DYNAMIC BEHAVIOR OF OFF-SHORE STORAGE TANKS

by

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SUMMARY

The dynamic behavior of off-shore oil storage tanks resulting from the sea wave action and earthquakes depends on the sloshing of the contained liquids. In order to maintain the static stability of the system a layer of sea water introduced, thus inside each tank there are two liquids (oil and water). The horizontal reservoir displacement involves forced internal oscillations, analogous to a discret lumped mass system.

Knowledge of the potential flows of two liquid layers leads straight to the eigenvalues, the pressures on the walls, total force on the tank as well as the dynamic elements of the equivalent lumped mass system.

The theoretical results are compared with those obtained with the laboratory rectangular tank.

INTRODUCTION

The off-shore storage tank are subjected simultaneusly to wave and earthquake dynamic action.

The coupling effect between the structure, the soil and the sea has been studied, and the analytical formulation for the exterior hydrodynamic force on cylindrical structures is now known (3).

Also, for a tank containing one liquid, the sloshing phenomena is understood and, for a reservoir of regular geometry, the practical lumped mass formulas are known (1,4,5).

In order to maintain static stability, a tank is often ballasted with a sea water. This modifies the sloshing phenomena and therefore also the lumped mass values.

ANALYTICAL MODEL

Let us consider two liquids filling a horizontally moving tank (fig.1). The internal fluid flow, for both layers (i = 1,2) is described by the potential (ψ) (2), which is decomposed into two parts ; the first one is due to the gravitational field (ϕ) and the second one to the horizontal tank displacement (φ) :

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$$\Psi(P,t) = \phi(P,t) + \varphi(P,t)$$

$$\Delta \phi_i (P,t) = 0 \qquad \Delta \varphi_i (P,t) = 0 \qquad (1)$$

$$\frac{\partial \phi_i}{\partial n} = 0 \qquad \frac{\partial \varphi_i}{\partial n} = \vec{V}.\vec{n} \qquad P \in \Sigma_V \quad (2)$$

$$\frac{\partial \phi_i}{\partial n} = 0 \qquad \frac{\partial \varphi_i}{\partial n} = 0 \qquad P \in \Sigma_H \quad (3)$$

$$\frac{p}{\rho_i} = -\frac{\partial \Psi}{\partial t} - gz \qquad P \in S \quad (4)$$

$$V_{z,1} = V_{z,2} \qquad P \in S \quad (5)$$

The elliptic problem we are dealing with (Eqs.1-5) represents the internal flow (Ψ) ; the conditions (Eq.2) show that the vertical walls are moving the fluid particles horizontally, (Eq.3) is the bottom opacity, (Eq.4) is the pressure (dynamic condition) on the separation surface, and (Eq.5) is the vertical velocity continuity (V_Z Kinematic condition).

For the rectangular tank the modal oscillation potentials ($\phi_1^{(n)}$), with frequency σ_n, are expressed by the solutions of the system (Eqs.1,2,3,5) as follows :

$$\phi_1^{(n)}(P,t) = \frac{a_\circ \sigma_n}{k} \frac{ch\, k(z+h)}{sh\, kh} . \cos kx . \cos \sigma_n t$$

$$\phi_2^{(n)}(P,t) = \frac{a_\circ \sigma_n}{k} \frac{ch\, k(z-h')}{sh\, kh'} . \cos kx . \cos \sigma_n t \qquad (6)$$

$$k = n.\pi a^{-1}$$

By using the pressure relation (Eq.4) the rectangular tank frequency can be written explicitely :

$$\sigma_n^2 = g.k \frac{\rho - \rho'}{\rho\, cth\, kh + \rho'\, cth\, kh'} \qquad (7)$$

Horizontal tank displacement, with the velocity V (t), defines the second flow potential :

$$\varphi_i (P,t) = V(t)\, x \qquad i = 1,2 \qquad (8)$$

also the total potential (Ψ), in both layers, could be expressed by the functions :

$$\Psi_i (P,t) = \sum_{n=1}^{\infty} \dot{f}_n(t) . B(kz) \cos kx + V(t)\, x \qquad (9)$$

Time dependent functions $f_n(t)$ are described by the dynamic pressure condition on the separation surface (Eq.4) rewritten in the form of the forced vibrations equation :

$$\ddot{f}_n(t) + \sigma_n^2 f_n(t) = -\dot{V}(t) . \mathcal{F}\, \mathcal{D} \qquad (10)$$

$$\mathcal{F} = (x, \cos kx) \qquad \mathcal{D} = \sigma_n^2 g^{-1} k^{-1} \qquad n = 1,3,5 ..$$

Assuming that the function $f_n(t)$ and the total potential flow (Ψ, Eq.9) are known, the pressures on the walls and the total force are obtained through the classical formulae :

$$p_i(P,t) = -\rho^{(i)} \frac{\partial \Psi_i}{\partial t} \qquad \vec{F}(t) = \int_{\Sigma_V \cup \Sigma_H} p \, \vec{n} \, ds \qquad (11)$$

As an example, the total force acting on the rectangular tank moving horizontaly, was calculated :

$$F(t) = \dot{V}(t) M \left(1 + \frac{1}{M} \sum_{n=1,3..}^{\infty} \frac{8 a^3 b}{\pi^4 n^4} \cdot \frac{\sigma_n^2}{g} [\rho - \rho'] \frac{-\omega^2}{\sigma_n^2 - \omega^2} \right) \quad (12)$$

where

$\dot{V}(t)$ is the horizontal ground acceleration, with ω the frequency

$M = ab(\rho h + \rho' h')$ is the total contained liquid mass.

Considering a discret lumped mass oscillating system (Fig.2) analogous to liquid forces, the mobile mass (m_n), the fixed mass (m) and the spring (Kn) expressions are as follows :

$$\frac{m_n}{M} = \frac{8}{\pi^3 n^3 r} \cdot \frac{(\rho - \rho')^2}{(\alpha\rho + \beta\rho')(\rho \, \mathrm{cth}\, kh + \rho' \, \mathrm{cth}\, kh')} \qquad (13)$$

$$m = M - m_n \qquad 2K_n = \sigma_n^2 \, m_n$$

$$r = H a^{-1} \quad \alpha = h H^{-1} \quad \beta = h' H^{-1} \quad M = ab(\rho h + \rho' h')$$

RESULTS

The following practical results were compared with laboratory experimental values obtained for a rectangular storage tank (ρ=1.49, ρ'= 1.0 a=0.6, H=0.75, b=0.35m).

The first step in the analysis shows that, in general, the period of one liquid is lower than the coupled system of two liquids (Fig.3).

It is observed that the theoretical pressure and period expressions were in good agreement with the experimental tests (Fig.3 and 4). The lumped masses (m_n) are lower than those obtained for a single liquid thank (4,5). In fact this effect is due to the phase lag existing between the upper and the lower liquid forces.

Finally it should be noted that the resonant characteristics of the system (σ_n, m_n) are favourable, when the storage tank is completed filled with two liquids.

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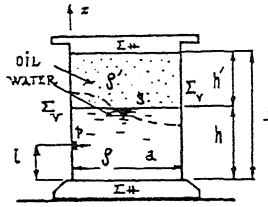

FIG. 1

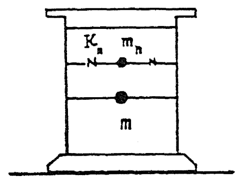

FIG. 2

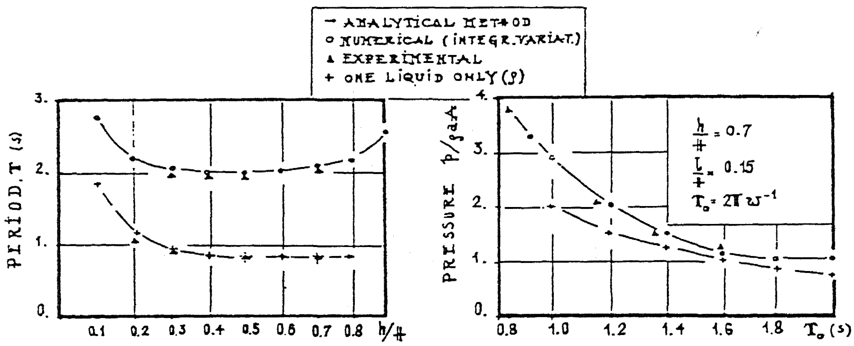

FIG.3

FIG.4

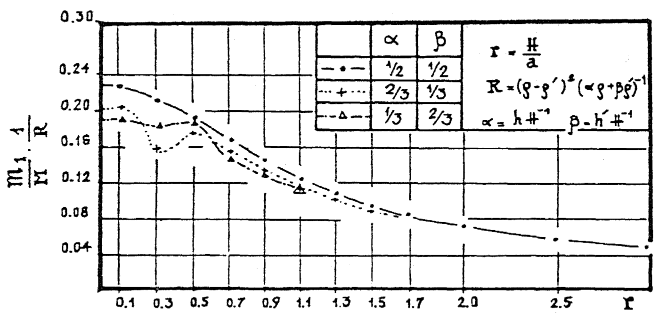

FIG.5