ON SEISMIC RESPONSE OF PRESSURE VESSELS WITH CUTOUTS AND CRACKS

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ABSTRACT

In this paper, the elasto-dynamic response of a finite circular cylindrical shell representing a pressure vessel containing an elliptical cutout or a crack is investigated. The vessel is subjected to seismic vibrations of the form $q_z = q(x,\phi)\cos(\omega t + \psi)$, where $q(x,\phi)$, and ω are amplitude and frequency of the seismically induced load and ψ is the phase constant. The distributions of stress concentration and stress intensity factors around and away from the cutout region are examined.

INTRODUCTION

Recently there has been a keen interest to analyze the effects of dynamically (and, in particular, seismically) induced vibrations, on pressure vessels and pipes with cutouts and cracks in order to achieve a cost-optimized structural design in accordance with the operational safety requirements. Moreover, the existence of cutouts and cracks due to operational necessity and/or imperfect manufacturing may further complicate and alter considerably the free and forced vibration characteristics as well as the stress field in the structure. For any given material under a specified stress field, there is a crack length of a certain initial value for which the crack length becomes self-propagating under cyclic loading conditions. If this length is ever reached, either by penetration or by the growth of a small crack complete loss of the structure may occur. Seismic motion is but one example of a possibly dangerous loading condition.

In this paper the dynamic forms of Morley's equations are developed. These equations are utilized to investigate the behavior of a finite elastic circular cylindrical shell with cutouts and cracks under the seismically induced dynamic load. The stress concentration factors around and away from an elliptical cutout are illustrated as a function of shell geometric, material and loading parameters, forcing frequency ω, shell curvature parameter βa, Poisson's ratio ν, and minor to major axis ratio $d=b/a$ of the elliptical cutout. The important case of an axial crack as the limiting form of an elliptical cutout is also investigated and stress intensity factors around the crack tip as well as their attenuation away from the crack zone are examined and illustrated.

Governing Equations

The shell coordinates $\bar{X},\bar{Y},\bar{Z}$, associated displacements $\bar{U},\bar{V}$ and $\bar{W}$ are

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shown in Fig. 1a Fig. 1b represents the elliptical coordinate system.

The dynamic form of Morley's equations are developed [1,2] through the equations of motion, kinematics of deformation of a shell element and the stress resultant-displacement relations in the form:

$$\nabla^4 u = \frac{\partial^3 w}{\partial x \partial \phi^2} - \nu \frac{\partial^3 w}{\partial x^3} - \frac{1}{\nu_m} \frac{\partial^2 q_x^*}{\partial \phi^2} - \frac{\partial^2 q_x^*}{\partial x^2} + \frac{\nu_p}{\nu_m} \frac{\partial^2 q_o^*}{\partial x \partial \phi} + G \frac{3-\nu}{2\nu_m} \frac{\partial^2}{\partial t^2} (\nabla^2 u)$$

$$+ \frac{G}{\nu_m} \left(\nu \frac{\partial^3 w}{\partial x \partial t^2} + \frac{\partial^2 q_x^*}{\partial t^2} - G \frac{\partial^4 u}{\partial t^4}\right) \quad (1)$$

$$\nabla^4 v = -(2+\nu) \frac{\partial^3 w}{\partial x^2 \partial \phi} - \frac{\partial^2 q_\phi^*}{\partial \phi^2} - \frac{1}{\nu_m} \frac{\partial^2 q_\phi^*}{\partial x^2} + \frac{\nu_p}{\nu_m} \frac{\partial^2 q_\phi^*}{\partial x^2} +$$

$$G \frac{3-\nu}{2\nu_m} \frac{\partial^2}{\partial t^2} (\nabla^2 v) - \frac{\partial^3 w}{\partial \phi^3} + \frac{G}{\nu_m} \left(\frac{\partial^3 w}{\partial \phi \partial t^2} - G \frac{\partial^4 v}{\partial t^4} + \frac{\partial^2 q_\phi^*}{\partial t^2}\right) \quad (2)$$

$$\nabla^4 (\nabla^2+1)^2 w + 12(1-\nu^2) \left(\frac{R}{h}\right)^2 \frac{\partial^4 w}{\partial x^4} = \frac{G}{\nu_m} (\nabla^2+1)^2 \frac{\partial^2}{\partial t^2} \left(\frac{3-\nu}{2} \nabla^2 w - G \frac{\partial^2 w}{\partial t^2}\right)$$

$$+ \frac{G}{k\nu_m} \frac{\partial^2}{\partial t^2} \left(\frac{3-\nu}{2} \nabla^2 w - \frac{\partial^2 w}{\partial \phi^2} - \nu^2 \frac{\partial^2 w}{\partial x^2} - \nu_m \nabla^4 w\right) - \frac{G^2}{k\nu_m} \frac{\partial^4}{\partial t^4} \left(w + G \frac{\partial^2 w}{\partial t^2} - \frac{3-\nu}{2} \nabla^2 w\right)$$

$$+ \frac{1}{k}\left[\nabla^4 q_z^* + \nu \frac{\partial^3 q_x^*}{\partial x^3} + (2+\nu) \frac{\partial^3 q_\phi^*}{\partial x^2 \partial \phi} + \frac{\partial^3 q_\phi^*}{\partial \phi^3} - \frac{\partial^3 q_x^*}{\partial x \partial \phi^2}\right] - \frac{G}{k\nu_m} \frac{\partial^2}{\partial t^2} \left(\frac{3-\nu}{2} \nabla^2 q_z^*\right.$$

$$\left.+ \nu \frac{\partial q_x^*}{\partial x} + \frac{\partial q_\phi^*}{\partial \phi} - G \frac{\partial^2 q_z^*}{\partial t^2}\right) \quad (3)$$

where $x=\bar{X}/R$, $\phi=\bar{Y}/R$, $z=\bar{Z}/R$, $u=\bar{U}/R$, $v=\bar{V}/R$, $w=\bar{W}/R$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \phi^2}, \quad G = \frac{R^2 \rho h}{D}, \quad D = \frac{Eh}{1-\nu^2}, \quad \nu_m = \frac{1-\nu}{2}, \quad \nu_p = \frac{1+\nu}{2}$$

$$k = \frac{h^2}{12R^2}, \quad (q_x^*, q_\phi^*, q_z^*) = \frac{R}{D} (q_x, q_\phi, q_z)$$

Here E is Young's modulus, ν Poisson's ratio and ρ is the density of the shell. R and h represent the radius and thickness of the cylindrical shell respectively. Furthermore, q_x^*, q_ϕ^*, q_z^* are the components of the mechanical loading in the x, ϕ, and z directions.

The dynamic response of a finite circular cylindrical shell representing a pressure vessel is investigated elastically due to seismic vibrations of the form $q_z = q(x,\phi)\cos(\omega t + \psi)$ [2]. Here $q(x,\phi)$ is the amplitude of a seismically induced load, ω is the forcing frequency in rad/sec and ψ is the phase constant. The method of solution is based on superposition of the following two solutions [3]:

a. The Nominal Solution

The stress field caused by prescribed dynamic surface loads in a shell of finite length with no cutout.

b. The Residual Solution

The stresses in the shell caused by applied edge loads around the cutout. These loads are equal in magnitude but opposite in sign to those present in the shell without cutout at the cutout location.

Solution of the Problem

In accordance with the expression of q_z, the unknown displacements u,v, and w are assumed in the following form:

$$u = U(x,\phi)\ \cos(\omega t + \psi)$$

$$v = V(x,\phi)\ \cos(\omega t + \psi)$$

$$w = W(x,\phi)\ \cos(\ t + \psi)$$

The q_z, u, v, and w expressions are substituted in the uncoupled set of partial differential equations (1)-(3). Thus the nominal solution is obtained to describe the behavior of the finite shell with specified boundary conditions at both ends of the cylindrical shell. Then the residual solution to the homogeneous governing differential equations is obtained in such a way that when it is combined with the corresponding nominal solution the specified boundary conditions around the elliptical cutout are satisfied. A boundary point matching technique in the least squares sense is used to satisfy the boundary conditions along the cutout.

Numerical Results

Figure 2 shows the distribution of the membrane stress concentration and stress intensity factor S_c around one quadrant of the elliptical cutout. The cutout location is indicated by the polar coordinate ϕ for a circular cutout and elliptical coordinate η is employed in the case of the elliptical cutout. For the forcing frequency of 100 Hz, and for the curvature parameter $\beta = \frac{1}{2\sqrt{Rh}}\ [3(1-\nu^2)]^{\frac{1}{4}}$ various minor to major axis ratios d=b/a=1., .8, .6, .2, .08 are illustrated. First, there is a decrease in the stress concentration level when the ratio d attains smaller values from 1 down to .8. There is a notable shift of the curve for .8 relative to the curve for d=1 towards the location $\phi=\eta=0^o$, i.e., to the tip of the elliptical cutout. On the other hand, a sharp increase in the magnitudes are followed by a more significant shift towards the tip of the elliptical cutout is observed for the ratios d=.6, .2 and d=.08 (penny-shaped crack). This is accompanied by increased values of the curvatures at the maximas of the curves. Observing this trend of increasing stress values towards the crack tip $\phi=\eta=0^o$, we may extrapolate an approaching stress singularity for the limiting zero value of the ratio d, as dictated by the elastic solutions. There is also an overall stress relief observed in the region $\phi=\eta=60°$ to $90°$ when the ratio d decreases. This may be explained physically, since the curvatures of the cutout curve in this zone become larger as the ratio d decreases, approaching a line crack along the x axis.

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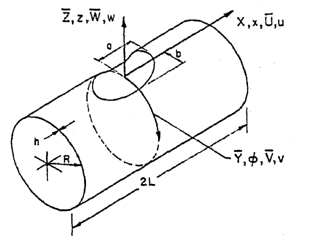

Figure 1a Geometry of the Shell with an Elliptical Cutout

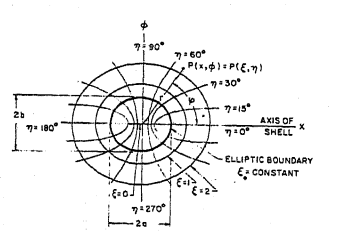

Figure 1b Elliptical Coordinate System

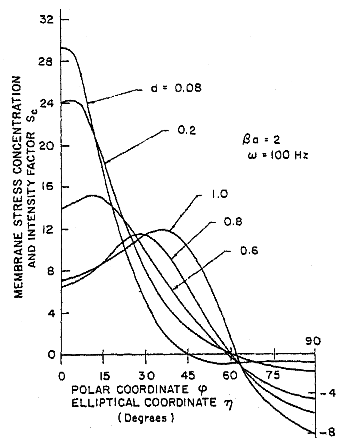

Figure 2 Effect of the minor to major axis ratio d on the Distribution of Membrane Stress Concentration and Intensity Factor S_c along φ or η axis around one quadrant of the Elliptical Cutout