J \tag{4}$$

where $\underset{\sim}{N}_{\alpha}(x_1,\ x_2)$ is the vector of trial functions, T denotes transpose, $\underset{\sim}{v}_j(t)$ is the time dependent vector of nodal displacements, and J denotes the total number of harmonic terms to be used in the approximation. If N is the number of mesh nodes corresponding to nonvanishing displacements, both vector fields $\underset{\sim}{N}_{\alpha}$ and $\underset{\sim}{v}_j$ are of dimension 2N as each node has two degrees of freedom.

After substituting (4) into (3), performing the indicated integrations over each element, summing over all the elements, and setting the first variation of Ω to zero there results

$$M\ \ddot{\underset{\sim}{v}}_j + [K + K_{oj}]\ \underset{\sim}{v}_j = \underset{\sim}{f}_j(t);\ j = 1,2,\ldots J \tag{5}$$

In this equation M and K are precisely the mass and stiffness matrices, respectively, for the plane-strain problem associated with the cross-sectional area of the dam. K_{oj} is a positive definite matrix that represents the effect of the third dimension on the total stiffness of the structure and $\underset{\sim}{f}_j$ is the effective modal excitation. Notice that *the effective stiffness of the dam, as measured by* $K + K_{oj}$ *increases with decreasing length and with the higher modes in the longitudinal direction* since K_{oj} is proportional [6] to $(j/L)^2$.

Natural frequencies and mode shapes are found from the solutions to the eigenvalue problem resulting from eliminating the excitation term from eq. (5).

If the fill material of the dam is uniform it can be shown [6] that

$$\omega_{nj} = [\omega_n^2 + (\pi j V_s/L)^2]^{1/2} \tag{6}$$

$$\underset{\sim}{v}_{nj} = \underset{\sim}{v}_n \sin[\pi j(x_3 + L/2)/L] \tag{7}$$

where $V_s = \sqrt{\mu/\rho}$ is the shear wave velocity in the medium and $\underset{\sim}{v}_n$ is the eigenmode associated to ω_n, the nth natural frequency of a dam of infinite length.

These equations represent a generalization for three dimensional dams of a result obtained by Hatanaka [1] and Ambraseys [7] for a two-dimensional shear wedge. Equations (6) and (7) show that the approximate natural frequencies and modes for transverse vibration of a uniform prismatic dam with rigid vertical abutments can be obtained readily from the corresponding values for the dam of unlimited length. Equation (6) can be rewritten [6] as

$$\omega_{nj} = \omega_n\ [1 + (\pi j H/a_n L)^2]^{1/2} \tag{8}$$

where $a_n = \omega_n H/V_s$ is a dimensionless coefficient that increases with n and depends on the geometry of the cross-section. From (8) we observe that whereas the increase in the natural frequencies due to the finite separation of the abutments becomes more pronounced for the higher harmonics of the longitudinal oscillation, the first mode is the most significant in the transverse direction, decreasing in importance with the higher modes. As for the two-dimensional shear wedge, the frequencies decrease with increasing L/H.

With the natural frequencies and normal modes established, modal superposition is used to analyze the transient response. Stresses at various points within the dam can be calculated once the nodal displacements have been evaluated. The method developed herein can be extended to dams with inclined abutments, provided the structure has a plane of symmetry perpendicular to the longitudinal axis. The procedure remains essentially the same as for the dam with vertical end walls, with L_e, the average length of the dam associated with each element e, replacing L. For details see [6]

NUMERICAL SOLUTIONS

As an example of the results which may be obtained by the proposed method, the earthquake analysis of homogeneous dams with symmetrical triangular cross-sections will be described. Attention will be devoted mostly to dams with vertical abutments (Fig. 1) —for a wide range of values of the length to height ratio, L/H— but triangular and trapezoidal valley cross-sections also will be investigated. All the calculations will be performed for $\nu = 0.45$, and two different side slopes, 3 on 1 and 1.5 on 1, will be considered. The limiting two-dimensional problem for an infinitely long dam has been studied by Chopra, Clough, and others [8, 9, and the references cited therein], and the results have been compared with those for a shear wedge. We will concentrate on the effects of the third dimension on the response. Natural frequencies for a two-dimensional shear wedge also will be presented for comparison.

The four sets of curves in Fig. 2 illustrate the effects of the length-height ratio L/H and side slope B/H on the natural frequencies of dams with rectangular valley cross-section. Shown in continuous lines are dimensionless frequencies a_{nj} —defined by $\omega_{nj}H/V_s$— for various transverse (n) and longitudinal (j) modes. The limiting values $a_n = \omega_n H/V_s$ corresponding to dams of infinite length are also given in the figure. Mode shapes for several values of n and j=1 are depicted in Fig. 3 for L/H = 3 and B/H = 1.5. Notice that only the first antisymmetrical transverse mode resembles a pure shear distortion. There are significant vertical displacements involved in the other antisymmetrical transverse mode, and the symmetrical mode has no resemblance to the shear-type mode. These observations are in agreement with the differences that result by comparing the natural frequencies a_{nj} of the three-dimensional body with those corresponding to a two-dimensional shear wedge-depicted in Fig. 2 by dashed lines and denoted by $a_{n'j}$. The values of a_{nj} shown in Fig. 2 were calculated from a structural idealization of 36 linear triangular elements and a lumped mass matrix. Upper bounds for these frequencies derived from a finer mesh and a consistent mass matrix are given in [6]. The values of $a_{n'j}$ for the two-dimensional shear wedge are in all cases greater than the corresponding upper bounds for a_{nj}. The differences become significant for the second and higher transverse modes.

To test the validity of the assumption that the axial displacement u_3 is negligible, the first two natural frequencies and mode shapes for a dam with relative dimensions L/H = 1.5 and B/H = 3 have been evaluated by means of a full three-dimensional finite element analysis. Using tetrahedral elements and the

SAP IV algorithm with lumped masses these quantities were calculated first by setting u_3 to zero everywhere in the dam and then by imposing the sole condition that u_3 vanishes on the boundary S_1. The dimensionless natural frequencies are shown on Table I together with the corresponding values from the present method. Mode shape calculations registered differences of less than 10% for the transverse modal displacements, while axial displacements along the crest for the complete problem ($u_3 \neq 0$) were less than 1% of the maximum transverse displacements for the first mode, and 30% for the second. Calculations on a Burroughs B6700 computer took 24 and 36 times longer for the three-dimensional finite element solutions with $u_3 = 0$ and $u_3 \neq 0$, respectively, than for the corresponding solution with the present semi-discrete method.

The three curves in Fig. 4 illustrate the effect on the fundamental natural frequency, ω_{11}, of the valley cross-section. Shown are curves for dams with rectangular, triangular, and trapezoidal valley cross-sections, and a fixed lateral slope ratio, B/H, of 1.5. The effect of the inclined abutment is, as expected, to increase the structural stiffness, and hence, ω_{11}.

Having determined the natural frequencies and mode shapes of the various dams, the structures were subjected to the NS component of the El Centro earthquake of 18 May 1940. The following dam properties were used: H=91.5 m, B/H = 1.5, V_s = 305 m/s, and ρ = 2030 kg/m³; as in [8] damping in the structures was assumed to be viscous in nature and equivalent to 20% of critical in each mode to simulate the large amount of energy absorbed in the structure in the nonlinear range of deformations. The contributions of the first 6 transverse and 5 longitudinal modes were included in the analysis. The initial, dead weight, stresses were not included, so that the computed values are the dynamic stresses only. The time history of the shear stresses τ_{xy} at a point A 30.5 m high, on the vertical axis of symmetry of a cross-sectional plane located at midlength on the dam is sown in Fig. 5 for three valley cross-sections: limiting two-dimensional dam of infinite length, rectangular valley cross-section with L/H = 1, and symmetrical triangular canyon with the same value of L/H. The coordinate system xyz is that shown in Fig. 3. These time histories exhibit the distinct reduction of the fundamental natural period for the shorter dams with respect to that of the limiting plane-strain case. In addition, whereas the infinitely long dam responds primarily in the fundamental mode, contributions from higher longitudinal and transverse modes are apparent for the dams located on narrow canyons.

The most noticeable difference in the response of the three cases considered lies, however, in the significant reduction of the maximum value of the stresses with decreasing length. Two factors are responsible for this reduction. First, the response spectrum for the earthquake record under consideration decreases with decreasing natural period within the range of natural periods of interest. Second, the natural periods corresponding to the higher harmonics in the longitudinal direction increase rapidly as the length of the dam increases; for very long dams there is effectively an infinite number of repeated natural frequencies and associated longitudinal modes contributing to the total response corresponding to each mode since ω_{nj} approaches ω_n for all n and j as L/H tends to infinity. For shorter dams ω_{nj} becomes larger as j increases. Therefore, *only a small number of longitudinal modes is practically involved in the response; the higher modes result in negligible deformations.*

In Fig. 6 contour lines are shown for the instantaneous shear stresses and principal normal stresses on the plane xy for the entire cross-section for the three dams under study at an instant t = 2.26 s, for which the shear stress at point A attains a maximum value. The reductions of the maximum amplitude of the stresses on the midplane are similar to those observed at point A. Notice that normal stresses vanish only on the vertical axis of symmetry and that shear stresses vary along horizontal planes. This indicates that, as remarked previously in [8], a shear wedge cannot adequately represent the true behavior of an earth dam. Additional results described in [6] show that for a dam in a narrow canyon the remaining components of stress can be of the same order as the plane stresses depicted here.

CONCLUSIONS

1. Three-dimensional deformation behavior increases the stiffness characteristics of earthfill dams compared to those of a two-dimensional idealization. This increase can have an important effect on earthquake response, especially if the dam is located in a narrow canyon.

2. An approximate method suitable for preliminary design has been developed for translating two — into three— dimensional solutions without excessive loss of accuracy. Along with giving physical insight into the dynamics of three-dimensional dams, this method allows the expedient evaluation of the natural frequencies and mode shapes of the three-dimensional inhomogeneous dam by means of calculations that are equivalent to those required for a two-dimensional structure. Explicit formulas are derived for these quantities for the case of a uniform prismatic dam. Response to transient excitation is obtained by the method of modal superposition. The method should prove most useful when the response is dominated by only a few modal components, as was the case for the examples considered herein.

3. Numerical results of the present study are based on a simple three-dimensional earthfill model. The observation that consideration of the third dimension tends to reduce the maximum response with respect to that of a plane-strain idealization, however, would seem to remain valid for more complex linear models, whenever the ordinates in the design spectrum do not increase with decreasing period in the region of interest. This effect could be important in design, as it would allow the structure to be designed for smaller excitations than would be required under the assumption of plane-strain behavior. Opposite effects might result if the spectral ordinates increased with decreasing period within the region of interest.

This paper has been limited to the linear range of behavior. It would be desirable to develop a more general approach to model materials with nonlinear properties, as well as hydrodynamic pressure and soil-structure dynamic interaction.

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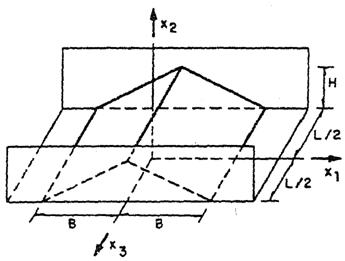


Fig 1. Example of earthfill dam model

Table I. Dimensionless natural frequencies of dam in rectangular canyon, B/H = 1.5, L/H = 1.5

Mode	SAP IV $u_3 \neq 0$	SAP IV $u_3 = 0$	Present Method
1	2.859	2.890	2.988
2	3.556	3.599	3.838

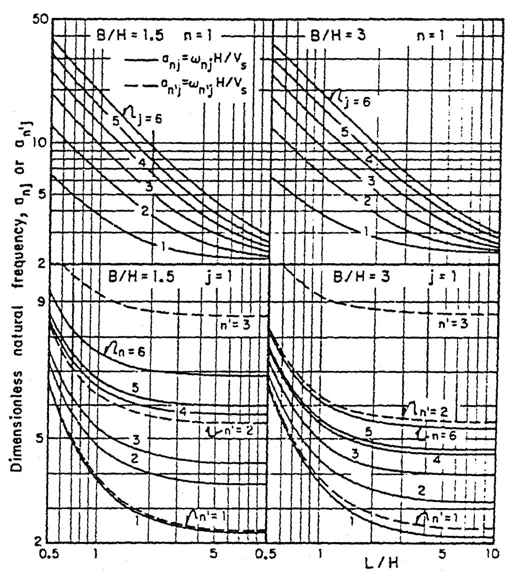
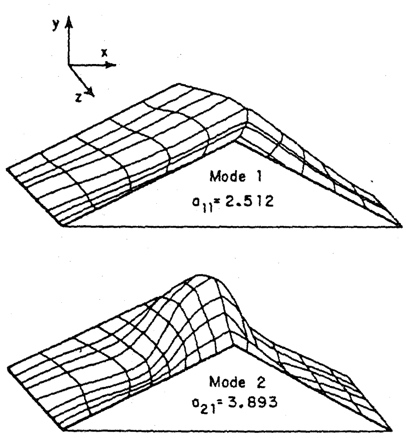


Fig 2. Natural frequencies for homogeneous dam in rectangular canyon



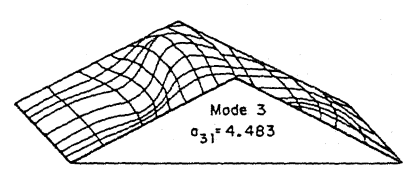


Fig 3. Modal shapes for dam in rectangular canyon, B/H = 1.5, L/H = 3, j = 1

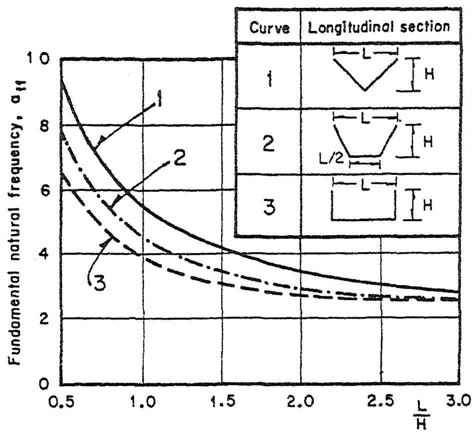


Fig 4. Fundamental natural frequency for dams of various longitudinal cross-sections; B/H = 1.5

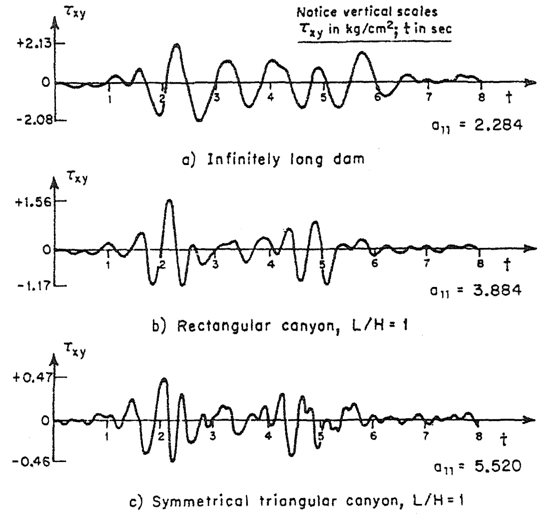


Fig 5. Time history of shear stress at a point on vertical axis of symmetry, 30.5 m high; B/H = 1.5

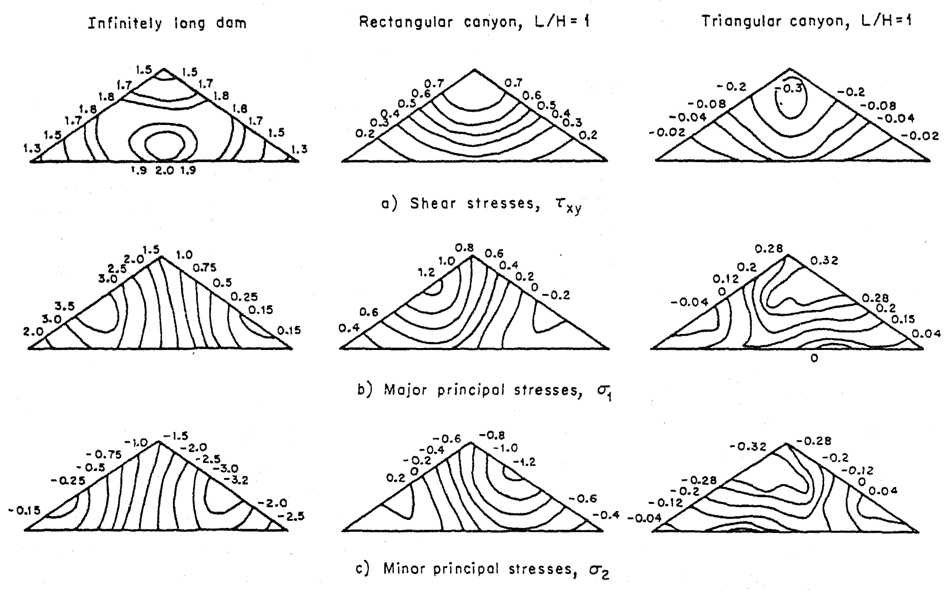


Fig 6. Stress contours in kg/cm² for midplane for a given instant of time, t = 2.26 sec; B/H = 1.5